Online Appendix to:

Conformity Voting and the Value of Public Information

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A Proofs

Proof of Lemma 1.

Voters support the incumbent only if their posterior expectation $\overline{\theta}_i$ is lower than or equal to the strategic cutoff μ

$$\overline{\theta}_i \equiv \frac{\sigma^2 y + \tau^2 \zeta_i}{\sigma^2 + \tau^2} \leq \mu \tag{A.1}$$

which, solving for the private signal ζ_i, gives the condition

$$\zeta_i \leq \mu + \frac{\sigma^2}{\tau^2}(\mu - y) \tag{A.2}$$

Assuming all voters follow the threshold rule given by μ and the challenger's valence equals θ, denote as Γ the fraction of voters that support the incumbent

$$\Gamma(\theta, \mu) \equiv \Phi\left(\frac{\mu + \frac{\sigma^2}{\tau^2}(\mu - y) - \theta}{\sqrt{\sigma^2}}\right) \tag{A.3}$$

The **fundamental cutoff** θ^* is such that the incumbent wins whenever the challenger's valence θ is below θ^*. When $\theta = \theta^*$, both candidates obtain the same fraction of votes

$$\Gamma(\theta^*) \equiv \Phi\left(\frac{\mu + \frac{\sigma^2}{\tau^2}(\mu - y) - \theta^*}{\sqrt{\sigma^2}}\right) = \frac{1}{2} \tag{A.4}$$

Solving for θ^*, we obtain

$$\theta^* = \mu + \frac{\sigma^2}{\tau^2}(\mu - y) \tag{A.5}$$

Each voter, given her information, forms subjective expectations about θ as well as each candidate's winning chances. Recall that the posterior distribution over θ of a voter who observes ζ_i and y takes the following form

$$\theta \mid \zeta_i, y \sim \mathcal{N}\left(\frac{\sigma^2 y + \tau^2 \zeta_i}{\sigma^2 + \tau^2}, \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}\right)$$

Thus, a voter's subjective probability that the incumbent wins the election equals

$$\alpha_i \equiv Pr(Incumbent\ wins \mid \zeta_i, y) = \int_\theta \mathbb{1}\{\Gamma(\theta) > 0.5\} \mathrm{d}F(\theta \mid \zeta_i, y) =$$

$$= Pr(\theta < \theta^* \mid \zeta_i, y) = \Phi\left(\frac{\theta^* - \overline{\theta}_i}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}\right) \tag{A.6}$$

where α_i denotes the probability that a voter with posterior expectation $\overline{\theta}_i$ attaches to the incumbent's victory.

The indifferent voter—whose posterior expectation about θ is equal to μ—obtains the same utility from voting for the incumbent and challenger. The expected utility of voting for the incumbent is (recall that $\theta_I = 0$)

$$\hat{\alpha}_i \gamma$$

where $\hat{\alpha}_i$ denotes the indifferent voter's subjective probability that the incumbent wins the election. By contrast, the expected utility of voting for the challenger is given by

$$\hat{\alpha}_i \mu + (1 - \hat{\alpha}_i)(\mu + \gamma)$$

Equating the two expressions and solving for $\hat{\alpha}_i$, we obtain

$$\hat{\alpha}_i = \frac{1}{2} + \frac{\mu}{2\gamma} \tag{A.7}$$

Combining equations A.6 and A.7, we get

$$\Phi\left(\frac{\theta^* - \mu}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}\right) = \frac{1}{2} + \frac{\mu}{2\gamma} \tag{A.8}$$

We are thus left with two equations (A.5 and A.8) and two unknown variables (μ, θ^*). Replacing θ^* in equation A.8, the final equation is equal to

$$\Phi\left(\beta(\mu - y)\right) = \frac{1}{2} + \frac{\mu}{2\gamma} \tag{A.9}$$

where

$$\beta \equiv \frac{\sigma^2}{\tau^2}\sqrt{\frac{\sigma^2+\tau^2}{\sigma^2\tau^2}} \tag{A.10}$$

Equation A.9 implicitly defines the strategic cutoff μ.

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Proof of Lemma 2.

From the equilibrium equation A.9, define the following function

$$\nu(\mu) \equiv \Phi\left(\beta(\mu - y)\right) - \frac{1}{2} - \frac{\mu}{2\gamma} \tag{A.11}$$

A unique equilibrium prevails if $\nu(\mu)$ crosses zero once. If $\nu(\mu)$ crosses zero more than once, we have multiplicity of equilibria. β is large if public information is precise relative to private information, and vice versa.

First, I prove that an equilibrium always exists. The strategic cutoff must be between $\mu_{min} = -\gamma$ and $\mu_{max} = \gamma$. If $\mu > \gamma$, for instance, the voter always prefers to vote for the challenger, regardless of her chances of winning the election. Thus, this voter cannot be the indifferent voter.

