Supplemental Appendix for “Political Corruption Traps”

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A1 Persistence in Corruption Measures

While it is difficult to measure persistence with existing data, Figure A1 shows that after controlling for common covariates, the correlation between a country’s initial and latest score on the well-known perceptual indices – spanning between 20 and 30 years – is positive, statistically significant, and quite strong.

Figure A1: Persistence in corruption perception indices

Each panel is a partial residual plot showing the correlation between the initial and the latest value of the indicated corruption perception index (the Transparency International’s Corruption Perception Index, the World Bank’s Control of Corruption Governance Indicator, and the International Country Risk Guide’s (ICRG) corruption index, from left to right respectively), after controlling for the common correlates of corruption. The dashed line is the 45-degree line.

A2 Background for Key Model Assumptions

A2.1 The Electoral Environment

Our main model assumes a country with N geographic units, each containing a representative voter, an incumbent politician, and a challenger. This environment could correspond

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1The correlates, taken from Treisman (2007), are: GDP (PPP), geographical size (logged area), British, French and other colonial legacies, British, French, Scandinavian, and German legal legacies, the share of Protestants, Catholics and Muslims, ethnolinguistic fractionalization, length of democracy, world region, and whether a country is a federal state.

2Pritchett, Woolcock, and Andrews (2010) argue that even if today’s countries with the weakest governance were to improve in line with the past experience of the 20 fastest improving countries, getting to the level of a median country such as Tanzania or Guatemala would take between 15 and 35 years.
to an electoral system where legislators are elected by plurality vote in single-member constituencies. In some countries, such legislators are vehicles for corruption. For example, while corruption appears to increase in magnitude up the political hierarchy in India, even state legislators on average extract substantial rents, as judged by their absolute and relative wealth accumulation (Chauchard, Klašnja, and Harish, 2016; Fisman, Schulz, and Vig, 2014).

However, legislators are often not the main corrupt actors. They are frequently not given a large amount of resources to work with, particularly in proportional representation systems, where backbenchers are often little more than party foot soldiers (e.g. Golden and Picci, 2015). In such circumstances, our model better describes the choice of directly elected local executives, such as governors or mayors. Directly elected local executives are quite common, and there is a wealth of evidence of their involvement in corruption. Examples abound. Ferraz and Finan (2011) estimate more than $550 million in corruption from federal transfers (about 15% of all transfers) in Brazilian municipalities in 2004 alone. Coviello and Gagliarducci (2014) document widespread manipulation of public procurement in Italian municipalities, especially by longer-tenured mayors. In Indonesia, prominent local politicians running on anti-corruption platforms have been highlighting the entrenched local corruption, such as the former mayor of Soko and governor of Jakarta (and now president) Joko Widodo, Bu Risma in Surabaya, or Ridwan Kamil in Bandung. The Romanian integrity agency and its anti-corruption directorate initiated more than 1,000 corruption cases against county and local politicians between 2008 and 2013 (Klašnja, 2015). Cases of corrupt local governments abound in China, such as in Shenyang, the capital of Liaoning Province, where almost an entire local administration was engaged in malfeasance, from the mayor, deputy mayor, the chiefs of the tax, price control, land resource, construction, and state assets bureaus, to the head of the city’s prosecutor’s office (Sun, 2004, p. 127). More than two dozen municipalities in Hungary were recently involved in suspicious tenders for public lighting renovation, awarded uniformly to a firm belonging to the Prime Minister’s son-in-law.  

A2.2 Voter Utility

A key but debateable assumption of the model is that voters can sometimes obtain a share of the benefit that politicians derive from corruption, which is the case when $b \in (0, 1)$.

