## Regression Tables

The following table provides the coefficients, standard errors, and model fit information for the four specifications we present as our primary results. See the replication materials for the results of our robustness tests.

### Table 1. Robustness Tests

<table>
<thead>
<tr>
<th>Measurement strategy</th>
<th>EFFECTIVENESS, SUPPORT, INFLUENCE</th>
<th>EFFECTIVENESS, DISTANCE, INFLUENCE</th>
<th>EXPENDITURES, SUPPORT, INFLUENCE</th>
<th>EXPENDITURES, DISTANCE, INFLUENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>( q_0 )</td>
<td>0.173***</td>
<td>0.0783***</td>
<td>0.250***</td>
<td>0.228***</td>
</tr>
<tr>
<td></td>
<td>(0.0264)</td>
<td>(0.0253)</td>
<td>(0.0262)</td>
<td>(0.0248)</td>
</tr>
<tr>
<td>( b )</td>
<td>0.259***</td>
<td>0.276***</td>
<td>0.268***</td>
<td>0.292***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.0276)</td>
<td>(0.0265)</td>
<td>(0.0292)</td>
</tr>
<tr>
<td>( E[c] )</td>
<td>0.162***</td>
<td>−0.116***</td>
<td>0.112***</td>
<td>−0.0815***</td>
</tr>
<tr>
<td></td>
<td>(0.0290)</td>
<td>(0.0281)</td>
<td>(0.0279)</td>
<td>(0.0281)</td>
</tr>
<tr>
<td>( E[c] \times b )</td>
<td>0.0782***</td>
<td>−0.0960***</td>
<td>0.0833***</td>
<td>−0.111***</td>
</tr>
<tr>
<td></td>
<td>(0.0279)</td>
<td>(0.0298)</td>
<td>(0.0278)</td>
<td>(0.0316)</td>
</tr>
<tr>
<td>( q_0 \times E[c] )</td>
<td>0.151***</td>
<td>−0.179***</td>
<td>0.144***</td>
<td>−0.0939***</td>
</tr>
<tr>
<td></td>
<td>(0.0229)</td>
<td>(0.0286)</td>
<td>(0.0261)</td>
<td>(0.0280)</td>
</tr>
<tr>
<td>PILOT</td>
<td>−0.108*</td>
<td>−0.306***</td>
<td>−0.166***</td>
<td>−0.298***</td>
</tr>
<tr>
<td></td>
<td>(0.0591)</td>
<td>(0.0567)</td>
<td>(0.0596)</td>
<td>(0.0569)</td>
</tr>
<tr>
<td>LENGTH</td>
<td>0.348***</td>
<td>0.422***</td>
<td>0.360***</td>
<td>0.432***</td>
</tr>
<tr>
<td></td>
<td>(0.0274)</td>
<td>(0.0331)</td>
<td>(0.0278)</td>
<td>(0.0335)</td>
</tr>
<tr>
<td>COMMISSION</td>
<td>−0.679***</td>
<td>−0.677***</td>
<td>−0.686***</td>
<td>−0.664***</td>
</tr>
<tr>
<td></td>
<td>(0.0672)</td>
<td>(0.0670)</td>
<td>(0.0683)</td>
<td>(0.0680)</td>
</tr>
<tr>
<td>ADDRESSEE</td>
<td>−0.148</td>
<td>−0.155</td>
<td>−0.153</td>
<td>−0.137</td>
</tr>
<tr>
<td></td>
<td>(0.186)</td>
<td>(0.185)</td>
<td>(0.187)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>SCOPE</td>
<td>−0.00668***</td>
<td>−0.119***</td>
<td>−0.00694***</td>
<td>−0.122***</td>
</tr>
<tr>
<td></td>
<td>(0.00163)</td>
<td>(0.0296)</td>
<td>(0.00169)</td>
<td>(0.0301)</td>
</tr>
<tr>
<td>Constant</td>
<td>−3.637***</td>
<td>−3.796***</td>
<td>−3.749***</td>
<td>−3.808***</td>
</tr>
<tr>
<td></td>
<td>(0.245)</td>
<td>(.0370)</td>
<td>(0.248)</td>
<td>(0.0376)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marginal Effects</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>0.0308***</td>
<td>0.014***</td>
<td>0.0456***</td>
<td>0.0405***</td>
</tr>
<tr>
<td></td>
<td>(0.00492)</td>
<td>(0.00455)</td>
<td>(0.00478)</td>
<td>(0.00444)</td>
</tr>
<tr>
<td>( b )</td>
<td>0.0493***</td>
<td>0.0496***</td>
<td>0.0480***</td>
<td>0.0518***</td>
</tr>
<tr>
<td></td>
<td>(0.00495)</td>
<td>(0.00501)</td>
<td>(0.00487)</td>
<td>(0.00520)</td>
</tr>
<tr>
<td>( N )</td>
<td>9,415</td>
<td>9,361</td>
<td>9,289</td>
<td>9,235</td>
</tr>
<tr>
<td>(Pseudo) ( R^2 )</td>
<td>0.0767</td>
<td>0.0738</td>
<td>0.0827</td>
<td>0.0774</td>
</tr>
</tbody>
</table>

**Notes:** Logit regressions with heteroskedasticity-robust standard errors in parentheses. Marginal effects calculated holding all other variables at their means. * \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \).
Proof: Equilibrium Solution

Proof. Assume continuous, unbounded support for all $k_1$ and $c$.