Notice that

$$\lim_{\mu \to \mu_{min}} \nu(\mu) = \Phi\left(\beta(\mu - y)\right) \geq 0$$

and

$$\lim_{\mu \to \mu_{max}} \nu(\mu) = \Phi\left(\beta(\mu - y)\right) - 1 < 0$$

which, together with continuity of $\nu(\mu)$, proves the existence of equilibrium.

Next, I prove that the number of equilibria always equals one or three. Note that the right-hand side of equation A.9 is equal to zero when $\mu = \mu_{min} = -\gamma$, and one when $\mu = \mu_{max} = \gamma$. As for the left-hand side, it is between zero and $\frac{1}{2}$ when $\mu = \mu_{min}$, and between $\frac{1}{2}$ and one when $\mu = \mu_{max}$. This rules out the existence of two equilibria. If $\nu(\mu)$ crosses zero twice, there must be a third instance given that the left-hand side of equation A.9 is lower than the right-hand side when $\mu = \mu_{max}$.

Can there be more than 3 equilibria? Taking the derivative of $\nu(\mu)$ with respect to μ in equation A.11 and equating it to zero, we get

$$v'(\mu) = \beta\phi\left(\beta(\mu - y)\right) - \frac{1}{2\gamma} = 0 \Leftrightarrow$$

$$\phi\left(\beta(\mu - y)\right) = \frac{1}{2\gamma}\frac{1}{\beta}$$

As the standard normal distribution is symmetric, $v'(\mu)$ equals zero for at most two different values of μ. By Rolle's theorem, $v(\mu)$ is equal to zero for at most three different values of μ.

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Proof of Proposition 1.

A sufficient condition for uniqueness of equilibrium is that the derivative of $\nu(\mu)$ with respect to μ is non-positive at any value of μ

$$\frac{\partial \nu(\mu)}{\partial \mu} = \beta\phi\left(\beta(\mu - y)\right) - \frac{1}{2\gamma} \leq 0$$

which simplifies to

$$\frac{1}{\beta} \geq 2\gamma\phi\left(\beta(\mu - y)\right) \tag{A.12}$$

The probability distribution function of the standard normal distribution is maximized at 0, where it is equal to $\frac{1}{\sqrt{2\pi}}$. This is the case when $\mu = y = 0$. The indifferent voter expects a close race and considers both candidates to be of similar valence. For fixed precision of public and private signals, the closest a race is expected to be, the higher the degree of strategic complementarity, and the more prevalent multiplicity of equilibria becomes.

Replacing $\phi()$ by its maximum possible value, $\frac{1}{\sqrt{2\pi}}$, and rearranging, the inequality of equation 4 in the main text is obtained.

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Proof of Proposition 2.

The strategic cutoff is equal to

$$\mu = \frac{\sigma^2 y + \tau^2 \hat{\zeta}_i}{\sigma^2 + \tau^2}$$

where $\hat{\zeta}_i$, with a slight abuse of notation, denotes the private signal received by the indifferent voter. Replacing μ in equation A.9, we get

$$\Phi\left(\beta\left(\frac{\sigma^2 y + \tau^2 \hat{\zeta}_i}{\sigma^2 + \tau^2} - y\right)\right) - \frac{\sigma^2 y + \tau^2 \hat{\zeta}_i}{\sigma^2 + \tau^2}\frac{1}{2\gamma} - \frac{1}{2} = 0$$

Totally differentiating with respect to the private and public signals ζ_i and y, we get

$$\frac{d\zeta_i}{dy} = -\frac{\frac{\sigma^2}{\tau^2}\frac{1}{2\gamma} + \beta\phi()}{\frac{1}{2\gamma} - \beta\phi()}$$

The above expression measures how much the private signal would have to decrease (increase) to make up for an increase (decrease) in the public signal, such that the indifferent voter is still indifferent between voting for the incumbent and the challenger.

Next, I take the indifferent voter's posterior expectation over θ,

$$\bar{\theta}_i \equiv \frac{\sigma^2 y + \tau^2 \zeta_i}{\sigma^2 + \tau^2} = \mu$$

and totally differentiate it with respect to the private and public signals

$$\frac{d\zeta_i}{dy} = -\frac{\sigma^2}{\tau^2}$$

This second expression measures how much the private signal needs to decrease (increase) to compensate for an increase (decrease) in the public signal, so that the voter still holds the same posterior expectation over θ.

The multiplier effect of public information can be defined as the ratio of the previously calculated two effects

$$M = \frac{\frac{1}{2\gamma} + \frac{\tau^2}{\sigma^2}\beta\phi\left(\beta(\mu - y)\right)}{\frac{1}{2\gamma} - \beta\phi\left(\beta(\mu - y)\right)}$$

The strategic effect of the public signal manifests itself in that the strategic cutoff μ is endogenous to the public signal. The multiplier measures the responsiveness of the strategic cutoff μ to changes in the public signal y, controlling for the responsiveness of the posterior expectation $\overline{\theta}_i$ to changes in y.

