OBITUARY

JOHN ARTHUR TODD

John Arthur Todd was one of the last survivors of the school of classical algebraic geometry that flourished in Cambridge under H. F. Baker, FRS. But, unlike most of his contemporaries, with the notable exception of W. V. D. Hodge, FRS, Todd made seminal contributions to the more modern theory, and his name is now enshrined in the literature and widely known.

Todd was born in Liverpool, son of a schoolmaster, also John Arthur Todd, and Agnes (Perfect) Todd. Educated at Liverpool Collegiate School, he won an entrance scholarship in Natural Science to Trinity College, Cambridge, and went up in October 1925 to read Mathematics. He had a distinguished undergraduate record, and graduated before he was 20. Already at this stage he had started to specialize in geometry, initially stimulated by the lectures of F. P. White and subsequently coming under the influence of the Lowndean Professor H. F. Baker, who became his research supervisor. He was supported by a Trinity College research scholarship, and in 1930 he won the coveted Smith's Prize.

Todd's contemporaries at Trinity included a notable group of geometers (H. S. M. Coxeter, FRS, W. L. Edge, P. Du Val), all of whom went on to have distinguished careers. At Trinity they formed a closely knit group, and all participated regularly in the famous 'Baker tea parties'. These gatherings were officially advertised in the *Reporter* as 'Conferences on Geometry, Saturday 4.15 p.m.'. They were clearly the centre of social and intellectual life, and were always fondly remembered in later years by the young men who took part.

After his success with the Smith's Prize, a Research Fellowship at Trinity would have been a natural consequence, but in the annual highly competitive Fellowship competitions, Todd was unsuccessful at all his three attempts. He lost out to his friend Coxeter and also to the brilliant analyst R. E. A. C. Paley, who died tragically young in a mountaineering accident.

In 1931 Todd was appointed to an Assistant Lectureship at the University of Manchester, one of the active centres of mathematics outside Cambridge. In fact, there was a strong Cambridge–Manchester link, and many Cambridge mathematicians spent years at Manchester before returning to Cambridge. The head of department, L. J. Mordell, FRS, was a notable example.

In 1933 Todd went to Princeton for a year, on a Rockefeller Fellowship. There the dominant figure was Solomon Lefschetz (For.Mem.R.S.), who was applying modern topological methods to algebraic geometry. Under Lefschetz's influence, Todd saw that there was more to geometry than the traditional projective geometry that the Baker school represented. As a result, over the next few years Todd's research moved in a different direction and led him to the work for which he was eventually elected to the Royal Society, and which made his name familiar to the succeeding generation of geometers.

In 1937 Todd moved back to Cambridge as a University Lecturer. By this time, Hodge occupied the Lowndean Chair and was steering Cambridge geometry into the

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mainstream of international work. Todd's new contributions marked him out as the obvious person to assist Hodge in modernizing Cambridge geometry. The outbreak of war no doubt prevented or postponed the full achievement of this objective and, when a new golden age of geometry began in the postwar years, it was a younger generation that inherited the task. Although Todd continued to take an interest in modern developments he felt, probably rightly, that his own contributions lay in a different direction, and he reverted to more classical topics, albeit with a modern algebraic flavour related to group theory.

Todd's academic career continued in Cambridge until his retirement. He was extremely pleased to be made Cayley Lecturer, a title with substantial historical overtones. In due course he became a Reader but, to his disappointment, he was passed over for professorial promotion and, as a result, he opted to take early retirement.

Despite his association with Trinity, and the distinction of an FRS, Todd was never elected a Fellow of the College. This did not prevent him from working hard and unselfishly for the College, supervising large numbers of Trinity students. There is no doubt that he was deeply aggrieved by the behaviour of his old College, and his view was widely shared by mathematical colleagues. Attempts were made, on several occasions, to put him forward for a teaching Fellowship at Trinity, and these moves had the backing of the senior mathematicians in the College. Unfortunately, opposition in other quarters, based no doubt on the perception of Todd as socially gauche, was too strong. In a college as large as Trinity there should have been room for a mathematician as distinguished and dedicated as Todd, even if he did not have all the social graces.

In 1958, Todd gave up waiting for the Trinity Fellowship and accepted a post at Downing, remaining a Fellow until his retirement in 1973, at which point the College elected him to an Honorary Fellowship. The move to Downing was a great success, and there is no doubt that the friendly environment he found there enabled him to emerge from his shell and develop a side of his personality that had been repressed in earlier years. Students from his time at Downing speak in altogether more effusive terms about his friendly nature than do those who met him only when he lived in the appropriately named 'Hermitage', on the site of what is now Darwin College.

