Bayesian chronological modelling

We have used the Bayesian chronological modelling program OxCal v4 (Bronk Ramsey 2009) to interpret $^{14}$C results, and to estimate the date of initial settlement, at five sites in Venice. Here, we provide the essential details to allow these estimates to be reproduced or revised. Two models are provided below. Model 1 (shown in Figure 3) is a wiggle-match of the Eastern Veneto oak chronology (Martinelli & Kromer 2002) to the IntCal13 calibration curve (Reimer et al. 2013), which was not available in 2002. Model 2 contains the five site sequences, which are fully independent of each other, except that Ca’ Foscari, Teatro Malibran and ex-Cinema San Marco are linked by dendrochronological cross-matching. Part of the Model 2 output is shown in Figure 4.

Bayesian models combine independent information about the chronology of individual samples and events – such as calibrated $^{14}$C dates, sapwood estimates, calendar date ranges for coins etc. (known as likelihoods) – with observations or suppositions about the relative dates of these samples and events (prior information), which may be based on stratigraphic sequences, dendrochronological cross-matching, or even a basic assumption that their dates are not independent of each other.

Several programs are available which use computationally intensive algorithms to repeatedly sample all the likelihoods in a model, and reject the resulting dates if they are incompatible with the prior information provided. This process therefore retains only the potential dates of samples and associated events which are consistent with the relative dating constraints imposed by the prior information, creating probability distributions known as posterior density estimates of the dates of these samples and events. Such estimates are not
independent of each other, and will change if any of the likelihoods or priors changes, but Bayesian modelling is a robust and reproducible method for estimating the real dates of individual samples, and of significant events related to those samples.

**Likelihoods: \(^{14}\text{C} \) data and dendrochronological cross-matching**

The \(^{14}\text{C} \) ages and dendrochronological cross-matching positions at Ca’ Foscari, Teatro Malibran and ex-Cinema San Marco are identical to those given in Meadows *et al.* (2012), except that the felling dates of dendrochronologically cross-matched timbers are expressed as offsets relative to the posterior estimate generated by Model 1 for “year 0” in the Eastern Veneto chronology, rather than relative to the last surviving annual ring of Ca’ Foscari US1879 (year 199); the 2012 model also used the previous calibration curve. A wood-age offset has been applied to the unidentified wood sample from coring at Ca’ Foscari in 2002.

All the other samples from these sites are from area excavations.

The Piazza San Marco (St Mark’s Square) model is based on \(^{14}\text{C} \) ages of samples obtained by coring, reported by Martinelli (2003) and Ammerman *et al.* (2017; Table S1), which incorrectly gives the \(^{14}\text{C} \) age of OxA-4521 as 1400±60 BP; the correct result, 1440±60 BP (Hedges *et al.* 1995), was given in earlier publications (e.g. Ammerman *et al.*, 1995). The San Lorenzo di Castello \(^{14}\text{C} \) results (Ammerman *et al.* 1992; Hedges *et al.* 1992; Bronk Ramsey *et al.* 2002) are from samples obtained by a combination of area excavation and coring. There is no relevant dendrochronological cross-matching at Piazza San Marco or San Lorenzo di Castello.

**Wood-age offsets**

Most of the \(^{14}\text{C} \) results from these five sites are on samples of wood, which can be misleadingly old if the sample consists of tree-rings formed long before the tree-fall or felling date. If no correction is made for wood-age offsets, model estimates of the initial dates of settlement will be too early. Almost all samples at Ca’ Foscari, Teatro Malibran and ex-Cinema San Marco were selected by a dendrochronologist; the number of tree rings in each sample, and the interval between the midpoint of the sample and the felling date of the timber, is usually known exactly. These offsets can be coded in OxCal; e.g. R_Date("Hd-24673", 1418, 17)+3; generates a probability density function for the felling date, by shifting the probability density function of the calibrated date exactly 3 years later (in this case, the sample was complete to the bark edge and contained 6 annual rings). If the wood-age offset is not known exactly, a likelihood can be specified for the number of missing annual rings.
between the midpoint of the sample and its felling date. In the special case of mature oak-wood retaining the heartwood-sapwood transition, a sapwood estimate (Corona, 1974) can be applied (e.g. \( +U(11,40) \) would shift the date by a uniform probability distribution of 11–40 years).

Martinelli (2003) reported two series of \(^{14}\text{C}\) results on wood from coring at St Mark’s Basilica: seven samples dated by AMS (OxA-) and ten radiometric dates from Heidelberg (Hd-). The AMS samples were not identified before submission and although it can be assumed that only the outermost surviving rings were dated, appropriate wood-age offsets may be quite variable, as they depend on the number of missing outer rings. The Heidelberg samples were identified, and the number of annual rings present was estimated. We assume that the entire sample was processed in these cases. This means that the average intrinsic age of the radiometric samples may be larger but less uncertain, as part of the offset (half the number of annual rings present) is known. All the Ammerman et al. (2017) samples were dated by AMS. Some were short-lived (the peach stones, a bone, a reed), while others were identified as elm or oak wood, with no information about maturity.

