Appendix: Conference Votes as a Measure of Bicameralism—A Formal Treatment

This Appendix provides a more formal treatment to supplement our earlier discussion of why Binder’s (2003) measure of bicameral differences, the absolute difference in average approval percentage of the two chambers in voting on conference reports, will almost certainly not capture true bicameral differences, the absolute preference difference between the House and Senate medians.

To show this, we formally define a bicameral difference measure and its conference-based analogue in the context of a legislative model. We then provide some illustration of the relationship between these measures and define how they could be equivalent. We subsequently analyze the conditions for equivalence to be realized and conclude with a discussion of how our analysis makes Binder’s (2003) findings more understandable.

A1. Formal Definitions

Assume a bicameral legislature with $n$ Senators and $m$ Representatives. Also assume that, for each $k^{th}$ Congress, these legislators have ideal points $S_{ki}$ and $H_{kj}$ respectively, with $S_k = \{S_{k1}, \ldots, S_{kn}\}$, where $S_{k1} \leq S_{k2} \leq \ldots \leq S_{kn}$ and $S_{k1} < S_{kn}$, and where $H_k = \{H_{k1}, \ldots, H_{km}\}$, where $H_{k1} \leq H_{k2} \leq \ldots \leq H_{km}$ and $H_{k1} < H_{km}$. Let $L_k = \min\{S_{k1}, H_{k1}\}$ and $U_k = \max\{S_{kn}, H_{km}\}$ and denote $M_{Sk}$ and $M_{Hk}$ as the Senate and House medians in the $k^{th}$ Congress.
Denote $q_v$ as the exogenous status quo and $p_v$ as the proposed change to the status quo in conference report $v$, where $v = 1, \ldots, \mu_k$ in the $k^{th}$ Congress (i.e., there are $\mu_k$ conference votes in the $k^{th}$ Congress). Without a loss of generality, assume that $p_v < q_v$ for each $v$.

Assume quadratic legislator utility functions and, for simplicity, a one-dimensional policy space. As such, a senator (representative) votes for $p_v$ if $S_{ki} (H_{kj}) < (p_v + q_v)/2$, with the cutpoint $Y_{kv}$ being $(p_v + q_v)/2$.

Now denote $\Phi_{Sk}(Y_{kv})$ and $\Phi_{Hk}(Y_{kv})$ as the Senate and House approval percentages for conference report $v$, where $\Phi_{Sk}(Y_{kv}) = |\{S_{ki} \leq Y_{kv}\}|/n$ and $\Phi_{Hk}(Y_{kv}) = |\{H_{kj} \leq Y_{kv}\}|/m$. Both $\Phi_{Sk}(\cdot)$ and $\Phi_{Hk}(\cdot)$ are cumulative density functions, with $\Phi_{Sk}(Y_{kv}) = 0$ for $Y_{kv} < S_{k1}$, $\Phi_{Hk}(Y_{kv}) = 0$ for $Y_{kv} < H_{k1}$, $\Phi_{Sk}(Y_{kv}) = 1$ for $Y_{kv} \geq S_{km}$, and $\Phi_{Hk}(Y_{kv}) = 1$ for $Y_{kv} \geq H_{km}$. Note that $\Phi_{Sk}(M_{Sk})$ and $\Phi_{Hk}(M_{Hk})$ will be close to 1/2 if $Y_{kv}$ is discretely distributed and will exactly equal 1/2 if $Y_{kv}$ is continuously distributed.

We can now define both the true bicameral difference, which we denote as $D_k$, and its conference-based analogue, which we denote as $I_k$. Specifically, $D_k = |M_{Sk} - M_{Hk}|$, and $I_k = (|\Phi_{Sk}(Y_{k1}) - \Phi_{Hk}(Y_{k1})| + |\Phi_{Sk}(Y_{k2}) - \Phi_{Hk}(Y_{k2})| + \ldots + |\Phi_{Sk}(Y_{k\mu_k}) - \Phi_{Hk}(Y_{k\mu_k})|)/\mu_k$.

**A2. Relationship between the Measures and the Equivalence Condition**

To see the relationship between these two measures, suppose that there are 3 Senators and 5 Representatives in the $k^{th}$ Congress so that $D_k$ will be the absolute distance between the two chamber medians, $H_{k3}$ and $S_{k2}$. Assume that members locate themselves, and that cutpoints are distributed uniformly, between 0 and 1 in the policy space. Finally,
assume that $H_{k2} < S_{k2} < H_{k4}$ so that House members fall on both sides of the Senate median.

Figures A-1 and A-2 illustrate this relationship. Figure A-1 demonstrates the resulting correspondence between $D_k$ and $I_k$ when we assume a discrete distribution of member preferences, with $H_{k1} = S_{k1} = 0$ and $H_{k3} = S_{k3} = 1$. While $D_k$ is simply the difference between $S_{k2}$ and $H_{k3}$, $I_k$ is the sum of all the shaded areas. These areas reflect the different approval percentages of the two chambers on their respective conference votes, assuming that the relevant cutpoints are uniformly distributed. Figure A-2 provides an analogous illustration for the case of continuously distributed preferences and of cutpoints uniformly distributed between $Y_M$ and $Y_m$, where $D_k$ is the difference between $M_{S_{k}}$ and $M_{H_{k}}$, and $I_k$ is the sum of all the shaded areas divided by $(Y_M - Y_m)^{1}$.