The Commission issues a referral to the Court when $EU_C(RF) = E[q_3(c) \mid c \leq c^o]b - k_1 - k_2 - k_3 \geq EU_C(-RF) = -k_1 - k_2$, where $c^o$ is the cost below which the government will not comply with a reasoned opinion in equilibrium. This condition simplifies to $k_3 \leq k_3^o = E[q_3(c) \mid c \leq c^o]b$. As long as the government cutpoint strategy, $c^o$, exists, because $q_3(c)$ is monotone in $c$ by assumption, the best response function $k_3^o$ exists. Define $\ell_3(c^o)$ as the belief of the government that the Commission will make a referral. By Bayes’ Rule $\ell_3(c^o) = Pr(RF) = Pr(k_3 \leq E[q_3(c) \mid c \leq c^o]b)$.

The government complies with a reasoned opinion when $EU_G(C_{RO}) = c \geq EU_G(-C_{RO}) = \ell_3(c^o)(q_3(c)(c-j) + (1-q_3(c))(0)) + (1-\ell_3(c^o))(0)$. This condition simplifies to $c \geq c^o = \frac{1-\ell_3(c^o)q_3(c^o)}{1-\ell_3(c^o)q_3(c^o)} \cdot 1$. Because $\frac{dq_3(c)}{dc} > 0$, $\lim_{c\to\infty} q_3(c) \to 0$, and $\lim_{c\to0} q_3(c) \to 1$ by assumption, $\frac{d\ell_3(c^o)}{dc} > 0$, and $j > 0$ by assumption, $\lim_{c\to-\infty} -\frac{\ell_3(c^o)q_3(c^o)}{1-\ell_3(c^o)q_3(c^o)} \to 0$. Since $\frac{d\ell_3(c^o)q_3(c^o)}{dc}$ is decreasing as $\lim_{c\to-\infty}$, for $\ell_3(c^o)$ sufficiently large (i.e., as long as the distribution of $k_3 \leq E[q_3(c) \mid c \leq c^o]b$ has sufficient density) a $c^o < 0$ exists. Otherwise $c^o = 0$. Define $p_2$ as the belief of the Commission that the government will comply with a reasoned opinion. By Bayes’ Rule $p_2(c^o) = Pr(C_{RO}) = Pr(c \geq c^o \mid c < 0)$. We demonstrate below that the government complies with letters of formal notice whenever $c \geq 0$, thus the conditional probability.

The Commission issues a reasoned opinion when $EU_C(RO) = p_2(c^o)(b - k_1 - k_2) + (1-p_2(c^o))(\ell_3(c^o)(E[q_3(c) \mid c \leq c^o]b - k_1 - k_2 - E[k_3 \mid k_3 \leq k_3^o]) + (1-\ell_3(c^o))(-k_1 - k_2)) \geq EU_C(-RO) = -k_1$. Solving for $k_2$ yields the Commission’s best reply function $k_2 \leq k_2^o = p_2(c^o)b + (1-p_2(c^o))\ell_3(c^o)(E[q_3(c) \mid c \leq c^o]b - E[k_3 \mid k_3 \leq k_3^o])$. Define $\ell_2(c^o)$ as the belief of the government that the Commission will bring a reasoned opinion. By Bayes’ Rule $\ell_2(c^o) = pr(RO) = pr(k_2 \leq b(p_2(c^o) + \ell_3(c^o)E[q_3(c) \mid c \leq c^o] - p_2(c^o)\ell_3(c^o)E[q_3(c) \mid c \leq c^o]) - \ell_3(c^o)(1-p_2(c^o))E[k_3 \mid k_3 \leq k_3^o])$.

The government complies with a letter of formal notice when $EU_G(C_{LFN}) \geq EU_G(-C_{LFN})$. Note that $EU_G(C_{LFN}) = c$ and $EU_G(-C_{LFN}) = \ell_2(c^o)(\max\{EU_G(C_{RO}), EU_G(-C_{RO})\}) + (1-\ell_2(c^o))(0)$. If $c \geq 0$ it is a weakly dominant strategy for the government to comply, because $c \geq \max\{EU_G(C_{RO}), EU_G(-C_{RO})\}$ when $c \geq 0$. $EU_G(C_{RO}) = c$, and $EU_G(-C_{RO}) < c$ (EU$_G$(-C$_{RO}$) is a convex combination of payoffs of 0 and $-j$). If $c < 0$, it is a strictly dominant strategy for the government not to comply. Because $EU_G(C_{RO}) = c$, the government can assure itself at least a convex combination of payoffs of $c$ and 0 by playing $EU_G(-C_{LFN})$. Define $p_1$ as the belief of the Commission that the government will comply with a letter of formal notice. By Bayes’ Rule, $p_1(c^o) = Pr(C_{LFN})$, where $Pr(C_{LFN}) = Pr(c > 0 \mid C_0) = \frac{q_3Pr(c^o)0}{1-p_0}$. We demonstrate below that the government
plays \( C_0 \) whenever \( c \geq 0 \), and thus \( c \geq 0 \) only occurs here if there is an accidental instance of noncompliance.