To be clear, however, our modeling approach does not assume that $b$ is positive in all contexts, only that it is reasonable to expect that $b$ could be either positive or negative depending on the circumstances. The possibility of a positive $b$ is consistent with a widespread view that when corruption is prevalent, corrupt politicians may be better able to achieve results valued by their constituents than honest politicians. For example, in Russia in the early 1990s, getting a business started often required bribing an entire chain of administrators and

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4 Research by Transparency International indicated that in Poland and Ukraine, out of a 5% kickback, about 4.5% ended up in private accounts, while the rest was likely channeled for reelection purposes (Transparency International, 2004, p.23).
authorities, from the local legislature, to the local executive branch, the central ministry, the fire authorities, the water authorities, and so on (Shleifer and Vishny, 1993). It is easy to imagine that a corrupt politician could be better able to navigate such an environment than an honest one. In a generally corrupt system of private infrastructure construction through city-granted franchise in the U.S. at the end of the 19th Century, streetcar lines were built more quickly and covered more miles in those U.S. cities where private streetcar companies were able to speculate in real estate along the streetcar lines (such as the dealings of Henry Huntington in Los Angeles, and Francis Newland in Maryland; Jackson, 1985).

Note that this result would only be reinforced if the returns to non-corrupt activities are diminishing as more politicians are corrupt and allocate their effort to corrupt behavior. That is, in our formulation the payoff to having a clean politician is constant, and the results in the text highlight the case when the citizen payoff to having a corrupt representative is higher than this constant (and potentially increasing in the level of corruption). As discussed above, it is plausible that the returns to non-corrupt activity are lower in a very corrupt environment, also raising the relative value of having a corrupt representative if others are corrupt as well.

It is also plausible that when corruption is rare citizens benefit little from corruption, but when corruption is more pervasive and open citizens benefit more from it. This need not even mean that citizens prefer corrupt politicians to non-corrupt politicians in general, but simply that they are more likely to view corruption as less of a concern vis-à-vis other factors affecting the vote decision when corruption is more prevalent (e.g. Klašnja and Tucker, 2013).

Sayings in several languages echo this notion: “el que no tranza, no avanza,” Spanish for “he who does not cheat, does not get ahead;” “soldi e amicizia vincono la giustizia” Italian, loosely meaning that money and friendship beat justice, and “Не подмажешь не поедешь,” Russian, loosely meaning “no grease, no go” (all quoted in Simpser 2013, p. 3).

For example, high corruption may also reinforce the demand for clientelism. de la O (2015) demonstrates that when corruption is high, voters in Mexico are more likely to believe that vote selling is prevalent in their communities and more willing to sell their own votes.

While we assume that a $c_i = 1$ type politician is predisposed to corruption, in reality a corrupt politician need not necessarily be the one that personally profits from bribes, but one who is willing to work within the confines of a corrupt system. For example, Mayor Daley of the infamous Chicago machine, President Harry Truman, a loyal member of the notoriously corrupt Pendergast machine in Kansas City, Manmohan Singh, the 14th Prime Minister of India during such high-profile corruption scandals as the 2G spectrum case, or Dilma Rousseff, the current president of Brazil and the former board chairman of Petrobras before the eruption of the ongoing Petrobras scandal, have all been thought to be personally honest.
A3 Proofs of Results in the Text

A3.1 Proof of Proposition 1

For part (i), implicitly differentiating the equilibrium condition gives:

\[
\frac{\partial x^*}{\partial \rho} = -\frac{x^* g_{12}(x^*, \rho x^*)}{g_{11}(x^*, \rho x^*) + \rho g_{12}(x^*, \rho x^*)} = -\frac{x^* g_{12}(x^*, \rho x^*)}{g_{11}(x^*, \rho x^*) + \rho g_{12}(x^*, \rho x^*)}
\]

The numerator is negative since \( g_{12} > 0 \) and the denominator is negative by the assumption that the diminishing returns to corruption outweigh the strategic complementarities, so the expression is positive. For part (ii), corrupt politicians must get a higher payoff since they could choose \( x_i = 0 \) and get the same payoff as a non-corrupt politician, but strictly prefer a higher \( x_i \) and hence must get a higher payoff. The payoff for a non-corrupt politician is constant in \( \rho \), and the envelope theorem combined with the fact that \( g_2 > 0 \) implies the equilibrium payoff for a corrupt politician is increasing in \( \rho \).

