Appendix

A1. Proof of the proposition

**Proposition.** In any local equilibrium, a) the platform mean of any party is located between the competition weighted mean and the decisive party member; b) the platform variance of any party is bound from above by \(|x_{m_j} - z_j|/\sqrt{3} + \omega\).

**Proof.** Suppose \(z\) is an equilibrium platform profile and there is at least one platform \(\hat{z}_j = (z_j, \sigma_j)\) so that \(z_i > x_{m_j} > EM_j\). Then there exist some \(\delta_1, \delta_2 \geq 0\) with at least one of them strictly positive, so that for the platform \(\hat{z}_j\) with \(z_j' = z_j - \delta_1\) and \(\sigma_j' = \sigma_j - \delta_2\), it holds that \(v_j^*(\hat{z}_j') > v_j^*(\hat{z}_j)\) and thus \(p_{2j}(\hat{z}_j') > p_{2j}(\hat{z}_j)\). As \(z_j > z_j' > EM_j\) or \(\sigma_j' < \sigma_j\), hence \(p_{1j}(\hat{z}_j') > p_{1j}(\hat{z}_j)\). For this reason, \(z\) cannot be an equilibrium strategy. The argument for \(z_i < x_{m_j} < EM_j\) is analogous which establishes the first claim. Next, suppose \(z\) is an equilibrium platform profile where for at least one platform \(\hat{z}_j = (z_j, \sigma_j)\) with \(x_{m_j} > z_j > EM_j\), it holds that \(\sqrt{3}\sigma_j > |x_{m_j} - z_j| + \sqrt{3}\omega\). Consider a platform \(\hat{z}_j\) so that \(z_j' = z_j\) and \(|x_{m_j} - z_j|/\sqrt{3} + \omega \leq \sigma_j' < \sigma_j\). Then the posteriors of both \(\hat{z}_j\) and \(\hat{z}_j'\) are uniformly distributed over \([x_{m_j} - \sqrt{3}\omega, x_{m_j} + \sqrt{3}\omega]\) and thus \(u_{m_j}(\hat{z}_j') = u_{m_j}(\hat{z}_j)\). As \(p_{1j}(\hat{z}_j') > p_{1j}(\hat{z}_j)\), \(z\) cannot be an equilibrium. The argument for \(z_j < x_{m_j} < EM_j\) is analogous which establishes the second claim. \(\square\)

A2. Proof of the theorem

We prove a theorem that establishes conditions for the existence and uniqueness of local (pure) Nash equilibria in open neighborhoods \(U\) of the equilibrium strategies. The proof is an application of the Banach fix point theorem that states that any contraction mapping \(T : X \rightarrow X\) on a complete metric space \(X\) has a unique fixed point \(x^* = T(x^*)\). We first have to consider an appropriate metric space \(X\) and a mapping \(T\) that maps \(X\) into \(X\) and translates the equilibrium into a fix point problem.

Consider two reasonable large but finite real numbers \(M_1\) and \(M_2\) such that \(M_1 > \max_{i,j}\{|x_i|, |x_{m_j}|\}\) and \(M_2 > \max_{i,j}\{|x_i| + \omega, |x_{m_j}| + \omega\}\). We define the two intervals \(I_1 = [-M_1, M_1]\) and \(I_2 = [0, M_2]\) and consider the strategy space \(I = I_1^P \times I_2^P\) where \(P\) is the number of parties. Next, we use equation (6) from the main text

\[
\begin{align*}
  z_j & = \frac{3 \sum (\rho_{ij} - \rho_{ij}^2) x_i + \alpha_n (p_{2j} - p_{2j}^2) (x_{m_j} \mp \sqrt{3}\sigma_j \pm \frac{1}{2}\sqrt{3}\omega)}{3 \sum (\rho_{ij} - \rho_{ij}^2) + \alpha_n (p_{2j} - p_{2j}^2)} \\
  \sigma_j & = \frac{\alpha_n (p_{2j} - p_{2j}^2) (\pm \frac{1}{\sqrt{3}} (x_{m_j} - z_j) + \frac{1}{2}\omega)}{\sum (\rho_{ij} - \rho_{ij}^2) + \alpha_n (p_{2j} - p_{2j}^2)}.
\end{align*}
\]

to define two vector-valued functions \(g\) and \(h\) on \(I\) with coordinate functions

\[
\begin{align*}
  g_j(z, \sigma) & := \frac{3 \sum (\rho_{ij} - \rho_{ij}^2) x_i + \alpha_n (p_{2j} - p_{2j}^2) (x_{m_j} \mp \sqrt{3}\sigma \pm \frac{1}{2}\sqrt{3}\omega)}{3 \sum (\rho_{ij} - \rho_{ij}^2) + \alpha_n (p_{2j} - p_{2j}^2)} \\
  h_j(z, \sigma) & := \frac{\alpha_n (p_{2j} - p_{2j}^2) (\pm \frac{1}{\sqrt{3}} (x_{m_j} - z_j) + \frac{1}{2}\omega)}{\sum (\rho_{ij} - \rho_{ij}^2) + \alpha_n (p_{2j} - p_{2j}^2)}.
\end{align*}
\]