This multiplier is always greater than one. As a result of an increase in the public signal y, the strategic cutoff μ always decreases. Therefore, the decrease in the private signal so that the indifferent voter continues to be indifferent is always bigger than the decrease needed for the expectation over θ to be the same.

For given values of σ^2, τ^2 and γ, the term is maximized when $\mu = y$, i.e. when the strategic cutoff equals the public signal (which happens when $y = 0$). When $y = 0$, the public information signals that the challenger's valence is very similar to the incumbent's. In this case a tight race is expected, and small perturbations to the public signal can induce drastic changes in voter sentiment, as all voters become more bullish about the chances of the candidate favored by the small change.

In addition, M is high when public information is precise relative to private information (β high). As β increases, the denominator of M decreases (in the unique equilibrium region, the denominator $\frac{1}{2\gamma} - \beta\phi()$ is positive). Moreover, $\frac{\tau^2}{\sigma^2}\beta$ is equal to $\sqrt{\frac{\sigma^2+\tau^2}{\sigma^2\tau^2}}$, which is decreasing in τ^2.

Finally, the multiplier M increases when the desire to conform with the majority increases (γ high).

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Proof of Lemma 3.

For given θ, define y^* as the threshold level of public signal such that the incumbent wins the election whenever y is smaller than y^*. Conditional on θ, the public signal y follows $y \sim N(\theta, \tau^2)$. Thus, the incumbent victory probability equals

$$Pr(I\ wins \mid \theta) = Pr(y < y^* \mid \theta) = \Phi\left(\frac{y^* - \theta}{\sqrt{\tau^2}}\right) \tag{A.13}$$

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Proof of Proposition 3.

To analyze how changes to information parameters affect ex-ante victory probabilities, we need the indifference condition of the threshold voter. Recall, from equation 1, that

$$\Phi\left(\frac{\sigma^2}{\tau^2}\sqrt{\frac{\sigma^2+\tau^2}{\sigma^2\tau^2}}(\mu - y)\right) = \frac{1}{2} + \frac{\mu}{2\gamma} \tag{A.14}$$

As the ex-ante probability of incumbent victory, equation 5 in the main text, depends on y^*, it is convenient to restate the above indifference condition of the threshold voter in terms of y^*. Recall that if $y = y^*$, the incumbent and challenger obtain the same fraction of votes. As private signals are centered in θ, this means that the indifferent voter is exactly the one that received a private signal equal to θ. Therefore, in this particular case the strategic cutoff

μ, the posterior mean of the indifferent voter, is given by

$$\mu(y^*) = \frac{\sigma^2 y^* + \tau^2\theta}{\sigma^2 + \tau^2} \tag{A.15}$$

Replacing $\mu(y^*)$ in equation A.14, and setting $y = y^*$, we obtain

$$\Phi\left(\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}}(\theta - y^*)\right) = \frac{1}{2} + \frac{1}{2\gamma}\frac{\tau^2\theta + \sigma^2 y^*}{\sigma^2+\tau^2} \tag{A.16}$$

To obtain the results of the proposition, first define the function H from equation A.16.

$$H(\theta, y^*, \gamma, \sigma^2, \tau^2) = \Phi\left(\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}}(\theta - y^*)\right) - \frac{1}{2} - \frac{1}{2\gamma}\frac{\tau^2\theta + \sigma^2 y^*}{\sigma^2+\tau^2} = 0 \tag{A.17}$$

For all subsequent proofs I assume that the incumbent is the better candidate, i.e. $\theta < 0$. When $\theta > 0$, all results are reversed.

i) $\frac{\partial P_I}{\partial \gamma}$

Given that the incumbent is assumed to be the best candidate ($\theta < 0$), we need to analyze the derivative of P_I with respect to γ. However, P_I only depends on γ through y^*, which positively affects P_I. Thus, the sign of $\frac{\partial y^*}{\partial \gamma}$ equals the sign of $\frac{\partial P_I}{\partial \gamma}$. The former is equal to

$$\frac{\partial y^*}{\partial \gamma} = -\frac{\frac{\partial H}{\partial \gamma}}{\frac{\partial H}{\partial y^*}} = \frac{\frac{\partial H}{\partial \gamma}}{\phi()\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}} + \frac{1}{2\gamma}\frac{\sigma^2}{\sigma^2+\tau^2}}$$

Thus, the sign of $\frac{\partial y^*}{\partial \gamma}$ is equal to the sign of $\frac{\partial H}{\partial \gamma}$. Below I restate the H function from equation A.17

$$H = \Phi\left(\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}}(\theta - y^*)\right) - \frac{1}{2} - \frac{1}{2\gamma}\frac{\tau^2\theta + \sigma^2 y^*}{\sigma^2+\tau^2} = 0$$