As most of the identified samples from coring were of oak (Quercus sect. ROBUR) (or less commonly elm, Ulmus sp., or ash, Fraxinus sp., which are similarly long-lived), we assume that most of the unidentified wood was also oak. Typically we have applied offsets of 20–60 years to such samples, unless the timber was described as a pole (i.e. shorter-lived) or very slow-grown (longer-lived). These species can live much longer, as shown by the dendrochronological cross-matching, but unless the median intrinsic age of such samples is assumed to be more than e.g. 100 years, our model favours a seventh-century date for the start of settlement, both at Piazza San Marco and at San Lorenzo di Castello. If the estimated wood-age offsets are less precise (e.g. 10–70 rather than 20–60 years), the uncertainty in the model output increases. However, even if unrealistically small and precise wood-age offsets (e.g. 10–20 years) are applied to wood samples from coring, it is unlikely, based on our model, that settlement at either location began much before the mid-seventh century.

**Prior information**

The prior information in the Ca’ Foscari, Teatro Malibran and ex-Cinema San Marco models is identical to that in Meadows et al. (2012). The Ca’ Foscari model incorporates the stratigraphic sequence (the Harris matrix), whereas the ex-Cinema San Marco results are placed in simple bounded-phase model, as the relevant stratigraphic relationships are
unpublished. For Teatro Malibran, only the dendrochronologically cross-matched timbers are included, and the only prior information is therefore their cross-matching positions.

Our model for Piazza San Marco assumes that there was a pre-Basilica settlement phase, which ended when construction of the first Basilica began in AD 828 (when, according to tradition, the relics of St Mark were brought to Venice). The Ammerman et al. samples are placed in the pre-Basilica phase, while all but four of the Martinelli samples are placed after the start of the Basilica; their dates are consistent with the original interpretation that they came from structural timbers predating the current, eleventh-century, Basilica. The four oldest Martinelli samples are potentially much earlier, and may not be anthropogenic at all, although with even modest wood-age offsets, two or three could have been felled at the same time as many of the Ammerman et al. samples. In our model, we have effectively excluded the four oldest samples, by treating them only as termini post quos for the Basilica, because otherwise the estimated start of settlement would be overly sensitive to which of these samples was included in the pre-Basilica phase, and what wood-age offsets were applied. Our model gives an unrealistically precise estimated date for the start of settlement at Piazza San Marco, because it incorporates as prior information the nominal sequences of samples within cores, when 2–3 samples have been dated per core. In each case, it is possible that the stratigraphically earlier sample predates the stratigraphically later sample, but the reverse sequence is usually also possible, and all samples may be redeposited. Removing this prior information only increases the uncertainty in the estimated start date by 5–10 years, however.

It was possible to create a less subjective model for the San Lorenzo di Castello sequence, using information provided at the time of 14C sample submission, and subsequently published in Archaeometry datelists (Hedges et al. 1992; Bronk Ramsey et al. 2002). The 1992 list is actually headed San Lorenzo di Ammiana, correctly described as being near Torcello in the northern Lagoon, but incorrectly located at 45°12’N 12°21’E (in the Adriatic, east of Chioggia). The description of the church by ‘AJA’ is consistent with San Lorenzo di Castello, however, and with the publication of the same dates in Ammerman et al. (1992), where they are clearly attributed to Castello. The second datelist (Bronk Ramsey et al. 2002), which repeats the incorrect coordinates, attributes both sets of samples to San Lorenzo di Castello.

Our model assumes that a reed, OxA-3338 (‘from a natural horizon in the lagoonal (barena) sediments at a depth of about one metre below the mortar fragments’), predates all the anthropogenic samples, while the first settlement phase includes OxA-3339 (‘a reed structure observed at the base of the trial sounding’), mortar encountered by coring (‘OxA-3334 and -3335 bracket a level with mortar fragments in core 13. OxA-3336 to -3337 do the same for a
similar level in core 13’), and what may be volparoni (‘a dense layer of twigs and canes…
[which] represents a phase of land reclamation on the site’s west side at a time prior to the
construction of the first church’ [OxA-7012, 8199] and ‘a small wooden stake driven into a
canal bank’ [OxA-6769]). There are also dates on three structural timbers and three burials
associated with the church itself, believed to originate in the early ninth century. In the model
code, it is assumed that the burials took place after the church was built, and before the latest
structural timber, timber 8, was felled. Although no palaeodietary information is given, it is
unlikely that these individuals had fully terrestrial diets if they lived in Venice; their 
$^{14}$C ages
are therefore calibrated assuming that 25±10% of protein consumed was from marine species.
However wrong this estimate may be, the burial dates do not affect the chronology of the
initial settlement phase; in fact, our estimated date of the start of settlement at San Lorenzo
would not be affected if the church phase was omitted entirely.