A3.2 Proof of Proposition 2

Expanding the expression for \( \frac{d}{d\rho} [dV(\rho)] \):

\[
\frac{d}{d\rho} [dV(\rho)] = -\frac{\partial x^*}{\partial \rho} + b \frac{d}{d\rho} [g(x^*, \rho x^*)] = -\frac{\partial x^*}{\partial \rho} + b \left( g_1(x^*, \rho x^*) \frac{\partial x^*}{\partial \rho} + \left[ \rho \frac{\partial x^*}{\partial \rho} + x^* \right] g_2(x^*, \rho x^*) \right).
\]

Rearranging and substituting \( g_1(x^*, \rho x^*) = 1 \), \( dV(\rho) \) is increasing if \( b > \tilde{b} \), where

\[
\tilde{b} = \frac{\frac{\partial x^*}{\partial \rho}}{\frac{\partial x^*}{\partial \rho} + \left[ \rho \frac{\partial x^*}{\partial \rho} + x^* \right] g_2(x^*, \rho x^*)}.
\]

All of the terms in the numerator and denominator are positive, and the denominator is strictly higher than the numerator, so this threshold is strictly between zero and one.

A3.3 Proof of Proposition 3

The behavior in the politician stage in any PBE is demonstrated in the main text, so showing the existence of an equilibrium requires that there are sequentially rational citizen strategies given the politician stage. For any fixed distribution of incumbent types (which are common knowledge among the voters), the voters are playing the equivalent of a simultaneous move game with binary actions. The payoff to each voter can be written:

\[
\mathbb{E}[u_i^Y(\sigma)] = \begin{cases} 
  a_i + c_i \mathbb{E}_\rho[dV(\rho)] & \text{if keeping} \\
  \mathbb{E}[a] + q \mathbb{E}_\rho[dV(\rho)] & \text{if replacing.}
\end{cases}
\]
The strategy used by others affects the voters’ payoff by changing the distribution of $\rho$, which is equal to $1/(N - 1)$ times the number of other corrupt politicians who are kept plus a binomial random variable with a “trial” for every replaced politician and success rate $q$.

So in an $N$ player simultaneous move game where voters keep or replace their politician and have payoffs given by equation 1, for each revelation of types there exists a mixed strategy profile which is a Nash Equilibrium (Nash, 1950). Since these payoffs are equivalent to the expected payoffs in the full extensive game with incomplete information given the politician’s sequentially rational strategies, the voters using this mixed strategy for each type revelation and the politicians choosing $x^*$ identified in the analysis of the politician stage constitute a PBE to the model.

Further, note that given a distribution of politician types and strategy used by others, voters with a corrupt politician strictly prefer to keep her if and only if:

$$a_i \geq \mathbb{E}[a] - (1 - q)\mathbb{E}_\rho[d^V(\rho)],$$

strictly preferring to keep if the inequality is strict and are indifferent if met with equality. Similarly, voters with a clean politician prefer to keep her if:

$$a_i \geq \mathbb{E}[a] + q\mathbb{E}_\rho[d^V(\rho)].$$

So, for any distribution of incumbent types, in any PBE there are two critical ability levels, one for those with a corrupt politician and one for those with a clean politician, where voters are indifferent and potentially play a mixed strategy, otherwise they must use a pure strategy.

For part (i), suppose there are $n' + 1$ corrupt incumbents, two of which have ability $a' = \mathbb{E}[a] - (1 - q)d^V(n'/(N - 1)) + \epsilon$ for a small $\epsilon > 0$. Let other politicians have a high enough ability that they are kept regardless of the strategies used by others. To prove the result, it is sufficient to show that there is an equilibrium where both of the corrupt politicians with ability $a'$ are kept and an equilibrium where both are replaced. By construction, all incumbents other than those with ability $a'$ are kept. If both of the corrupt politicians with ability $a'$ are kept, then all politicians are kept, resulting in corruption level $n'/(N - 1)$ (with certainty) from the perspective of both voters in question. And since $a' > \mathbb{E}[a] - (1 - q)d^V(n'/(N - 1))$, it is a best response to keep the politician at this corruption level.