The derivative of H with respect to γ equals

$$\frac{\partial H}{\partial \gamma} = \frac{1}{2\gamma^2}\frac{\tau^2\theta + \sigma^2 y^*}{\sigma^2+\tau^2} < 0$$

Notice that $\frac{\tau^2\theta+\sigma^2 y^*}{\sigma^2+\tau^2}$ equals μ, the strategic cutoff, when the public signal is such that the incumbent and challenger obtain the same fraction of votes ($y = y^*$). As we assumed the incumbent was the better candidate, $\theta < 0$ and $y^* > \theta$. The indifferent voter receives a negative private signal equal to θ and a public signal y^* that is higher than θ. For this voter to be indifferent, her posterior expectation about θ must be negative. She needs to be confident enough about the incumbent's superior capabilities.

To prove this point more rigorously, assume that $\mu = 0$. In this case, the right-hand side in equation A.9 that determines the equilibrium equals $\frac{1}{2}$, whereas the left-hand side is lower than $\frac{1}{2}$ due to $y > \theta$. If $\mu > 0$, on the other hand, the right-hand side is bigger than $\frac{1}{2}$. For the equality to hold, we would need $\mu > y$ in the left-hand side. This is a contradiction, as μ is formed as a linear combination of y and the private signal ζ_i, which for the indifferent voter in this particular case equals $\zeta_i = \theta < 0$. Thus, the only way that equation A.9 can be satisfied is if $\mu < 0$.

Therefore, as $\frac{\partial H}{\partial \gamma} < 0$, so is $\frac{\partial P_I}{\partial \gamma} < 0$. The better candidate's winning chances decrease when voters' desire to conform with the majority increases. As voters increasingly value voting for the winner at the expense of voting for the better candidate, they become more prone to backing a candidate whom they think is not so competent, but who seems to have the support of other voters.

ii) $\frac{\partial P_I}{\partial \sigma^2}$

As in the case of γ, note that P_I (defined in equation A.13 above) does not directly depend on σ^2, and it positively depends on y^*, provided that $\theta < 0$.

Thus, I calculate the derivative of y^* with respect to σ^2, which is equal to

$$\frac{\partial y^*}{\partial \sigma^2} = -\frac{\frac{\partial H}{\partial \sigma^2}}{\frac{\partial H}{\partial y^*}} = \frac{\frac{\partial H}{\partial \sigma^2}}{\phi()\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}} + \frac{1}{2\gamma}\frac{\sigma^2}{\sigma^2+\tau^2}}$$

This means that the sign of $\frac{\partial y^*}{\partial \sigma^2}$ is equal to the sign of $\frac{\partial H}{\partial \sigma^2}$. The latter derivative is given by

$$\frac{\partial H}{\partial \sigma^2} = -\frac{y^* - \theta}{(\sigma^2 + \tau^2)^2}\left[\phi()\frac{1}{2}\sqrt{\frac{\tau^2(\sigma^2+\tau^2)}{\sigma^2}} + \tau^2\right]$$

As $\theta < 0$, we obtain that $\frac{\partial y^*}{\partial \sigma^2} < 0$. In turn, this implies that $\frac{P_I}{\partial \sigma^2} < 0$. The better candidate's (in this case the incumbent) winning chances decrease if the precision of private information decreases.

When the incumbent is the better candidate, the challenger needs a strong favorable public signal to win the election. If the precision of private information decreases, voters have more dispersed and less reliable private information about candidates' valence. As a consequence, they attach more weight to the public signal. In this situation, a not-so-stellar public signal favoring the challenger might suffice for her to win the election.

iii) $\frac{\partial P_I}{\partial \tau^2}$

First I calculate $\frac{\partial y^*}{\partial \tau^2}$

$$\frac{\partial y^*}{\partial \tau^2} = -\frac{\frac{\partial H}{\partial \tau^2}}{\frac{\partial H}{\partial y^*}} = \frac{\frac{\partial H}{\partial \tau^2}}{\phi()\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}} + \frac{1}{2\gamma}\frac{\sigma^2}{\sigma^2+\tau^2}}$$

Thus, the sign of $\frac{\partial y^*}{\partial \tau^2}$ equals the sign of $\frac{\partial H}{\partial \tau^2}$. The latter derivative is equal to

$$\frac{\partial H}{\partial \tau^2} = (y^* - \theta)\frac{1}{2}\frac{\sigma^2}{(\sigma^2+\tau^2)^2}\left[\phi()\sqrt{\frac{\tau^2(\sigma^2+\tau^2)}{\sigma^2}}\frac{\sigma^2+2\tau^2}{\tau^4} + \frac{1}{\gamma}\right]$$

Therefore, $\frac{\partial y^*}{\partial \tau^2} > 0$ when $\theta < 0$. When the incumbent is the better candidate, most private signals favor her. To win the election, the challenger needs a very favorable public signal. If the precision of the public signal decreases, as voters attach less weight to them, the pool of public signals that gives rise to the challenger's victory shrinks. However, a change in τ^2 also implies a change in the distribution of y.