I remember supervisions with Todd in the early 1950s quite vividly. He was efficient and helpful in dealing with technical mathematical difficulties; in fact, he prided himself on being able to solve all questions in the Mathematical Tripos. But when these had been dealt with, an awful silence would descend and could last unbroken until the end of the supervision period. A few experiences of this sort meant that I always came with additional questions or topics of conversation that could fill the dreaded silence. My experience was typical: all my friends had to adopt the same defensive tactics.

As a budding geometer, I was supervised frequently by Todd, and I also attended many of his lectures, including the more advanced specialized ones for Part III of the Tripos. During one of these lectures, he remarked on the absence of a geometric proof of a certain algebraic result in invariant theory. I was able to provide such a proof, and he responded in a very encouraging and helpful way, enabling me to publish my first short paper. I have always been grateful for the psychological boost this provided, and it made a sharp contrast with the somewhat different treatment I was given a few years later from a much more famous mathematician.

Todd's undergraduate lectures were distilled into his textbook *Projective and* *analytical geometry* [**44**]. I particularly remember this book and its influence on me since it was part of the preparatory reading that was recommended when I emerged from National Service, prior to coming up to Cambridge. The book has a clear and elegant account of such topics as twisted cubics, quadric surfaces and line geometry, all of which have an engaging beauty when encountered for the first time. If there is such a thing as 'love at first sight' in mathematics, then I was smitten with it when reading Todd's book. Looking back on it now, I can see what a hybrid the book really is, and how it reflects Todd's position as a transitional figure in twentieth century geometry. Todd wanted to present the beautiful geometry of the traditional type, but his conscience told him that he had to modernize it and make it more respectable by talking more about the algebraic foundations. Here he was reflecting, at a more elementary level, the motives that led Hodge (in collaboration with Pedoe) to produce his three-volume work on *Methods of algebraic geometry*.

Todd's more advanced lectures varied from year to year, and he was able to present a wide range of interesting topics from classical geometry, such as invariant theory or higher-dimensional curves. The mixture, or alternation, of geometry and algebra reflected his own interests and skills. On the one hand, his heart was in the traditional synthetic geometry, with no equations (or only the most symbolic ones). On the other hand, he was an algebraic virtuoso of the highest standard. He was not afraid of equations, and prided himself, quite rightly, on his manipulative skill.

Despite his social limitations, Todd attracted a string of research students, carefully and conscientiously nurturing their development. Many of his students now hold chairs of mathematics (including G. C. Shephard, Sir Roger Penrose, FRS, G. Horrocks and R. Dye). Todd would have been the last to claim credit for all their subsequent success, but a helping hand in the early stages, together with the encouragement of originality, ensured that early talent was allowed to flourish. An indication of his relations with his research students is provided by Professor Dye, who wrote to me:

> Supervisions for all of us were friendly talkative affairs, ranging over many issues. We regarded Todd as a friendly, benevolent, interested uncle; puffing his pipe, occasionally nodding off, interested in our social life, what jobs we might get, etc. There was nothing of the recluse in his relationship with us. Sometimes, in the street, we would have to make sure he saw us, as he hurried along, looking through his powerful and aperture-limited specs!

Of his time at Downing, the following extract from an article that appeared in the College magazine provides a vivid picture, and illustrates the warm feelings that he evoked in the College.

> But what of him as a Fellow of the College? In a word, he was splendid. When first he arrived, as the present writers believe, he was withdrawn and difficult to get to know; perhaps his essentially donnish and reclusive nature had not been helped in that regard by Trinity's not having embraced its own. But within the writers' affectionate memories of him, he grew into an active and genuinely loved person. To his pupils, we think he seemed a benevolent if mildly austere grandfather, complete, or perhaps replete, with pipe and moustache. To his colleagues in College, he became a mildly eccentric uncle whose company we enjoyed and indeed sought. His admitted enthusiasms—philately and following the fortunes of the stock-exchange—were of limited conversational interest. But his knowledge of and genuine

> enthusiasm for music—and his occasional impishness—were a different matter. He needed friendship; he found it and gave it as a Fellow. He became the moving spirit in organising a number of us to go to the admirable performances of the Chelsea Opera Group (then under the baton of a youthful Colin Davis) and to other concerts too. He regaled us with his memories to attending in his youth performances of *Tristan and Isolde* ('it had a very funny effect upon me') and other operas under the baton of Beecham. With arthritic fingers he played and explained features of Beethoven's late piano sonatas. Every well-equipped College should have a John Todd and we were privileged to have had him and to have enjoyed his company.
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> Yet at the same time, he was not unworldly; his robust Lancastrian commonsense had much to do with guiding the College through the difficult days of what one might describe as the Leavis crisis. And his wisdom—the word is appropriate—in helping members of the College through difficult days in the early 1970s, the personal distress which he felt over some of the events of that time notwithstanding, was invaluable. He was also, it should be added, a remarkably effective contributor to discussions of the Governing Body; in that forum, he was a man of few words—but he was adept in seeing to the heart of a problem and in pricking the balloon of the pompous and the self-important.