If both voters with the politician with ability $a'$ choose to replace them, then the new level of corruption will either remain at $d^V(n'/(N - 1))$ if the new other politician is corrupt (probability $q$) or become $d^V((n' - 1)/(N - 1))$ if the new other politician is not corrupt (probability $1 - q$). So these voters prefer to remove their politician if:

$$a' = \mathbb{E}[a] - (1 - q)d^V(n'/(N - 1)) + \epsilon$$

$$\leq \mathbb{E}[a] - (1 - q)(qd^V(n'/(N - 1)) + (1 - q)d^V((n' - 1)/(N - 1))).$$

In particular, suppose the corrupt incumbents have ability above $\mathbb{E}[a] - (1 - q)\min_{\rho \in [0,1]} d^V(\rho)$ and the clean incumbents have ability above $\mathbb{E}[a] + q\max_{\rho \in [0,1]} d^V(\rho)$.
Rearranging gives:
\[ \epsilon \leq (1 - q)^2(d^V(n'/ (N - 1)) - d^V((n' - 1)/(N - 1))), \]
which since \( d^V(n'/ (N - 1)) > d^V((n' - 1)/(N - 1)) \) holds for sufficiently small \( \epsilon \). So, there are multiple equilibria to the model, one where all politicians are kept, and one where all are kept except the two corrupt politicians with ability \( a' \).

For part (ii), suppose there are \( n' + 1 \) corrupt incumbents, all of whom have ability \( a' \) defined in part i. The remaining \( N - n' - 1 \) politicians are clean and have sufficiently high ability to be kept regardless of the strategies used by others, as defined above.

If all of the voters with corrupt politicians choose to keep them, then the corruption level from their perspective will be \( n'/(N - 1) \) with certainty and all strictly prefer to keep by an identical calculation to part (i).

If all corrupt politicians are removed then the number of corrupt politicians (write this \( C \)) is a binomial random variable with \( n' \) trials with “success” rate \( q \). The expected value of \( d^V(\rho) \) if the \( n' \) corrupt politicians are replaced is then:

\[
\mathbb{E}_\rho[d^V(\rho)|\text{all corrupt replaced}] = \sum_{C=0}^{n'} \Pr(C)d^V(C/(N - 1))
\]

\[
< \sum_{C=0}^{n'} \Pr(C)d^V(n'/(N - 1)) = d^V(n'/(N - 1))
\]

where the inequality comes from the fact that \( d^V(\rho) \) is increasing and \( \Pr(C = n') < 1 \). All voters with a corrupt politician prefer to replace them if

\[
a' = \mathbb{E}[a] - (1 - q)d^V(n'/ (N - 1)) + \epsilon
\]

\[
\leq \mathbb{E}[a] - (1 - q)\mathbb{E}_\rho[d^V(\rho)|\text{all corrupt replaced}].
\]

Rearranging gives:
\[ \epsilon \leq (1 - q)(d^V(n'/ (N - 1)) - \mathbb{E}_\rho[d^V(\rho)|\text{all corrupt replaced}]), \]
which holds for sufficiently small \( \epsilon \). □

### A3.4 Proof of Proposition 4

The existence of an equilibrium follows from the fact that the right-hand side of the equilibrium condition is a continuous function from \([0, 1]\) to \([0, 1]\) and hence must have a fixed point. When \( \frac{dV}{d\rho} < 0 \), the right-hand side is decreasing, ensuring this intersection is unique. If not, the right-hand side is increasing, and as shown in the illustrations in the main text, may have multiple intersections.
A4 Entry Model

Here, we present a model where the composition of the pool of those running for office changes as a function of how corrupt they expect politics to be.

To build on the politician model presented in the main text, suppose that for each of the $N$ districts there are $M > 1$ candidates. The type space is the same as in the politician model: each candidate has an ability $a_i$ drawn from $F$, and is either predisposed to corruption ($c_i = 1$) with probability $q$, or averse to corruption ($c_i = 0$).\textsuperscript{10} Potential candidates do not know the type of other potential candidates.

Candidates pay a cost $k$ to run for office. Those who do not enter office get a payoff normalized to zero, and those who win office observe the type of other winners and simultaneously choose a corruption level $x_i$ with the same utility function as in the main model. We also assume that the ability distribution either has no upper bound, or if it is bounded the upper bound is sufficiently high as formally defined below.