The Commission issues a letter of formal notice when \( EU_C(LFN) = p_1(b - k_1) + (1 - p_1)(\ell_2(\ell_3(p_2(\ell_3(c^0)(b - k_1 - E[k_2 | k_2 \leq k_2^\circ]))) + (1 - p_2(c^0))(\ell_3(c^0)(E[q_3(c) | c \leq c^0])b - k_1 - E[k_2 | k_2 \leq k_2^\circ] - E[k_3 | k_3 \leq k_3^\circ] + (1 - \ell_3(c^0))(-k_1 - E[k_2 | k_2 \leq k_2^\circ]) + (1 - \ell_2(c^0))(-k_1)) \geq EU_C(-LFN) = 0 \). Solving for \( k_1 \) yields the Commission’s best reply function \( k_1 \leq k_1^0 = p_1b + (1 - p_1)(\ell_2(\ell_3(p_2(c^0)b + (1 - p_2(c^0))(\ell_3(c^0)(E[q_3(c) | c \leq c^0]b - E[k_3 | k_3 \leq k_3^\circ])) - E[k_2 | k_2 \leq k_2^\circ]) \). Define \( \ell_1(c^0) \) as the belief of the government that the Commission will bring a letter of formal notice. By Bayes’ Rule, \( \ell_1(c^0) = Pr(LFN) = Pr(k_2 \leq k_2^\circ) \).

The government ex ante complies when \( EU_G(C_0) \geq EU_G(-C_0) \). Note that \( EU_G(C_0) = c \) and \( EU_G(-C_0) = \ell_1(\max\{EU_G(C_{LFN}), EU_G(-C_{LFN})\}) + (1 - \ell_1)(0) \). If \( c \geq 0 \) it is a weakly dominant strategy for the government to comply. \( c \geq \max\{EU_G(C_{LFN}), EU_G(-C_{LFN})\} \) when \( c \geq 0 \) since \( EU_G(C_{LFN}) = c \), \( EU_G(C_{RO}) = c \), and \( EU_G(-C_{RO}) < c \). If \( c < 0 \), it is a strictly dominant strategy for the government not to comply. Because \( EU_G(C_{LFN}) = c \), the government can assure itself at least a convex combination of payoffs of \( c \) and \( 0 \) by playing \( EU_G(-C_0) \). Define \( p_0 \) as the belief of the Commission that the government will ex ante comply. By Bayes’ Rule, \( p_0 = Pr(C_0) = Pr(c \geq 0)(1 - q_0) \).

This system of best replies defines the Perfect Bayesian Equilibrium for the game. It is summarized below.

\[
C_0^* = \begin{cases} 
  C_0 & \text{if } c \geq 0 \\
  -C_0 & \text{otherwise}
\end{cases}
\]

\[
LFN^* = \begin{cases} 
  LFN & \text{if } k_1 \leq k_1^* \\
  -LFN & \text{otherwise}
\end{cases}
\]

\[
C_{LFN}^* = \begin{cases} 
  C_{LFN} & \text{if } c \geq 0 \\
  -C_{LFN} & \text{otherwise}
\end{cases}
\]

\[
RO^* = \begin{cases} 
  RO & \text{if } k_2 \leq k_2^* \\
  -RO & \text{otherwise}
\end{cases}
\]

\[
C_{RO}^* = \begin{cases} 
  C_{RO} & \text{if } 0 > c \geq c^* \\
  -C_{RO} & \text{otherwise}
\end{cases}
\]
\[ RF^* = \begin{cases} 
RF & \text{if } k_3 \leq k_3^* \\
-RF & \text{otherwise} 
\end{cases} \]

\[ c^* = -\frac{\ell_3(c^*)q_3(c^*)j}{1 - \ell_3(c^*)q_3(c^*)} \]

\[ k_1^* = p_1 b + (1 - p_1)(\ell_2(c^*)q_2(c^*)b + (1 - p_2(c^*)) \]

\[ (\ell_3(c^*)E[q_3(c) \mid c \leq c^*]b - E[k_3 \mid k_3 \leq k_3^*]) - E[k_2 \mid k_2 \leq k_2^*]) \]

\[ k_2^* = p_2(c^*)b + (1 - p_2(c^*))\ell_3(c^*)E[q_3(c) \mid c \leq c^*]b - E[k_3 \mid k_3 \leq k_3^*]) \]

\[ k_3^* = E[q_3(c) \mid c \leq c^*]b \]

\[ \ell_1(c^*) = \Pr(k_1 \leq k_1^*) \]

\[ \ell_2(c^*) = \Pr(k_2 \leq k_2^*) \]

\[ \ell_3(c^*) = \Pr(k_3 \leq k_3^*) \]

\[ p_0 = \Pr(c \geq 0)(1 - q_0) \]

\[ p_1 = \frac{q_0 \Pr(c \geq 0)}{1 - p_0} \]

\[ p_2(c^*) = \Pr(c \geq c^* \mid c < 0) \]
Proof: Comparative Statics

Result 1

Proof. To demonstrate $\frac{\partial (1-p_2(c^*))\ell_3(c^*)}{\partial q_0} = 0$, simply note that both $\ell_3(c^*)$ and $p_2(c^*)$ are independent of $q_0$. We prove the remaining components of the result by first examining how $\ell_3(c^*)$ and then $p_2(c^*)$ change in $b$ and $E[c]$, respectively.