The derivative of P_I with respect to τ^2 is equal to

$$\frac{\partial P_I}{\partial \tau^2} = \phi\left(\frac{y^* - \theta}{\sqrt{\tau^2}}\right)\left[\frac{\frac{\partial y^*}{\partial \tau^2}\sqrt{\tau^2} - \frac{y^*-\theta}{2\sqrt{\tau^2}}}{\tau^2}\right]$$

Therefore, the sign of $\frac{\partial P_I}{\partial \tau^2}$ equals the sign of $\frac{\partial y^*}{\partial \tau^2}\sqrt{\tau^2} - \frac{y^*-\theta}{2\sqrt{\tau^2}}$.

$$\frac{\partial P_I}{\partial \tau^2} > 0 \Leftrightarrow$$

$$\frac{\partial y^*}{\partial \tau^2} > \frac{y^* - \theta}{2\tau^2} \Leftrightarrow$$

$$\frac{\tau^2}{\sigma^2+\tau^2}\left[\phi\left(\right)\sqrt{\frac{\tau^2(\sigma^2+\tau^2)}{\sigma^2}}\frac{\sigma^2+2\tau^2}{\tau^4} + \frac{1}{\gamma}\right] > \phi\left(\right)\sqrt{\frac{\sigma^2+\tau^2}{\sigma^2\tau^2}} + \frac{1}{2\gamma} \Leftrightarrow$$

$$\phi\left(\right)\sqrt{\frac{\sigma^2+\tau^2}{\sigma^2\tau^2}}\left(\frac{\sigma^2+2\tau^2}{\sigma^2+\tau^2} - 1\right) > \frac{1}{\gamma}\left(\frac{1}{2} - \frac{\tau^2}{\sigma^2+\tau^2}\right) \Leftrightarrow$$

$$\phi\left(\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}}(\theta - y^*)\right)\sqrt{\frac{\tau^2}{\sigma^2(\sigma^2+\tau^2)}} > \frac{1}{2\gamma}\left(\frac{\sigma^2-\tau^2}{\sigma^2+\tau^2}\right)$$

The above condition is satisfied when the precision of public information is low (τ^2 high), the precision of private information is high (σ^2 low), and both candidates' valences are similar (θ close to zero). As for voters' desire to conform with the majority, γ, the condition is more easily satisfied when γ is high. The reason is that even if the right-hand side is increasing in γ when $\tau^2 > \sigma^2$, in that case the right-hand side is always negative and, thus, smaller than the left-hand side.

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B Alternative Payoff Function

In this section I analyze how the equilibrium and main results change when an alternative payoff function is considered. The voter's utility from voting for a loser is normalized to zero. Voting for a winning incumbent yields utility γ to the voter, and voting for the winning challenger yields utility $\theta + \gamma$. Figure B.1 below shows the strategic form of this modified game. The only modification with respect to Figure 1 pertains to the quadrant where the voter supports the challenger and the incumbent wins the election.

	C wins	I wins
Vote for C	$\theta + \gamma$	0
Vote for I	0	γ

Figure B.1: The Modified Game in Strategic Form

Equilibrium Analysis

Following steps analogous to the main text equilibrium characterized by equation 1, the equilibrium equation pertaining to the modified game is equal to

$$\Phi\left(\beta(\mu - y)\right) = \frac{\mu + \gamma}{\mu + 2\gamma} \tag{B.1}$$

Lemma B.1 *There is always an equilibrium where everybody votes for the incumbent.*

Proof of Lemma B.1.

In this game the upper-dominance region no longer exists. Consequently, there always exists an equilibrium where every voter supports the incumbent: if voters believe the incumbent will win with probability 1, it is in their best interest to vote for the incumbent regardless of their beliefs about the challenger's valence. In a threshold equilibrium notation, this equilibrium is given by $\mu = \infty$.

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In addition to this equilibrium, however, sometimes there exist other two interior "threshold equilibria". While in the main text the strategic cutoffs could lie between $-\gamma$ and γ, in this modified game the cutoffs can lie between $-\gamma$ and ∞.

Lemma B.2 *There can be at most two interior threshold equilibria.*

Proof of Lemma B.2.

To show there can be at most 3 equilibria (one of them being the one defined in Lemma B.1), define $\nu(\mu) = \Phi\left(\beta(\mu - y)\right) - \frac{\mu+\gamma}{\mu+2\gamma}$ from equation B.1 above. Taking the derivative of $\nu(\mu)$ with respect to μ and equating it to zero, we obtain

$$\frac{\partial \nu(\mu)}{\partial \mu} = \beta\phi\left(\beta(\mu - y)\right) - \frac{\gamma}{(\mu + 2\gamma)^2} = 0$$

which simplifies to

$$\beta\phi(\beta(\mu - y)) = \frac{\gamma}{(\mu + 2\gamma)^2} \tag{B.2}$$

As the support of μ goes from $-\gamma$ to ∞, the right-hand side of equation B.2 is monotonically decreasing in μ. By contrast, the standard normal distribution is symmetric. Therefore, $v'(\mu)$ can equal zero at most at two different values of μ, and $v(\mu)$ can equal zero at most at three different values of μ.