A strategy in the entrant model consists of a mapping from the potential candidate type (i.e., the pair $(a_i, c_i)$) to a decision to run, and a mapping from the type to a corruption level if elected. Our equilibrium definition (formalized below) will require that the strategies chosen are mutual best responses given a long-run stable proportion of corruption using a definition analogous to that in the main model.

Let $Pr(\text{win}; \cdot)$ be the probability of winning, conditioned on the strategies used by others. For simplicity we assume that there is a lottery where every candidate wins with equal probability, so the probability of winning if $m$ other candidates in the district run is $1/(1+m)$.

For a fixed proportion of corrupt (winning) politicians, a candidate prefers to enter if:

$$Pr(\text{win}; \cdot)(a_i + c_i d_P(\rho)) \geq k,$$

where $d_P(\rho)$ is the difference between the equilibrium payoff for a corrupt versus clean politician as defined in the text. Since this payoff is increasing in $a_i$, any optimal entrance strategy can be characterized by a threshold for non-corrupt candidates $\hat{a}_P^0$ and corrupt candidates $\hat{a}_P^1$ such that potential candidates run if and only if $a_i \geq \hat{a}_P^c_i$.\textsuperscript{11}

Since the candidates do not know the others’ types, the number who run from their perspective is a binomial random variable with $M - 1$ trials and success rate $\pi(\hat{a}_P^0, \hat{a}_P^1)$, where $\pi(\cdot)$ is the unconditional probability of running:

$$\pi(\hat{a}_P^0, \hat{a}_P^1) = (1 - q)(1 - F(\hat{a}_P^0)) + q(1 - F(\hat{a}_P^1)).$$

The expected probability of winning is then obtained by summing over the probability

\textsuperscript{10}We refer to a generic candidate with a subscript $i$ without referring to their district; given the symmetry of the districts it is not necessary to distinguish between them with additional notation.

\textsuperscript{11}We use $\hat{a}_P^c_i$ when referring to the ability threshold used by a voter from a particular constituency, and $\hat{a}_P^c$ when referring to a common threshold used by voters in all constituencies.
of winning against $m$ candidates times the probability that this many others run:

$$Pr(win|\hat{a}_0^P, \hat{a}_1^P) = \mathbb{E}_m[1/(m + 1)]$$

$$= \sum_{m=0}^{M-1} \frac{1}{m + 1} \frac{(M - 1)!}{(M - m - 1)!m!} \pi(\hat{a}_0^P, \hat{a}_1^P)^m (1 - \pi(\hat{a}_0^P, \hat{a}_1^P))^{M-m-1}$$  \hspace{1cm} (2)

If $m_1$ is a binomial random variable with $M - 1$ trials and success rate $\pi_1$ and $m_2$ is a binomial random variable with $M - 1$ trials and success rate $\pi_2 > \pi_1$, then the distribution of $m_2$ first-order stochastic dominates the distribution of $m_1$.\footnote{This follows from the regularized incomplete beta function representation of the cumulative distribution function of a binomial random variable with $M - 1$ trials and success rate $\pi$: $Pr(m < j|M - 1, \pi) = (n - k)(n\choose k) \int_{t=0}^{1-\pi} t^{M-1-j-1}(1 - t)^j dt$, which is decreasing in $\pi$. Since the cumulative distribution function is decreasing in $\pi$, for $\pi_1 < \pi_2$ the distribution with $\pi_2$ first-order stochastic dominates the distribution under $\pi_1$.} And since the probability of winning is a decreasing function of $m$, if the distribution of $m_2$ first-order stochastic dominates the distribution of $m_1$ then $\mathbb{E}_{m_2}[1/(m_2 + 1)] > \mathbb{E}_{m_1}[1/(m_2 + 1)]$. In words, anything that increases the propensity of politicians to run decreases the probability that any individual politician wins. Since $\pi(\hat{a}_0^P, \hat{a}_1^P)$ is decreasing in both $\hat{a}_0^P$ and $\hat{a}_1^P$ and strictly decreasing where $f(\hat{a}_c^P) > 0$, $\mathbb{E}_m[1/(m + 1)]$ is increasing in $\hat{a}_0^P$ and $\hat{a}_1^P$, and strictly increasing where $f(\hat{a}_c^P) > 0$.\footnote{E.g., for $\hat{a}_c^P < 0$, $\pi$ is constant in $\hat{a}_c^P$ (since there is no density on negative ability), and hence so is $\mathbb{E}_m[1/(m + 1)]$.}