Todd's services to mathematics were not confined to his teaching and research. He served for sixteen years (1951–67) as Secretary of the London Mathematical Society, seeing it through a period of rapid expansion, culminating in its receiving a Royal Charter in 1965. His unique services to the Society were recognized by his election as President in 1967. He was also a regular attender at the annual British Mathematical Colloquium, probably participating in all its meetings for the first 25 years. He was similarly conscientious at the International Mathematical Congresses. He was at the Oslo meeting in 1936, and I remember him at the Moscow Congress in 1966.

To all his colleagues and students in Cambridge, Todd was the archetypal bachelor, so it was a considerable surprise to hear that, on his retirement, he had married and simultaneously retreated from the mathematical world. In fact, he married the widow of a former friend whose home provided him with regular hospitality and where it was always planned he would retire. In the circumstances, marriage was the natural solution, but the new lifestyle meant that his former colleagues heard little of him in subsequent years. Perhaps he felt he had devoted enough of his life to mathematics. Few were aware that, sadly, his wife died in 1981.

Mathematical work: classical geometry

Geometry has always oscillated between the synthetic and the algebraic approach. Diagrams, pictures and mental visions are the heart of the synthetic method, where intrinsic geometric concepts and constructs are the only tools used. By contrast, ever since the time of Descartes, algebraic formulae and manipulation have provided a mechanical alternative, technically powerful but lacking in insight. As with all fundamental dichotomies, the reality is much more complicated than antagonists allow. Geometrical argument can become lost in its own intricacies, and elegance in algebra can be cultivated. Moreover, most new developments involve an appropriate combination of geometric and algebraic ideas, notation and techniques.

These general remarks are highly pertinent to all aspects of Todd's work. His main interests can be summarized under three general headings: (i) invariant theory, (ii) group theory, (iii) canonical systems. In each case there is a major algebraic component but, as represented in Todd's work, the essential insight, interest and emphasis was on the geometric meaning behind the formulae. Todd was a superb technician and manipulator of formulae, but he also brought to bear a keen appreciation of the underlying geometry. This provided the impetus to the algebra, and helped to guide it in productive directions.

Invariant theory flourished in the nineteenth century in the hands of mathematicians such as Gordan, Cayley, Sylvester and Salmon. The aim was to exhibit those expressions in the algebraic description of geometrical configurations or systems that are invariant under changes in coordinates. In principle, any geometric condition should be expressible by the vanishing of a suitable invariant. Traces and determinants of symmetric matrices provide initial examples of invariants of conics and quadrics. The general enthusiasm for invariants was halted in its tracks by the famous proof, by the young David Hilbert, that there is always a finite basis for any system of invariants. Hilbert's theorem was also the starting point for much of modern algebra.

But despite the collapse of the great industry that produced invariants, interest in the subject did not totally disappear. Todd wrote many papers on the subject, dealing mainly with binary forms and linear systems of quadrics. His technical virtuosity enabled him to correct or complete earlier work, and he always enjoyed producing unexpected results that tied the geometry to the algebra. His contributions to invariant theory can best be described as a labour of love. His love of the subject drove him on, and he did not shirk the labour, but the results are useful rather than dramatic.

Todd's work on group theory was of greater significance, and played a part in the resurgence of the subject which culminated in the final classification of all finite simple groups. An early joint paper with Coxeter [**25**] produced the Todd–Coxeter algorithm for the enumeration of cosets in a group given by generators and relations. Coxeter has generously explained that the authors appear in reverse alphabetical order because his own contribution was rather marginal and concerned with notation.

Another significant contribution to group theory was Todd's joint paper [**72**] with his research student G. C. Shephard. This paper studied groups generated by complex reflections, analogues of the well-known Coxeter groups for the real case. These complex reflection groups have attracted considerable attention in recent years, and in particular inspired Coxeter's book *Regular complex polytopes*.

Direct collaboration with Coxeter led to the paper [**71**], in which the 'Coxeter–Todd lattice' K_{12} was first described. This remarkable lattice yields what is almost certainly the closest packing of congruent balls in euclidean 12-space, with kissing number 756. It has been extensively studied as part of the increasing interest in higher-dimensional lattices and their associated groups of automorphisms.