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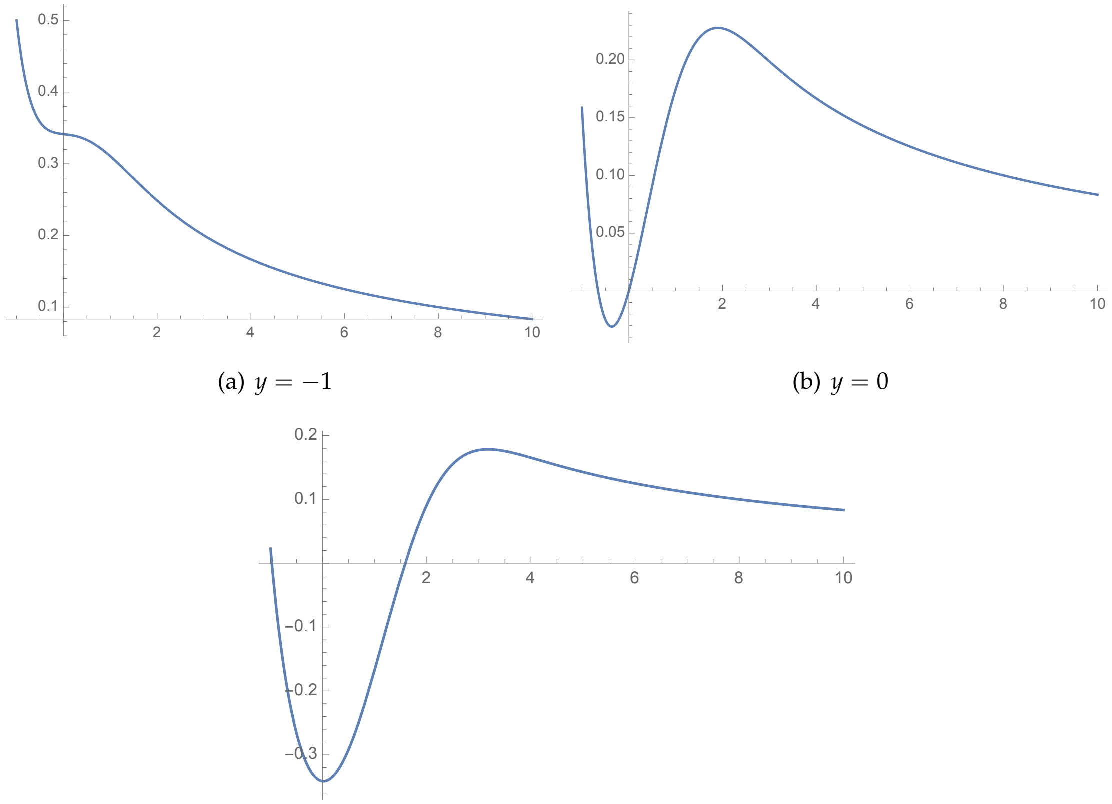
However, when the public signal y about the challenger's valence is sufficiently low, the only equilibrium that survives is the one where everybody votes for the incumbent. The intuition is that if the public signal is unfavorable to the challenger, even voters that receive a very positive private signal about the challenger's valence realize the incumbent will win. In such a case, given the structure of the payoff function, their optimal behavior is to support the incumbent, regardless of their beliefs about the challenger's valence.

To show this, assume for simplicity that $\gamma = 1$ and $\beta = 1$. Then, $v(\mu)$ equals

$$v(\mu) = \Phi\left(\mu - y\right) - \frac{\mu + 1}{\mu + 2}$$

In Figure B.2 I plot $v(\mu)$ as a function of μ assuming $\beta = 1$ and $\gamma = 1$. When the public signal y is equal to -1 (subfigure a), $v(\mu)$ only goes to zero in the limit as $\mu \to \infty$. By contrast, when y is equal to zero (subfigure b) $\nu(\mu)$ crosses zero twice, yielding two interior threshold equilibria. Finally, when $y = 1$ (subfigure c) $\nu(\mu)$ also crosses zero twice.



(a) $y = -1$

(b) $y = 0$

(c) $y = 1$

Figure B.2: $\nu(\mu)$ as a function of μ when $\beta = 1$ and $\gamma = 1$.

Comparative Statics

The next step is to verify whether the main result in Proposition 3—i.e. that the best candidate's winning chances, under certain conditions, are decreasing in the precision of public information—holds in this modified game. For that, I assume the parameters are such that interior equilibria exist.