Again, we save notation and write \(\pm\) for the two cases \(EM_j \leq z_j\). We can then state the theorem:
Theorem. (Existence and uniqueness of local Nash Equilibria) Let $T : I \rightarrow I$ defined by $T(s_1, ..., s_{2p}) = (g_1(z, \sigma), ..., g_p(z, \sigma), h_1(z, \sigma), ..., h_p(z, \sigma))$, and $\{U_1, U_2, ...\}$ be a collection of environments in $I$. For any such $U_i$, if there exists a real number $\lambda < 1$ so that for the supremum norm $||\cdot||$ of the Jacobian $DT$ of $T$ it holds that $||DT|| \leq \lambda$ in $U_i$, then there exists a fixed point $s^*$ in $U_i$. At least one of these fixed points is a unique local Nash equilibrium $s^* = (z^*, \sigma^*)$ of party leader strategies in some $U_i$.

Proof. From the definition of $\rho_{ij}$ and $p_{2j}$ we know that $0 < (\rho_{ij} - \rho^2_{ij}) < 1$ and $0 < (p_{2j} - p^2_{2j}) < 1$. From equation (A1) it is clear that $z_j$ is a convex combination of all voter ideal points $x_i$ and $x_m_j \pm \sqrt{3} \sigma \pm \frac{1}{2} \sqrt{3} \omega$. Likewise, $\sigma_j$ is a convex combination of zero and $\pm \frac{1}{\sqrt{3}} (x_m_j - z_j) + \frac{1}{2} \omega$. Therefore $T$ maps into $I$.

The possibility that unique local Nash equilibria exist in some environments $U$ is an immediate consequence of the application of the Banach fix point theorem (see also Merrill and Adams, 2001). Using the mean value theorem, we can infer that $T$ is a $\lambda$-contraction in some $U$ so that there is a unique fixed point $s^*$ of $T$ in $U$. This is also an extrema of $u_j$. We still need to show that at least one of the critical points (there could be many, one for each $U$) is a maximum. To this end, we note that $\frac{\partial u_j}{\partial z_j} < 0$ if $z_j > M_1$, and $\frac{\partial u_j}{\partial z_j} > 0$ if $z_j < -M_1$. Moreover, $\frac{\partial u_j}{\partial \sigma_j} < 0$ if $\sigma_j > M_2$ and $\frac{\partial u_j}{\partial \sigma_j}(0+) \geq 0$. Thus, there is at least one $s^*$ that is a local Nash equilibrium. □

Supplementary materials

S1. Monte Carlo simulation

The theorem in the Appendix of the article relates the existence of local Nash equilibria $z^*$ to the existence of some environment $U$ of $z^*$ where $T$ is a contraction. Whether or not such an environment exists (or even many such environments) is difficult to establish in general terms. Fortunately, the Banach fix point theorem provides a constructive algorithm to find these fix points. Specifically, for any arbitrary starting point $s_0$ in $U$, the sequence $s_n$ defined by $s_n = T(s_{n-1})$ converges to the fix point $s^*$ of $U$. We here provide the R code of the simulation.

```r
# MC simulation ambiguity model
rm(list = ls())
library(foreign)

# utility functions and voting probabilities
uij <- function(xi,zj,sigmaj) {
    if (length(xi) > 1 | length(zj) > 1) {
        n.x <- length(xi)
        n.z <- length(zj)
        x <- matrix(rep(xi,n.z),n.x,n.z,byrow=FALSE)
        z <- matrix(rep(zj,n.x),n.x,n.z,byrow=TRUE)
        sigma <- matrix(rep(sigmaj,n.x),n.x,n.z,byrow=TRUE)
        t <- -sigma^2 - (x-z)^2
    } else {
        t <- -sigmaj^2 - (xi-zj)^2
    }
}
```