Model 1. Wiggle-match dating of the Eastern Veneto floating chronology and some
cross-matched timbers (Figure 3).
The following OxCal CQL code (equal-area font) will run in OxCal v4 (Bronk Ramsey
2009).
Options()
{
    Resolution=1;
};
Plot()
{
    //details in Martinelli and Kromer (2002)
    D_Sequence("Eastern Veneto oak chronology wiggle-match")
    {
        Date("year 0");
        Gap(51);
        R_Date("year 51: Hd-21355", 1588, 16);
        Gap(73);
        R_Date("year 124: Hd-20714", 1465, 24);
        Gap(74);
        R_Date("year 198: Hd-20713", 1439, 23);
        Gap(52);
        Date("year 250");
}
Line( );
//cross-matching positions shown in Figure 2
//further details in Meadows et al. (2012)
//post US1282 ends in year 201 at the heartwood-sapwood transition, and is estimated to be missing 11-40 sapwood rings, giving a felling date range of years 202-231
MCMC_Sample("felling date estimates for cross-matched timber structures")
{
  Label("Teatro Malibran, Structure E, year 212");
  Date("=year 0")+212;
  Label("Ca' Foscari, plank US177, year 226");  Date("=year 0")+226;
  Label("ex-Cinema San Marco, post US1282, year 202-231");
  Date("=year 0")+U(202,231);
};

The posterior estimate for "year 0" generated by this model should be saved as year_0.prior in the same folder as Model 2, in order for Model 2 to run properly.