(Figures A-1 and A-2 about here)

Figures A-1 and A-2 also provide intuition about how, given a fixed bicameral difference, changes in the preference distribution will translate into changes in the conference vote measure. For example, in Figure A-1, if $H_{k2}$ and $H_{k4}$, which help define the shape of the preference curvature, are more extreme than illustrated, then $I_k$ is smaller. Analogously, for Figure A-2, it should be obvious that the preferences in either chamber could vary in a manner that does not impact bicameral distance but does change the approval percentage (i.e., $\Phi_{S_{k}}(Y_{kv})$ and $\Phi_{H_{k}}(Y_{kv})$) and, thus, $I_k$. Put differently, the relationship between $D_k$ and $I_k$ crucially depends on the curvature of preferences.

1 Calculating $I_k$ does not require a denominator in the discrete case illustrated in Figure A-1 because we assume that the supports of the cutpoints are between 0 and 1, i.e., this is equivalent to dividing the numerator by 1.
More generally, the conference vote measure will only capture bicameral difference if it satisfies an equivalence condition, \( I_k = \alpha + \beta D_k \) for all Congresses, where \( \alpha \) and \( \beta \) are a real number and a positive real number respectively. As we will show, this will rarely be the case.

**A3. Meeting the Equivalence Condition**

To see how the equivalence condition might be satisfied, let us return to the simple case of a 3 Senator, 5 Representative legislature. Assume that there are \( k \) Congresses with different preference distributions and suppose that in Congress \( k = 0, D_0 = 0 \), and that in Congress \( k = 1, D_1 > 0 \). In calculating the corresponding \( I_k \),

\[
I_k = H_{k2}(1/3 - 1/5) + (H_{k3} - H_{k2})(2/5 - 1/3) + (H_{k4} - H_{k3})(2/5 - 1/3) + (1 - H_{k4})(1/3 - 1/5).
\]

In this case, denoting \( T \) as any real number, for any Congress \( k \),

\[
D_k = (1-T)D_1 + TD_0.
\]

This, in turn, means that the equivalence condition will only hold if

\[
I_k = (1-T)I_1 + TI_0,
\]

which will require a very specific preference curvature, i.e., that

\[
(H_{k4} - H_{k2}) = T(H_{14} - H_{12}) + (1 - T)(H_{04} - H_{02}).
\]

For instance, if \( H_{14} - H_{12} = .5 \), \( H_{04} - H_{02} = .7 \) and \( T = 3 \), then \( H_{k4} - H_{k2} = .1 \) for equivalence to hold. It is clear, then, that the preference distribution for equivalence will be a knife-edged case. If this condition is not satisfied even for a single Congress, the resulting measure is invalid.

To this point, we have held the cutpoint distribution constant and have only examined how changes in the preference curvature impact equivalence. In fact, changes

\[2\] Because \( (H_{k4} - H_{k2}) \) must be nonnegative, the equivalence condition cannot hold if \( T > (H_{04} - H_{02}) /[(H_{04} - H_{02}) - (H_{14} - H_{12})] \).
in the distribution of cutpoints may also undermine the possibility of equivalence between our two measures.

To see this, suppose that in the $k^{th}$ Congress cutpoints are distributed uniformly on $(a_k, b_k) \supseteq (0, 1)$, i.e., unlike our earlier assumption, the support of the cutpoint distribution is broader than that of the preference distribution. Doing this means that the conference vote measures for the $k^{th}$ Congress is now $I_k = \left[ H_{k2}(1/3 - 1/5) + (H_{k3} - H_{k2})(2/5 - 1/3) + (H_{k4} - H_{k3})(2/5 - 1/3) + (1 - H_{k4})(1/3 - 1/5) \right] / (b_k - a_k)$. ³ This produces an even more complicated and restrictive condition than previously for equivalence to hold, $(2 + H_{k2} - 3H_{k3} - H_{k4} + 3S_{k2})/(b_k - a_k) = T(2 + H_{12} - H_{14} + 3S_{12})/( b_1 - a_1) + (1 - T)(2 + H_{02} - H_{04})/( b_0 - a_0)$.

While it is more complicated, generalizing from the discrete case to a full blown legislature and analytically dealing with continuous distributions for preferences and cutpoints yields analogous inferences about $I_k$ and $D_k$. ⁴ Assuming that legislators do not locate exactly evenly along one dimension—for instance, if approximately normal or bimodal distributions are posited (which are consistent with empirical results, e.g., Poole

³ Note, also, that as $(b_k - a_k)$ is larger, $I_k$ is smaller. This means that, other things being equal, unanimous conference votes, such as voice votes, will diminish the value of $I_k$ regardless of the value $D_k$.