The winners then choose corruption levels as in the politician model. Formally, our equilibrium definition is as follows:

**Definition** An equilibrium to the entry model comprises a corruption level $x^*$ and entry thresholds $\hat{a}_0^P$ and $\hat{a}_1^P$ that jointly solve:

$$x^* = \arg\max_{x_i} a_i - x_i + g(x_i, \bar{x})$$  \hspace{1cm} (3)

$$\hat{a}_0^P = k/Pr(win; \cdot)$$  \hspace{1cm} (4)

$$\hat{a}_1^P + d^P(\rho) = k/Pr(win; \cdot)$$  \hspace{1cm} (5)

$$\rho = \frac{q(1 - F(\hat{a}_1^P))}{q(1 - F(\hat{a}_1^P)) + (1 - q)(1 - F(\hat{a}_0^P))}$$  \hspace{1cm} (6)

And $Pr(win; \cdot)$ is the expected probability of winning derived above.

Condition 3 states that politicians choose a corruption level that is a mutual best response as derived in the main model. Conditions 4-5 imply that, given this behavior in the politician stage, potential entrants with exactly the threshold ability level $\hat{a}_c^P$ (given their corruption disposition $c_i$) are indifferent between running and not. That is, those with ability level above this threshold prefer to run and those below the threshold prefer to stay out of politics.
implicitly made by potential candidates approximates the PBE. A simpler justification for this assumption is that if those in office play the equilibrium to this stage, and candidates enter if and only if their ability is above $\hat{a}_c^P$, the expected proportion of corrupt politicians is in fact $\rho^*$.

The result which motivates our assumption about the changing pool of entrants in the main model is as follows:

**Proposition A1** For any fixed $\rho$:

(i) There is a unique pair of thresholds meeting equations 4-5, and

(ii) $\hat{a}_0^P$ is continuous and weakly increasing in $\rho$ and $\hat{a}_1^P$ is continuous and weakly decreasing in $\rho$.

**Proof** First, note that since $\hat{a}_0^P = \hat{a}_1^P + d^P(\rho)$, we can write the equilibrium probability of winning solely as a function of $\hat{a}_0^P$: $Pr(\text{win}; \hat{a}_0^P, \hat{a}_1^P - d^P(\rho))$. So finding a joint solution to these equations is equivalent to finding $\hat{a}_0^P$ that solves:

$$\hat{a}_0^P = k/Pr(\text{win}; \hat{a}_0^P, \hat{a}_1^P - d^P(\rho))$$

(7)

The right-hand side of equation 7 is strictly positive, continuous, and decreasing in $\hat{a}_0^P$. So the left-hand side of equation 7 is above the right-hand side at $\hat{a}_0^P = 0$. Further, since the left-hand side is increasing (without bound), there must exist a unique $a_0^*(\rho)$ where the equation is met with equality. So, a formal statement of the notion that the upper bound on ability must be sufficiently high is that for any $\rho$ the upper bound on the ability distribution is above $a_0^*(\rho)$.

For part (ii), consider any $\rho_1 < \rho_2$, and write the corresponding ability thresholds at each corruption level $\hat{a}_c^P(\rho)$. Since $\hat{a}_0^P(\rho) = \hat{a}_1^P(\rho) + d^P(\rho)$ for each $\rho$:

$$\hat{a}_0^P(\rho_2) - \hat{a}_0^P(\rho_1) = \hat{a}_1^P(\rho_2) - \hat{a}_1^P(\rho_1) + d^P(\rho_2) - d^P(\rho_1)$$