Consider $\frac{\partial \ell_3(c^*)}{\partial b}$. We employ proof by contradiction. Hypothesize $\frac{\partial \ell_3(c^*)}{\partial b} < 0$. Holding $c^*$ constant, because $k_3^* = E[q(c) \mid c \leq c^*]b$, increasing $b$ increases $k_3^*$, and therefore $\ell_3(c^*) = \Pr(k \leq k_3^*)$ increases for a fixed $c^*$.

Now consider $c^* = -\frac{\ell_3(c^*)q_0(c^*)}{1-\ell_3(c^*)q_3(c^*)}$. Increasing $\ell_3(c^*)$, holding $q_3(c^*)$ constant, decreases $c^*$. Note that $\frac{\partial q_3(c^*)}{\partial c^*} > 0$, and therefore the effect of a change in $\ell_3(c^*)$ on $c^*$ is moderated by $q_3(c^*)$. If this indirect effect yields $\frac{\partial c^*}{\partial \ell_3(c^*)} > 0$ we immediately have a contradiction since that would increase $\ell_3(c^*)$. Otherwise, $\frac{\partial \ell_3(c^*)}{\partial b} < 0$. We prove the remaining components of the result by first examining $\ell_3(c^*)$ caused by the indirect effect of $b$ on $\ell_3(c^*)$ through $c^*$ must be larger than $b$’s direct increase in $\ell_3(c^*)$. However, if $\frac{\partial \ell_3(c^*)}{\partial b} < 0$, increasing $b$ does not decrease $c^*$ and we have a contradiction. Therefore, $\frac{\partial \ell_3(c^*)}{\partial b} > 0$.

A nearly identical argument holds for $\frac{\partial \ell_3(c^*)}{\partial E[c]}$. Hypothesize $\frac{\partial \ell_3(c^*)}{\partial E[c]} < 0$. Because $E[q_3(c) \mid c \leq c^*]$ is increasing in $E[c]$, holding $c^*$ constant, $\ell_3(c^*)$ is also increasing in $E[c]$, holding $c^*$ constant. The remainder of the proof by contradiction from $\frac{\partial \ell_3(c^*)}{\partial b}$ follows as above. Thus, we have $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$.

Now consider $\frac{\partial p_2(c^*)}{\partial b}$. From the equilibrium proof, $p_2(c^*) = \Pr \left( c \geq \frac{\ell_3(c^*)}{1-\ell_3(c^*)q_3(c^*)} \mid c < 0 \right)$. Let $F$ be the CDF of $c$. Then, $p_2(c^*) = \frac{1-F(c^*)-(1-F(0))}{F(0)-F(c^*)} = \frac{F(0)-F(c^*)}{F(0)}$, or equivalently, $p_2(c^*) = \frac{\Pr(c<0)-\Pr(c<c^*)}{\Pr(c<0)}$. Because $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial c^*}{\partial \ell_3(c^*)} < 0$ (from above), and $\frac{\partial \Pr(c<c^*)}{\partial c^*} > 0$, $\frac{\partial p_2(c^*)}{\partial b} > 0$.

Finally, consider $\frac{\partial p_2(c^*)}{\partial E[c]}$. Because $\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0$ and $\frac{\partial c^*}{\partial \ell_3(c^*)} < 0$ (from above), $\frac{\partial c^*}{\partial E[c]} < 0$. Since $\frac{\partial c^*}{\partial E[c]} < 0$ and $\frac{\partial \Pr(c<c^*)}{\partial E[c]} > 0$, $\frac{\partial \Pr(c<c^*)}{\partial E[c]} < 0$. We also know by definition $\frac{\partial \Pr(c<0)}{\partial E[c]} < 0$.

Because $\frac{\partial c^*}{\partial E[c]} < 0$, $\frac{\partial \Pr(c<0)-\Pr(c<c^*)}{\partial E[c]} > 0$. That, combined with $\frac{\partial \Pr(c<0)}{\partial E[c]} > 0$, implies $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$. Since $\frac{\partial \ell_3(c^*)}{\partial b} > 0$ and $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$, the sign of $\frac{\partial (1-p_2(c^*))\ell_3(c^*)}{\partial E[c]}$ is ambiguous. Since $\frac{\partial c^*}{\partial b} > 0$ and $\frac{\partial p_2(c^*)}{\partial E[c]} > 0$, the sign of $\frac{\partial (1-p_2(c^*))\ell_3(c^*)}{\partial E[c]}$ is ambiguous. \qed
Result 2

Proof. From the equilibrium proof, \( p_1 = \frac{q_0 \Pr(c \geq 0)}{1 - p_0} = \frac{q_0 (1 - \Pr(c < 0))}{1 - (1 - \Pr(c < 0))(1 - q_0)} \). Then, \( \frac{\partial p_1}{\partial q_0} = \frac{\Pr(c < 0) - \Pr(c < 0)^2}{(\Pr(c < 0) + q_0 - \Pr(c < 0)q_0)^2} \). Both the numerator and denominator are positive; thus, \( \frac{\partial p_1}{\partial q_0} > 0 \). Note that \( \ell_2(c^*) \) does not contain \( q_0 \). Thus, \( \frac{\partial (1 - p_1)(\ell_2(c^*))}{\partial q_0} < 0 \). \( \square \)