The expression below denotes the ex-ante probability of an incumbent victory, which is the same as in the main text

$$Pr(I\, wins \mid \theta) = \Phi\left(\frac{(y^* - \theta)}{\sqrt{\tau^2}}\right) \tag{B.3}$$

Proposition B.1 *The better candidate's winning probabilities increase with:*
i) A decrease in the voters' desire to conform with the majority ($\gamma \downarrow$)
ii) An increase in the precision of private information ($\sigma^2 \downarrow$)
iii) A decrease in the precision of public information ($\tau^2 \uparrow$), provided that

$$\phi\left(\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}(y^*-\theta)\right)\sqrt{\frac{\tau^2}{\sigma^2(\sigma^2+\tau^2)}}\left[\frac{1-\sigma^2}{\tau^2}+\frac{1}{\sigma^2+\tau^2}\right] > \frac{\gamma\sigma^2(\sigma^2-\tau^2)}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}$$

Proof of Proposition B.1.

i) $\frac{\partial P_I}{\partial \gamma}$

The best candidate always benefits when voters' desire to conform with the majority decreases, as was the case in the main text. To show this, note that P_I depends on γ only through y^*, and under the assumption that the incumbent is the better candidate ($\theta < 0$), P_I depends positively on y^*. Thus, the sign of $\frac{\partial P_I}{\partial \gamma}$ is the same as the sign of $\frac{\partial y^*}{\partial \gamma}$.

To calculate $\frac{\partial y^*}{\partial \gamma}$, I define

$$H(\theta, y^*, \gamma, \sigma^2, \tau^2) = \Phi\left(\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}}(\theta - y^*)\right) - 1 + \gamma \frac{\sigma^2 + \tau^2}{\sigma^2(y^* + 2\gamma) + \tau^2(\theta + 2\gamma)} = 0 \quad \text{(B.4)}$$

which is the counterpart to equation A.17 for the game in the main text.

Thus,

$$\frac{\partial y^*}{\partial \gamma} = -\frac{\frac{\partial H}{\partial \gamma}}{\frac{\partial H}{\partial y^*}} = \frac{\frac{\partial H}{\partial \gamma}}{\phi()\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}} + \frac{\sigma^2(\sigma^2+\tau^2)\gamma}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}}$$

and

$$\frac{\partial H}{\partial \gamma} = \frac{(\sigma^2+\tau^2)(\sigma^2 y^* + \tau^2\theta)}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}$$

As shown in equation A.15, $\mu(y^*) = \frac{\sigma^2 y^* + \tau^2\theta}{\sigma^2+\tau^2}$ denotes the posterior of the indifferent voter when $y = y^*$ and the outcome ends in a tie. As $\theta < 0$, this implies that $y^* > \theta$. In turn, the same reasoning applied to the proof of the main text result leads to conclude that the posterior about θ of the indifferent voter must be negative. Therefore, $\frac{\partial P_I}{\partial \gamma} < 0$. The better candidate's electoral prospects decrease when voters' desire to conform with the majority increases.

ii) $\frac{\partial P_I}{\partial \sigma^2}$

For the same reasons as for γ, the sign of $\frac{\partial P_I}{\partial \sigma^2}$ is the same as the sign of $\frac{\partial H}{\partial \sigma^2}$. The latter equals

$$\frac{\partial H}{\partial \sigma^2} = (\theta - y^*)\left[\phi()\frac{1}{\sqrt{\tau^2\sigma^2(\tau^2+\sigma^2)}}\frac{1}{\sigma^2+\tau^2} + \frac{\gamma\tau^2}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}\right] < 0$$

as $\theta < 0$ and, correspondingly, $y^* > \theta$ and $\theta - y^* < 0$.

Therefore, $\frac{\partial P_I}{\partial \sigma^2} < 0$. The better candidate's electoral chances decrease when the precision of private information decreases.

iii) $\frac{\partial P_I}{\partial \tau^2}$

The derivative of P_I with respect to τ^2 is equal to

$$\frac{\partial P_I}{\partial \tau^2} = \phi\left(\frac{y^* - \theta}{\sqrt{\tau^2}}\right)\left[\frac{\frac{\partial y^*}{\partial \tau^2}\sqrt{\tau^2} - \frac{y^*-\theta}{2\sqrt{\tau^2}}}{\tau^2}\right]$$

Therefore, the sign of $\frac{\partial P_I}{\partial \tau^2}$ equals the sign of $\frac{\partial y^*}{\partial \tau^2}\sqrt{\tau^2} - \frac{y^*-\theta}{2\sqrt{\tau^2}}$. More specifically,

$$\frac{\partial P_I}{\partial \tau^2} > 0$$

iff

$$\frac{\partial y^*}{\partial \tau^2} > \frac{y^* - \theta}{2\tau^2}$$

Thus,

$$\frac{\partial y^*}{\partial \tau^2} = -\frac{\frac{\partial H}{\partial \tau^2}}{\frac{\partial H}{\partial y^*}} = \frac{\frac{\partial H}{\partial \tau^2}}{\phi()\sqrt{\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}} + \frac{\sigma^2(\sigma^2+\tau^2)\gamma}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}}$$