Suppose $\hat{a}_0^P(\rho_2) < \hat{a}_0^P(\rho_1)$. Since $d^P(\rho_2) > d^P(\rho_1)$, this requires that $\hat{a}_1^P(\rho_2) < \hat{a}_1^P(\rho_1)$. That is, if the clean types become more apt to run when $\rho$ increases, the corrupt types must become more apt to run as well. However, this makes running less desirable for both types as it decreases the chance of winning, contradicting the claim that the non-corrupt type is more apt to run. Formally, $Pr(\text{win}|\hat{a}_0^P, \hat{a}_1^P)$ is increasing in both $\hat{a}_c^P$’s, so:

$$Pr(\text{win}|\hat{a}_0^P(\rho_2), \hat{a}_1^P(\rho_2)) < Pr(\text{win}|\hat{a}_0^P(\rho_1), \hat{a}_1^P(\rho_1))$$

That is, potential entrants behave as if the proportion of corrupt politicians will be exactly equal to the probability of being corrupt if running, while in fact when the number of districts is finite the number of corrupt politicians is uncertain (following a binomial distribution). So, if the value of holding office is nonlinear in $\rho$ and the number of districts is finite, this solution concept is not exactly the same as Perfect Bayesian Equilibrium (PBE), which would require the politicians to explicitly sum over the potential realized values of $\rho$, rendering the solution far more analytically complex. However, when $N$ is large, the calculation implicitly made by potential candidates approximates the PBE. A simpler justification for this assumption is a behavioral one: citizens simply make an approximation of how corrupt politics will be by equation 6.
Combining this with the non-corrupt ability threshold condition implies:
\[
\hat{a}_0^P(\rho_1) = k/Pr(\text{win}; \hat{a}_0^P(\rho_1), \hat{a}_1^P(\rho_1)) < k/Pr(\text{win}; \hat{a}_0^P(\rho_2), \hat{a}_1^P(\rho_2)) = \hat{a}_0^P(\rho_2)
\]
contradicting \(\hat{a}_0^P(\rho_2) < \hat{a}_0^P(\rho_1)\). So \(\hat{a}_0^P\) must be weakly increasing in \(\rho\).

A similar argument holds to show \(\hat{a}_0^P\) is decreasing in \(\rho\): if \(\hat{a}_1^P\) is increasing, then \(\hat{a}_0^P\) must be increasing as well, meaning the chance of winning increases for both types, contradicting the indifference condition for the non-corrupt type.

For continuity, equation 7 is continuous in \(\hat{a}_0^P\) and \(dP(\rho)\) with a unique solution for all \(\rho\), and so the \(\hat{a}_0^P\) solving this equation must be continuous in \(\rho\). Since \(dP(\rho)\) is continuous, and \(\hat{a}_0^P = dV(\rho) + \hat{a}_1^P\), \(\hat{a}_1^P\) is continuous as well.

Combining these entry rules with equation 6, a fixed point \(\rho^*\) must solve:

\[
\rho^* = \frac{q(1 - F(\hat{a}_1^P(\rho^*)))}{q(1 - F(\hat{a}_1^P(\rho^*))) + (1 - q)(1 - F(\hat{a}_0^P(\rho^*)))}
\]

(8)

Our main result from the entry model is as follows:

**Proposition A2** There is at least one – and potentially more than one – \(\rho^*\) meeting equation 8 and hence at least one equilibrium and potentially multiple equilibria to the entry model.

**Proof** The right-hand side of equation 8 is continuous function from \([0, 1]\) to \([0, 1]\), so it must have a fixed point, proving the existence of an equilibrium. The possibility of multiple \(\rho^*\) and hence equilibria to the entry model is demonstrated by Figure A2.

Figure A2 shows examples of the entry model where there is a unique equilibrium (left panel) and multiple equilibria (right panel). In both panels, \(M = 10\), \(q = 1/2\), \(k = 1/2\) and ability follows a uniform distribution on \([0, 1]\).\(^{15}\) In the left panel \(dP(\rho) = \rho/2\), and in the right panel \(dP(\rho) = \Phi(20(\rho - 7/10))\). In both cases, the proportion of corrupt entrants is increasing in \(\rho\). In the right panel, the sharp increase in \(dP(\rho)\) around 7/10 leads to a dramatic increase in the proportion of entrants that are corrupt around this level, leading to multiple intersections and hence multiple equilibria.

**References**


\(^{15}\)For these parameters, the “sufficiently high ability” condition is met as the thresholds for running are always interior.
Figure A2: Examples with one (left panel) and multiple (right panel) equilibria in the entry model.