⁴ Relaxing assumptions about chamber preference and cutpoint distributions to derive the conditions under which the conference vote measure captures bicameral differences, e.g., using different bimodal preference distributions, such as a mixture of two logistic distributions and of two triangular distributions, produces very messy results that are not very illuminating.
and Rosenthal 1997)—the equivalency condition is unlikely to hold for at least three reasons:

(1) The curvature of the preference distributions across chambers and Congresses must meet a very restrictive (and unlikely to be satisfied) set of conditions. As implied by the discussion of Figure A-2, it is easy to illustrate that, due to varying preference curvatures, the conference vote measure can increase or decrease for a constant level of bicameral difference or it can remain the same or change in a manner not consistent with changes in bicameral difference. If this is a problem at one point in time, it will affect the conference vote measure for the entire time series.

(2) The reliance on unanimous votes (where both chambers agree) when there is real bicameral difference will skew $I_k$ toward zero. For example, in the extreme, if unanimous votes are used for all votes, and the chambers are considered to be in accordance in all instances, $I_k$ is necessarily 0, although $D_k$ may be positive.

(3) If conference votes are endogenously chosen, the equivalence condition is unlikely to hold given the resulting curvature of cutpoint distributions. Suppose, for example, only bills with a good chance to pass go to conference committee because only proposals outside the two chambers’ gridlock intervals are voted on. As such, the cutpoint distributions will also be accordingly outside the gridlock interval. Consequently, and as simulation results show (available from authors), the equivalence condition will be unlikely to hold.
To summarize, the extent to which $I_k$ and $D_k$ are correlated depends on the preference distribution curvatures of each chamber in one Congresses, their relationship across Congresses, and cutpoint distributions. Empirically, the frequent use of voice votes and the strong possibility of endogenous policy choices further diminish the likelihood of equivalence. Hence, the necessary conditions for the equivalence condition rarely hold, resulting in potentially severe measurement errors for analyses such as Binder’s (2003).


Recall that Binder finds that the relationship between her conference vote measure and the common space measure of bicameral difference is decidedly different if one uses 1970 as a historical break point. The implication is then made that using common space scores after 1970 is problematic given the assumption of equal salience of dimensions.

While somewhat speculative, there would seem to be at least three potential reasons for this correlative pattern that have nothing to do with this critique of common space scores:

(1) Small sample size. Given that there are only 12 observations before 1971 and 15 afterwards (13 for Binder’s 1999 analysis, which is our focus), correlations may vary greatly. For example, we simulated the correlation of $I_k$ and $D_k$ for 13 observations 100 times, assuming the number of party seats controlled by the Democrats and
Republicans in each chamber for the 92nd through 104th Congresses. Assuming different preference distribution curvatures and cutpoint distributions,\(^5\) we find that making the preference curvatures asymmetric yields a correlation less than -.4 about 20 to 25 percent of the time (results available from authors).

(2) *Endogeneity of conference votes.* If, in the spirit of pivotal politics models, we assume that endogenous conference votes produce polarized cutpoints, then the negative correlation between \(I_k\) and \(D_k\) becomes more likely. For example, when preference curvatures are asymmetric and cutpoints are polarized, we find via our simulations that the probability of the -.43 correlation found by Binder is not uncommon. Put differently, if forces such as greater party agenda control resulted in conference votes on issues falling outside the gridlock interval being more common after 1970, the correlation found between \(I_k\) and \(D_k\) would not be surprising.

(3) *Increase in proportion of recorded votes given trends in bicameral differences.* Likely the most important reason for the change in the correlation uncovered is the dramatic increase in the proportion of conference votes recorded after 1970 in the data used to calculate \(I_k\) (see Figure 1 in the main text). This decrease in the proportion of unanimous votes would drive the conference vote measure up at exactly the same time when the preference-based measure of bicameral difference, whether using W-NOMINATE or common space scores, indicates a decline. Hence, there would be a

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\(^5\) We use 13 Congresses to conform to Binder (1999). Including latter Congresses does not change our results. Note that we are not claiming to simulate the true relationship for these 13 Congresses, as we lack the requisite information about cutpoint distributions.
negative correlation between the conference vote measure and either the common space or the W-NOMINATE measure.
Figure A-1: Relationship between $D_k$ and $I_k$ with Discrete Preferences
(Legislature with 5 Representatives and 3 Senators)

Note: $I_k$ is the sum of the shaded areas. Cutpoints are assumed to be uniformly distributed between $S_{k1}$ and $S_{k3}$.
Figure A-2: Relationship between $I_k$ and $D_k$ with Continuous Preferences

Note: $I_k$ is the sum of the area between the Senate and House approval percentages, shown as $\square$, divided by $(Y_M - Y_m)$. Cutpoints are assumed to be distributed uniformly between $Y_m$ and $Y_M$. 