# utility functions and voting probabilities
uij <- function(xi,zj,sigmaj) {
    if (length(xi) > 1 | length(zj) > 1) {
        n.x <- length(xi)
        n.z <- length(zj)
        x <- matrix(rep(xi,n.z),n.x,n.z,byrow=FALSE)
        z <- matrix(rep(zj,n.x),n.x,n.z,byrow=TRUE)
        sigma <- matrix(rep(sigmaj,n.x),n.x,n.z,byrow=TRUE)
        t <- -sigma^2 - (x-z)^2
    } else {
        t <- -sigmaj^2 - (xi-zj)^2
    }
```
rohij <- function(xi,zj,sigmaj,z,sigma) {
  exp(u1j(xi,zj,sigmaj)) / rowSums(exp(u1j(xi,z,sigma)))
}

roh <- function(x,z,sigma) {
  exp(u1j(x,z,sigma)) / rowSums(exp(u1j(x,z,sigma)))
}

u2j <- function(xmj,hetj,zj,sigmaj) {
  if (zj <= xmj) {
    lower <- xmj-sqrt(3)*hetj
    upper <- zj + sqrt(3)*sigmaj
    mu.post <- (upper+lower)/2
    sigma.post <- (upper-lower) / (2 * sqrt(3))
  }
  if (xmj < zj) {
    lower <- zj-sqrt(3)*sigmaj
    upper <- xmj + sqrt(3)*hetj
    mu.post <- (upper+lower)/2
    sigma.post <- (upper-lower) / (2 * sqrt(3))
  }
  if (lower > upper) {
    u <- NA
  } else {
    u <- -sigma.post^2 - (xmj-mu.post)^2
  }
  u
}

roh2j <- function(xmj,hetj,zj,sigmaj) {
  exp(u2j(xmj,hetj,zj,sigmaj)) / ( exp(u2j(xmj,hetj,zj,sigmaj)) + exp(uc) )
}

p2 <- function(z,sigma) {
  t <- rep(0,length(z))
  for (j in 1:length(z)) t[j] <- roh2j(xm[j],hetj[j],z[j],sigma[j])
  t
}

# contraction mapping T

g <- function(x,z,sigma,EM) {
  t <- (EM <= z)*2-1
  zstar <- ( 6 * colSums( (roh(x,z,sigma)-roh(x,z,sigma)^2) * x) +
    alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) *
    ( (-t) * 2* sqrt(3) * sigma + t * sqrt(3) * hetj + 2* xm ) ) /
    ( 2 * alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) +
      6 * colSums(roh(x,z,sigma)-roh(x,z,sigma)^2) )
  list("z"=zstar)
h <- function(x,z,sigma,EM) {
    sstar <- ( alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) * 
        ( 3 * hetj + 2 * sqrt(3) * abs(xm -z)) ) / 
        ( 6 * (alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) + 
        colSums(roh(x,z,sigma)-roh(x,z,sigma)^2) ) ) )
    list("sigma"=sstar)
}
# Nash equilibrium
ne <- function(x,start.z,start.sigma,precision) {
    i <- 1
    z <- start.z
    sigma <- start.sigma
    d.z <- 1000
    d.sigma <- 1000
    h.z <- 0
    h.sigma <- 0
    while ((d.z > precision | d.sigma > precision) & i < 500) {
        i <- i+1
        z_1 <- z
        sigma_1 <- sigma
        okay <- FALSE
        ii <- 0
        while (okay == FALSE & ii < 100) {
            ii <- ii + 1
            EM <- colSums( (roh(x,z_1,sigma_1)-roh(x,z_1,sigma_1)^2)*x) / 
            colSums((roh(x,z_1,sigma_1)-roh(x,z_1,sigma_1)^2) )
            z <- g(x,z_1,sigma_1,EM)$z
            okay <- all(!is.na(z))
            if (okay==FALSE) {
                sigma_1[is.na(z)] <- sigma_1[is.na(z)]*1.1
            }
        }
        hh <- h(x,z_1,sigma_1,EM)
        sigma <- hh$sigma
        d.z <- sum(abs(z-z_1))
        d.sigma <- sum(abs(sigma-sigma_1))
        h.z <- c(h.z,d.z)
        h.sigma <- c(h.sigma,d.sigma)
    }
    list("z"=z,"sigma"=sigma,"it"=i,"start.z"=start.z,"start.sigma"=start.sigma,"EM"=EM)
}
# simulation with 100 runs and 10 starting values
sims <- 100
n.start <- 10
# fixed parameters
uc <- -1
n <- 1000
alpha <- 1
prec.pow <- 5
prec <- 10^(-prec.pow)
# matrices to collect results
mc <- array(list(NULL), c(sims,10))
mc.xm <- mc.x <- array(list(NULL), c(sims,1))
# Monte Carlo simulation, sims runs
for (ii in 1:sims) {
  cat(".")
  x <- rnorm(n,0,1)
p <- sum(rmultinom(1,size=1,prob=c(1,1,1)) * c(1,2,3))*2 + 1
  distinct.xm <- p-1
  while (distinct.xm < p) {
    xm <- round(rnorm(p,0,1),digits=3)
    distinct.xm <- length(table(xm))
  }
  xm <- sort(xm)
  hetj <- rep(0.2,p)
z.ne <- NULL
  sigma.ne <- NULL
  # run for n.start different starting values
  for (i in 1:n.start) {
    start.z <- runif(p,0,1)*xm
    start.sigma <- (1/sqrt(3)*abs(start.z-xm)+1/2*hetj)*runif(p,0,1)
    t1 <- ne(x,start.z,start.sigma,prec)
    z.ne <- rbind(z.ne,round(t1$z, digits=prec.pow))
    sigma.ne <- rbind(sigma.ne,round(t1$sigma, digits=prec.pow))
    mc[[ii,i]] <- t1
  }
  mc.xm[[ii]] <- list("xm"=xm)
  mc.x[[ii]] <- list("x"=x)
  # plot simulation for different starting values
  par(mfrow=c(2,1))
  par(mar=c(2,1,2,1))
  par oma=c(1,1,1,1))
  dotchart(z.ne, labels="",main="z",xlim=c(-3,3))
  dotchart(sigma.ne,labels="",ylab="sigma",main="sigma",xlim=c(0,3))
}
S2. List of parties


References