Result 3

Proof. From Result 2, \( \frac{\partial p_1}{\partial q_0} = \frac{\Pr(c < 0) - \Pr(c < 0)^2}{(\Pr(c < 0) + q_0 - \Pr(c < 0)q_0)^2} > 0 \). Taking the cross-partial with respect to \( \Pr(c < 0) \), we have \( \frac{\partial p_1}{\partial q_0 \partial \Pr(c < 0)} = \frac{\Pr(c < 0) - q_0 + \Pr(c < 0)q_0}{(\Pr(c < 0) - q_0 + \Pr(c < 0)q_0)^2} \). The sign of \( \frac{\partial p_1}{\partial q_0 \partial \Pr(c < 0)} \) depends on parameter values: \( \frac{\partial p_1}{\partial q_0 \partial \Pr(c < 0)} < 0 \) when \( \Pr(c < 0) > \frac{q_0}{1 + q_0} \). Thus, the positive effect of \( q_0 \) on \( p_1 \) is decreasing in \( \Pr(c < 0) \), which implies that the negative effect of \( q_0 \) on \( \Pr(RO | \text{LFN}) = (1 - p_1)\ell_2(c^*) \) is decreasing in \( \Pr(c < 0) \).

Note that \( q_0 \) is the probability of unintentional noncompliance conditional on the government choosing to comply. The unconditional probability of unintentional noncompliance is the joint probability that the government chooses to comply and unintentionally commits a violation, \( q_0 \Pr(c \geq 0) \). The probability of intentional noncompliance, \( \Pr(c < 0) \), is greater than the probability of accidental noncompliance when \( \Pr(c < 0) \geq q_0 \Pr(c \geq 0) \), which is equivalent to the condition under which \( \frac{\partial p_1}{\partial q_0 \partial \Pr(c < 0)} < 0 \). \( \square \)

Result 4

Proof. From the equilibrium proof, \( \ell_2(c^*) = \Pr(k_2 \leq k_2^*) \). Since \( \Pr(k_2 \leq k_2^*) \) is increasing in \( k_2^* \), we prove that \( \frac{\partial k_2^*}{\partial b} > 0 \). Substituting in \( k_2^* \), we have \( k_2^* = bp_2(c^*) + \ell_3(c^*)k_3^* - p_2(c^*)\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] \). We then take the derivative with respect to \( b \):

\[
\frac{\partial k_2^*}{\partial b} = \left( \frac{\partial p_2(c^*)}{\partial b}b + p_2(c^*) \right) + \left( \frac{\partial \ell_3(c^*)}{\partial b}k_3^* + \ell_3(c^*)\frac{\partial k_3^*}{\partial b} \right) - \left( \frac{\partial p_2(c^*)}{\partial b}\ell_3(c^*)k_3^* + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial b}k_3^* + p_2(c^*)\ell_3(c^*)\frac{\partial k_3^*}{\partial b} \right) - \left( \frac{\partial \ell_3(c^*)}{\partial b}E[k_3 \mid k_3 \leq k_3^*] + \ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b} \right) + \left( \frac{\partial p_2(c^*)}{\partial b}\ell_3(c^*)E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\frac{\partial \ell_3(c^*)}{\partial b}E[k_3 \mid k_3 \leq k_3^*] + p_2(c^*)\ell_3(c^*)\frac{\partial E[k_3 \mid k_3 \leq k_3^*]}{\partial b} \right).
\]
Reorganizing terms yields:

\[
\frac{\partial k_3^*}{\partial b} = p_2(c^*) + \frac{\partial p_2(c^*)}{\partial b} (b - \ell_3(c^*)k_3^*) + \frac{\partial \ell_3(c^*)}{\partial b} (1 - p_2(c^*)) (k_3^* - E[k_3 | k_3 \leq k_3^*])
\]

\[+ \ell_3(c^*)(1 - p_2(c^*)) \left( \frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b} \right) + p_2(c^*) \ell_3(c^*) \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}.\]

The first term, \(p_2(c^*)\), is positive because \(p_2(c^*) \in (0, 1)\).

The second term, \(\frac{\partial p_2(c^*)}{\partial b} (b - \ell_3(c^*)k_3^*)\), is strictly positive, because \(b\) is strictly greater than \(\ell_3(c^*)k_3^* = \ell_3(c^*)E[q_3(c) | c \leq c^*b]\) and \(\frac{\partial p_2(c^*)}{\partial b} > 0\) from Result 1.

The third term, \(\frac{\partial \ell_3(c^*)}{\partial b} (1 - p_2(c^*)) (k_3^* - E[k_3 | k_3 \leq k_3^*])\), is strictly positive, because \(\frac{\partial \ell_3(c^*)}{\partial b} > 0\) from Result 1, \((1 - p_2(c^*)) > 0\) since \(p_2(c^*) \in (0, 1)\), and \((k_3^* - E[k_3 | k_3 \leq k_3^*]) > 0\) because \(k_3\) is unbounded.