On the other hand,

$$\frac{\partial H}{\partial \tau^2} = (y^* - \theta)\left[\phi()\frac{1}{2\sqrt{\tau^2\sigma^2(\tau^2+\sigma^2)}}\left(\frac{1}{\tau^2} + \frac{1}{\sigma^2+\tau^2}\right) + \frac{\gamma\sigma^2}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}\right]$$

Following the proofs of the main text, I assume that the best candidate is the incumbent, i.e. that $\theta < 0$, which implies that $y^* - \theta > 0$. Given this, the first thing to note is that $\frac{\partial y^*}{\partial \tau^2} > 0$.

Thus, all the pieces have been calculated to establish conditions under which the best candidate's electoral chances decrease when the public information becomes more precise. Then, given that $\frac{\partial P_I}{\partial \tau^2} > 0$ iff $\frac{\partial y^*}{\partial \tau^2} > \frac{y^*-\theta}{2\tau^2}$, after some simplification we obtain that

$$\frac{\partial P_I}{\partial \tau^2} > 0 \Leftrightarrow$$

$$\phi\left(\frac{\sigma^2}{\tau^2(\sigma^2+\tau^2)}(y^*-\theta)\right)\sqrt{\frac{\tau^2}{\sigma^2(\sigma^2+\tau^2)}}\left[\frac{1-\sigma^2}{\tau^2} + \frac{1}{\sigma^2+\tau^2}\right] > \frac{\gamma\sigma^2(\sigma^2-\tau^2)}{(\sigma^2(y^*+2\gamma)+\tau^2(\theta+2\gamma))^2}$$

which is the condition under which the best candidate, in this case the incumbent, wants public information to be imprecise.

Although it is a messier inequality relative to the one in the main text, it holds under similar parameter configurations. In particular, an increase in the precision of public information

($\tau^2 \downarrow$) hurts the better candidate's electoral chances when the precision of public information is low (τ^2 high), the precision of private information is high (σ^2 low), and voters' desire to conform with the majority is high (γ high). In Figure B below I assume $\gamma = 1$ and $\theta \to 0$ from the left—i.e. both candidates' valences are almost equal but the incumbent is slightly better—and depict the region where the inequality is satisfied (in blue) as a function of σ^2 and τ^2, .

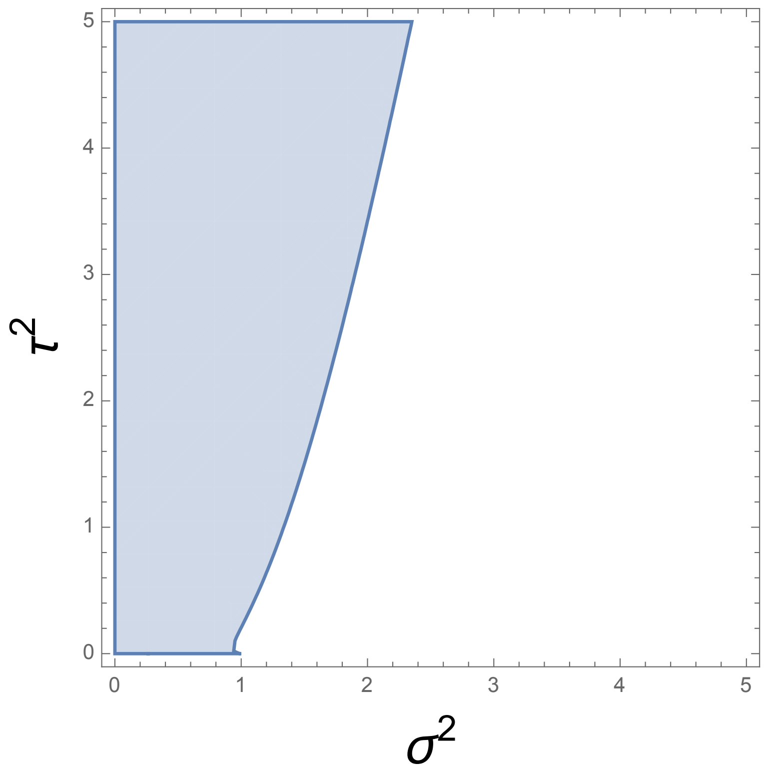


Figure B.3: $\frac{\partial P_I}{\partial \tau^2}$ as a function of σ^2 and τ^2 when $\gamma = 1$, and $\theta = 0$. The derivative is positive in the blue region.

■