**Model 2. Site chronologies for five locations (Figure 4).**

Plot()
{
  Prior("year 0", "year_0.prior");
  Phase("five early settlements")
  {
    Sequence("Ca' Foscari")//model incorporates stratigraphic sequence of structures
    {
      Boundary("Ca' Foscari start settlement");
      Phase("Ca' Foscari samples")
      {
        //additional code
      }
    }
  }
}
R_Date("2002 sample", 1460, 50)+U(20,60);//unidentified
wood from coring below limit of excavation
Sequence("2004 excavation")
{
    Phase("Wattle US 1660")
    {
        R_Date("Hd-24673", 1418, 17)+3;
    }
    Phase("Phase 2 onwards")
    {
        Sequence("US 1940 branch")
        {
            Phase("Wattle US 1940")
            {
                R_Date("Hd-24703", 1341, 17)+4;
            }
            Phase("Wattle US 1814")
            {
                R_Date("Hd-24684", 1321, 19)+2;
            }
            Phase("US 1757, 1871")
            {
                Phase("Wattle US 1757A")
                {
                    R_Date("Hd-24704", 1270, 17)+3;
                }
                Phase("Wattle US 1871")
                {
                    R_Date("Hd-24706", 1297, 18)+2;
                }
            }
        }
    }
    Sequence("US 1879 and later structures")
}
// oak heartwood, final ring is year 199; at least 11 sapwood rings missing
After("Plank US 1879")
{
    Date("=year 0")+210;
}
// elm complete to bark-edge, final ring is year 226
Phase("Plank US 177")
{
    Date("=year 0")+226;
}
Phase("US 1882, 2121")
{
    Phase("Wattle US 2121")
    {
        R_Date("Hd-24683", 1293, 21)+3;
    }
    Sequence("US 1882 onwards")
    {
        Phase("Wattle US 1882")
        {
            R_Date("Hd-24688", 1283, 19)+3;
        }
        Phase("Wattle US 2000")
        {
            R_Date("Hd-24687", 1278, 21)+2;
        }
    }
}
Boundary("Ca' Foscari end phase 3");
Phase ("Teatro Malibran")
{
    //multiple oak timbers complete to bark-edge, final ring is year 212
    Phase ("structure E")
    {
        Date ("=year 0") + 212;
    }
    //elm, no bark-edge, no sapwood estimate, last cross-matched ring is year 227
    After ("structure C")
    {
        Date ("=year 0") + 227;
    }
    First ("Teatro Malibran Structure E");
};
Line();
Sequence ("ex-Cinema San Marco") //simple bounded-phase model, as detailed stratigraphic sequence is not yet available
{
    Boundary ("ex-Cinema San Marco start settlement");
    Phase ("ex-Cinema San Marco samples")
    {
        //oak, heartwood only, final ring is year 134, minimum 11 sapwood rings missing
        After ("US 1317")
        {
            Date ("=year 0") + 145;
        }
        //oak, complete to heartwood/sapwood boundary at year 191, sapwood estimate 11-40 rings
        Phase ("US 1282 post")
        {
            Date ("=year 0") + U(202, 231);
};
//complete to bark-edge in year 241
Phase("US 1211 post")
{
  Date("=year 0")+241;
};
//one timber is complete to bark-edge at year 236
Phase("US 1212/29")
{
  Date("=year 0")+236;
  R_Date("Hd-22405", 1281, 19);
};
Phase("US 1223/8")
{
  R_Date("Hd-22399", 1267, 15);
};
Phase("US 1275/3")
{
  R_Date("Hd-22073", 1302, 12);
};
Phase("EUS 1330")
{
  R_Date("Hd-21983", 1305, 20);
};
};
Boundary("ex-Cinema San Marco end");
};
Line( );
Sequence("Piazza San Marco")
{
  Boundary("Piazza San Marco start settlement");
  Phase("Piazza San Marco ")
  {
    After("very early Basilica dates")//from Martinelli
    (2003)
Boundary("start building Basilica", 828);
Phase("structural timbers")
{
R_Date("Hd-21469 CF 1 oak wood", 1108, 17)+U(20,40);
R_Date("Hd-22117 CF 32 oak wood", 1085, 14)+U(30,60);
R_Date("Hd-21893 CF 61 oak wood", 1070, 23)+U(20,40);
R_Date("Hd-21595 CF 43 oak wood", 956, 27)+U(25,50);
R_Date("Hd-21459 CF 38 alder wood", 905, 19)+U(10,30);
R_Date("Hd-21454 CF 34 oak wood", 1176, 17)+U(25,50);
R_Date("OxA-10076 CF 42 wood", 1045, 34)+U(20,60);
R_Date("Hd-22116 CF 23 oak wood", 1036, 14)+U(25,50);
R_Date("OxA-9342 CF 45 wood", 1128, 38)+U(20,60);
R_Date("Hd-22120 CF 33 oak wood", 1126, 13)+U(20,40);
R_Date("Hd-21467 CF 41 oak wood", 1104, 19)+U(20,40);
R_Date("Hd-22112 CF 22 oak wood", 1097, 16)+U(20,40);
R_Date("OxA-10077 CF 19 wood", 965, 36)+U(20,60);
};
};
Line( );
Sequence("San Lorenzo di Castello")
{
R_Date("OxA-3338 6/9-16,-3.20m reed", 1340, 70);
Boundary("San Lorenzo di Castello start settlement");
Phase("before church")
{
R_Date("OxA-3339 17 reed structure", 1380, 55);
Sequence("core 13")
{
R_Date("OxA-3335 3/13-9b, -2.79m wood", 1415, 65)+U(20, 60);
    Date("mortar in core 13");
    R_Date("OxA-3334 2/13-7, -2.51m wood", 1440, 65)+U(20, 60);
};
Sequence("core 9")
{
    R_Date("OxA-3337 5/9-13, -2.39m wood", 1390, 70)+U(20, 60);
    Date("mortar in core 9");
    R_Date("OxA-3336 4/9-10, -2.12m wood", 1350, 70)+U(20, 60);
};
R_Date("US1764 twigs (OxA-7012, 8199)", 1283, 33);
R_Date("OxA-6769 No.10 wood", 1220, 50)+U(5, 20);
};
Boundary("end early settlement");
Combine("San Lorenzo first church")
{
    R_Date("OxA-3333 1/Nr.13, -1.28m wood", 1155, 65)+U(10, 20);
    R_Date("OxA-6768 No.9 wood", 1295, 55)+U(20, 60);
};
Phase("burials")
{
    Curve("IntCal13","IntCal13.14c");
    Curve("Marine13","Marine13.14c");
    Mix_Curve("Mixed","IntCal13","Marine13",25,10);
    R_Date("OxA-6747 No.12 bone", 1210, 50);
    R_Date("OxA-6661 No.13 bone", 1135, 45);
    R_Date("OxA-6557 No.11 bone", 1115, 45);
};
R_Combine("timber 8 OxA-7011, OxA-8198, Hd-17754") //the 3
14 ages from this structural timber are combined before
calibration
{
    R_Date("OxA-8198 No.8 wood", 1075, 35);
    R_Date("OxA-7011 No.8 wood", 1020, 70);
    R_Date("Hd-17754 outer rings", 1079, 26);
};
Boundary("rebuild church");
};
Page(); //Figure 4
Phase("estimated start of settlement at different sites")
{
    Date("=Ca’ Foscari start settlement");
    Date("=Teatro Malibran Structure E");
    Date("=ex-Cinema San Marco start settlement");
    Date("=Piazza San Marco start settlement");
    Date("=San Lorenzo di Castello start settlement");
};

References