The fourth term, \(\ell_3(c^*)(1 - p_2(c^*)) \left( \frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b} \right)\), is strictly positive on average. First, we know \(\ell_3(c^*)(1 - p_2(c^*)) > 0\), because \(\ell_3(c^*)\) and \(p_2(c^*)\) are probabilities. Second, from above, \(k_3^* > E[k_3 | k_3 \leq k_3^*]\), which means \(k_3^*\) and \(E[k_3 | k_3 \leq k_3^*]\) can not cross. Therefore, they cannot cross: \(\frac{\partial k_3^*}{\partial b} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b} > 0\) must hold on average.

The fifth term, \(p_2(c^*) \ell_3(c^*) \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}\) is strictly positive because \(p_2(c^*)\) and \(\ell_3(c^*)\) are probabilities and \(\frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}\) must be greater than zero since \(\frac{\partial k_3^*}{\partial b} > 0\) from Result 1.

Since all terms are strictly positive on average, \(\frac{\partial k_3^*}{\partial b} > 0\) on average. More generally, as long as the fourth term is not too negative at some point, the partial derivative is positive everywhere. Note that \(p_1\) does not contain \(b\). Thus, \(\frac{\partial ((1 - p_1)\ell_2(c^*))}{\partial b} > 0\).

**Result 5**

**Proof.** Consider \(\frac{\partial \ell_2(c^*)}{\partial E[c]}\). From the equilibrium proof, \(\ell_2(c^*) = \text{Pr}(k_2 \leq k_2^*)\). Since \(\text{Pr}(k_2 \leq k_2^*)\) is increasing in \(k_2^*\), we prove that \(\frac{\partial k_2^*}{\partial E[c]} > 0\). Substituting in \(k_3^*\), we have \(k_3^* = bp_2(c^*) + \ell_3(c^*)k_3^* - p_2(c^*) \ell_3(c^*)k_3^* - \ell_3(c^*)E[k_3 | k_3 \leq k_3^*] + p_2(c^*) \ell_3(c^*)E[k_3 | k_3 \leq k_3^*]\). We then take the derivative with respect to \(E[c]\):

\[
\frac{\partial k_2^*}{\partial E[c]} = \left( \frac{\partial p_2(c^*)}{\partial E[c]}b \right) + \left( \frac{\partial \ell_3(c^*)}{\partial E[c]}k_3^* + \ell_3(c^*) \frac{\partial k_3^*}{\partial E[c]} \right)
\]

\[- \left( \frac{\partial p_2(c^*)}{\partial E[c]} \ell_3(c^*)k_3^* + p_2(c^*) \frac{\partial \ell_3(c^*)}{\partial E[c]} k_3^* + p_2(c^*) \ell_3(c^*) \frac{\partial k_3^*}{\partial E[c]} \right)
\]

\[- \left( \frac{\partial \ell_3(c^*)}{\partial E[c]} \right) E[k_3 | k_3 \leq k_3^*] + \ell_3(c^*) \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} \right)
\]

\[+ \left( \frac{\partial p_2(c^*)}{\partial E[c]} \ell_3(c^*)E[k_3 | k_3 \leq k_3^*] + p_2(c^*) \frac{\partial \ell_3(c^*)}{\partial E[c]} E[k_3 | k_3 \leq k_3^*] \right) + \frac{p_2(c^*) \ell_3(c^*)}{\partial E[c]} \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} \right).\]
Reorganizing terms yields:

\[
\frac{\partial k_2^*}{\partial E[c]} = \frac{\partial p_2(c^*)}{\partial E[c]} (b - \ell_3(c^*)k_3^*) + \frac{\partial \ell_3(c^*)}{\partial E[c]} (1 - p_2(c^*)) (k_3^* - E[k_3 | k_3 \leq k_3^*]) + \ell_3(c^*) (1 - p_2(c^*)) \left( \frac{\partial k_3}{\partial E[c]} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} \right) + p_2(c^*) \ell_3(c^*) \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]}. 
\]

The first term, \(\frac{\partial p_2(c^*)}{\partial E[c]} (b - \ell_3(c^*)k_3^*)\), is strictly positive, because \(b\) is strictly greater than \(\ell_3(c^*)k_3^* = \ell_3(c^*)E[q_3(c) | c \leq c^*]b\) and \(\frac{\partial p_2(c^*)}{\partial E[c]} > 0\) from Result 1.

The second term, \(\frac{\partial \ell_3(c^*)}{\partial E[c]} (1 - p_2(c^*)) (k_3^* - E[k_3 | k_3 \leq k_3^*])\), is strictly positive, because \(\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0\) from Result 1, \((1 - p_2(c^*)) > 0\) since \(p_2(c^*) \in (0, 1)\), and \((k_3^* - E[k_3 | k_3 \leq k_3^*]) > 0\) because \(k_3\) is unbounded.

The third term, \(\ell_3(c^*) (1 - p_2(c^*)) \left( \frac{\partial k_3}{\partial E[c]} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} \right)\), is strictly positive on average. First, we know \(\ell_3(c^*) (1 - p_2(c^*)) > 0\), because \(\ell_3(c^*)\) and \(p_2(c^*)\) are probabilities. Second, from above, \(k_3^* > E[k_3 | k_3 \leq k_3^*]\), which means \(k_3^*\) and \(E[k_3 | k_3 \leq k_3^*]\) can not cross. Because they can not cross, \(\frac{\partial k_3}{\partial E[c]} - \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial E[c]} > 0\) must hold on average.

The fourth term, \(p_2(c^*) \ell_3(c^*) \frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}\) is strictly positive because \(p_2(c^*)\) and \(\ell_3(c^*)\) are probabilities and \(\frac{\partial E[k_3 | k_3 \leq k_3^*]}{\partial b}\) must be greater than zero since \(\frac{\partial k_3}{\partial E[c]} > 0\) from Result 1.

Since all terms are strictly positive on average, the derivative is strictly positive on average. More generally, as long as the third term is not too negative at some point, the partial derivative is positive everywhere.

Consider \(\frac{\partial p_1}{\partial E[c]}\). From the equilibrium proof, \(p_1 = \frac{q_0 \Pr(c^\geq 0)}{1 - q_0} = \frac{q_0 \Pr(c^\geq 0)}{1 - q_0 - q_0}\). The numerator is increasing in \(E[c]\), and the denominator is decreasing in \(E[c]\), so \(\frac{\partial p_1}{\partial E[c]} > 0\).

Since \(\frac{\partial \ell_3(c^*)}{\partial E[c]} > 0\) and \(\frac{\partial p_1}{\partial E[c]} > 0\), the sign of \(\frac{\partial [(1 - p_1)\ell_3(c^*)]}{\partial E[c]}\) is ambiguous.

We cannot feasibly sign the cross-partial with respect to \(b\). It is

\[
\frac{\partial^2 k_2^*}{\partial E[c] \partial b} = \frac{\partial^2 p_2(c^*)}{\partial E[c] \partial b} (b - \ell_3(c^*)X) + \frac{\partial p_2(c^*)}{\partial E[c]} (1 - \left( \frac{\partial \ell_3(c^*)}{\partial b} X + \ell_3(c^*) \frac{\partial X}{\partial b} \right)) + \frac{\partial^2 \ell_3(c^*)}{\partial E[c] \partial b} X + \frac{\partial \ell_3(c^*)}{\partial E[c]} \frac{\partial X}{\partial b} - \frac{\partial p_2(c^*)}{\partial E[c]} \frac{\partial^2 \ell_3(c^*)}{\partial E[c] \partial b} X - p_2(c^*) \frac{\partial^2 \ell_3(c^*)}{\partial E[c] \partial b} X + \frac{\partial \ell_3(c^*)}{\partial b} \frac{\partial X}{\partial E[c]} + \ell_3(c^*) \frac{\partial^2 X}{\partial E[c] \partial b} - \frac{\partial p_2(c^*)}{\partial E[c]} \ell_3(c^*) \frac{\partial X}{\partial E[c]} - p_2(c^*) \frac{\partial \ell_3(c^*)}{\partial b} \frac{\partial X}{\partial E[c]} - p_2(c^*) \ell_3(c^*) \frac{\partial^2 X}{\partial E[c] \partial b},
\]

where \(X = E[q_3(c) | c \leq c^*]b - E[k_3 | k_3 \leq k_3^*]\). Thus, we turn to a numeric solution. \(\square\)
Numeric Solution

We estimate the sign of the cross-partial in Result 5 using a numeric solution. In each round of the simulation, we randomly draw values of exogenous parameters from uniform distributions and calculate endogenous parameters numerically, providing functional forms of probability distributions where necessary. We perform one thousand iterations of the simulation, keeping only in-equilibrium combinations of parameter values, and estimate the effect of exogenous parameters on endogenous parameters using OLS models. To simulate changes in $E[c]$, we change the lower bound of its uniform distribution, as changes in the upper bound do not effect endogenous parameters. Replication code is provided below.

```r
# set up

# libraries
library(rootSolve)
library(numDeriv)
library(dplyr)
library(reshape2)

# set parameter values
c.lb <- -7
c.ub <- 7
k3.lb <- -5
k3.ub <- 5
b.par <- 3
j.par <- 1

# functions

u.pdf <- function(u.lower, u.upper) {
  1 / (u.upper - u.lower)
}

u.cdf <- function(value, u.lower, u.upper) {
  (value - u.lower) / (u.upper - u.lower)
}

q3 <- function(c, z = 1) {
  1 / (1 + exp(-z * c))
}

E.q3 <- function(c.star, c.lb, c.ub) {

```
integrand <- function(c, c.lb, c.ub) {
  q3(c = c) * u.pdf(u.lower = c.lb, u.upper = c.ub)
}

integrate(f = Vectorize(integrand), lower = c.lb, upper = c.star, c.lb = c.lb, c.ub = c.ub)$value / u.cdf(value = c.star, u.lower = c.lb, u.upper = c.ub)

l3 <- function(c.star, b.par, c.lb, c.ub, k3.lb, k3.ub) {
  integrand <- function(k3, k3.lb, k3.ub) {
    u.pdf(u.lower = k3.lb, u.upper = k3.ub)
  }
  integrate(f = Vectorize(integrand), lower = k3.lb, upper = E.q3(c.star = c.star, c.lb = c.lb, c.ub = c.ub) * b.par, k3.lb = k3.lb, k3.ub = k3.ub)$value
}

c.star <- function(b.par, j.par, c.lb, c.ub, k3.lb, k3.ub) {
  fun <- function (c.star = c.star, b.par = b.par, j.par = j.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) {
    - (l3(c.star = c.star, b.par = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) * q3(c = c.star) * j.par) / (1 - (l3(c.star = c.star, b.par = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) * q3(c = c.star))) - c.star
  }
  multiroot(fun, start = -1, b.par = b.par, j.par = j.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub)$root
}

p2 <- function(c.star, c.lb, c.ub) {
  (u.cdf(value = 0, u.lower = c.lb, u.upper = c.ub) - u.cdf(value = c.star, u.lower = c.lb, u.upper = c.ub)) / u.cdf(value = 0, u.lower = c.lb, u.upper = c.ub)
}

E.k3 <- function(c.star, b.par, c.lb, c.ub, k3.lb, k3.ub) {
  integrand <- function(k3, k3.lb, k3.ub) {
    k3 * u.pdf(u.lower = k3.lb, u.upper = k3.ub)
  }
  integrate(f = integrand, lower = k3.lb, upper = E.q3(c.star = c.star, c.lb = c.lb, c.ub = c.ub) * b.par, k3.lb = k3.lb, k3.ub = k3.ub)$value
}

k2 <- function(c.star, b.par, j.par, c.lb, c.ub, k3.lb, k3.ub) {
\[ \text{p2(c.star = c.star, c.lb = c.lb, c.ub = c.ub) \ast b.par + (1 - p2(c.star = c.star, c.lb = c.lb, c.ub = c.ub)) \ast l3(c.star = c.star, b.par = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub) \ast (E.q3(c.star = c.star, c.lb = c.lb, c.ub = c.ub) \ast b.par - E.k3(c.star = c.star, b.par = b.par, c.lb = c.lb, c.ub = c.ub, k3.lb = k3.lb, k3.ub = k3.ub))} \]

```
# set up simulation

# function to estimate one round of the simulation
sim.round <- function(b.par.range, j.par.range, c.lb.range, c.ub.range, k3.lb.range, k3.ub.range) {
  b.par.draw <- runif(1, b.par.range[1], b.par.range[2])
  j.par.draw <- runif(1, j.par.range[1], j.par.range[2])
  c.lb.draw <- runif(1, c.lb.range[1], c.lb.range[2])
  c.ub.draw <- runif(1, c.ub.range[1], c.ub.range[2])
  k3.lb.draw <- runif(1, k3.lb.range[1], k3.lb.range[2])
  k3.ub.draw <- runif(1, k3.ub.range[1], k3.ub.range[2])
  c.star.solution <- c.star(b.par = b.par.draw, j.par = j.par.draw, c.lb = c.lb.draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)
  E.q3.sim <- E.q3(c.star = c.star.solution, c.lb = c.lb.draw, c.ub = c.ub.draw)
  l3.sim <- l3(c.star = c.star.solution, b.par = b.par.draw, c.lb = c.lb.draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)
  p2.sim <- p2(c.star = c.star.solution, c.lb = c.lb.draw, c.ub = c.ub.draw)
  k2.sim <- k2(c.star = c.star.solution, b.par = b.par.draw, j.par = j.par.draw, c.lb = c.lb.draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw)
  output <- data.frame(b.par = b.par.draw, j.par = j.par.draw, c.lb = c.lb.draw, c.ub = c.ub.draw, k3.lb = k3.lb.draw, k3.ub = k3.ub.draw, c.star = c.star.solution, E.q3 = E.q3.sim, l3 = l3.sim, p2 = p2.sim, k2 = k2.sim)
  return(output)
}

# function to perform the full simulation
run.sim <- function(iterations, b.par.range, j.par.range, c.lb.range, c.ub.range, k3.lb.range, k3.ub.range) {
  output <- list()
  for(i in 1:iterations) {
    output[[i]] <- sim.round(b.par.range, j.par.range, c.lb.range, c.ub.range, k3.lb.range, k3.ub.range)
  }
  return(output)
}
```
output[[i]] <- sim.round(b.par.range = b.par.range, j.par.range = j.par.range, 
c.lb.range = c.lb.range, c.ub.range = c.ub.range, k3.lb.range = k3.lb. 
range, k3.ub.range = k3.ub.range)
}

rbind(output)

output <- do.call("rbind", output)

return(output)

# run simulation
# run simulation

# run simulation
output <- run.sim(iterations = 1000, b.par.range = c(0, 10), j.par.range = c(0, 
10), c.lb.range = c(-10, 0), c.ub.range = c(0, 10), k3.lb.range = c(0, 1), k3. 
ub.range = c(1, 10))

# keep in-equilibrium values
output <- filter(output, p2 > 0 & p2 < 1 & l3 > 0 & l3 < 1 & c.star > -10 & c.star < 0 & k2 > 0)

# check l3 comparative statics
f.l3 <- (l3 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.l3 <- lm(formula = f.l3, data = output)
summary(mod.l3)

# check p2 comparative statics
f.p2 <- (p2 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.p2 <- lm(formula = f.p2, data = output)
summary(mod.p2)

# estimate first-order derivatives
f.k2 <- (k2 ~ b.par + j.par + c.lb + c.ub + k3.lb + k3.ub)
mod.k2 <- lm(formula = f.k2, data = output)
summary(mod.k2)

# estimate cross-partial derivative
f.k2 <- (k2 ~ b.par * c.lb + j.par + c.ub + k3.lb + k3.ub)
mod.k2 <- lm(formula = f.k2, data = output)
summary(mod.k2)