In his paper [**57**], Todd discovered, by use of the character table, that $\mathrm{PSL}_3(4)$ is a maximal subgroup of $\mathrm{PSU}_4(9)$. This result was in advance of its time, as evidenced by more recent work on finite simple groups. It also attracted the attention of J. G. Thompson, FRS, when he first arrived in Cambridge.

Finally, there was Todd's paper [**79**] on the Mathieu group M_{24}. Conway and Sloane, in their book *Sphere packings, lattices and groups*, remark that 'There are almost as many different constructions of M_{24} as there have been mathematicians interested in that most remarkable of all finite groups'. Todd's paper was inspired by

Coxeter's geometric interpretation of the Mathieu group M_{12}, and Todd set out to do something analogous for M_{24}. He may not have totally succeeded, but he realized M_{24} as a group of projective transformations over the field of two elements, and produced what Conway referred to as 'Todd's beautiful paper'.

Mathematical work: topology

Although Todd spent most of his working life on classical projective geometry and associated group theory, it is his work on canonical systems that has had the biggest impact on the development of algebraic geometry. His contributions fall into two closely related parts. First there was his introduction of canonical systems for higher-dimensional varieties, and subsequently there were the results he obtained for the arithmetic genus in terms of the canonical systems. All of this was pioneering work of a high order, and it bridged the gap, of some 20 years, between the traditional work of the Italian school and the modern postwar developments.

To appreciate Todd's contributions, it is necessary to recall the state of algebraic geometry at the time, and then to follow through the subsequent development of the subject culminating in Hirzebruch's famous work on the Riemann–Roch theorem, as expounded in his book ⟨**3**⟩.

Intrinsic invariants of algebraic varieties, independent of any projective embedding, start with the genus g of a curve and the canonical divisor class K given by the zeros and poles of a meromorphic differential. They are related by

$$\deg K = 2g-2.$$

The canonical class has a natural extension to a non-singular variety X of any dimension d using meromorphic differential d-forms. In particular, the homology class

$$[K] \in H_{2d-2}(X)$$

is an intrinsic variant.

Getting beyond divisors to algebraic cycles of lower dimension is much less obvious, and progress in this direction was rather slow. On an algebraic surface X, Severi had introduced an invariant set of points K_0. These were defined, by using a generic pencil of curves $\{S\}$, by the formula

$$K_0(X) = D - 2K(S) - S^2,$$

where D is the set of double points of the pencil, $K(S)$ is the canonical class of a fixed member S, and S^2 is the self-intersection of S, that is, the base of the pencil. Severi showed that, up to an appropriate notion of algebraic equivalence, $K_0(X)$ is independent of the choice of the pencil $\{S\}$. In particular, he showed that the degree of $K_0(X)$ is a numerical invariant of X. In fact, this invariant had in essence been known earlier, and (up to a constant shift) is called the Zeuthen–Segre invariant.

Thus for an algebraic surface there were two numerical invariants: K^2 and K_0. In fact, as became clear later, these are actually topological invariants of X. Severi's work on K_0 was extended by B. Segre to varieties of dimension 3. Segre also introduced a canonical system of curves K_1 defined by somewhat similar methods. The extension of these ideas to define canonical classes $K_h(X)$ for all values of h $(0 \leqslant h \leqslant d)$ was clearly a natural objective, and Todd, on his return from Princeton, set out to develop the relevant theory.

In his first paper [**26**], he gave an elegant inductive definition of the canonical classes. The key ingredients are the Jacobians of linear systems and a generalized adjunction formula. If X is embedded in a projective space P_N, then we can consider the linear system (of dimension $h+1$) cut out by all hyperplanes through a fixed generic P_{N-h-2}. The Jacobian J_h of this linear system is the locus of the singular points, given by the tangential hyperplanes of the system. The adjunction formula relates the canonical classes of X and of a hyperplane section S:

$$K_h(S) = S[K_{h+1}(S) + K_{h+1}(X)],$$

where the right-hand side represents the intersection with S.

With this understood, Todd's inductive formula is

$$J_h = K_h[(1+S)^{h+2}], \tag{1}$$

where K_h is defined here by linearity, so that explicitly

$$K_h[(1+S)^{h+2}] = \sum_{i=0}^{h+2} \binom{h+2}{i} K_h(S^i)$$

and, by convention, $K_h(1) = K_h(X)$.

Since this definition of $K_h(X)$ involves the choice of linear system $\{S\}$, it is necessary to prove independence of this choice. Todd's original argument in [**26**] was, as he recognized, incomplete in that it assumed that algebraic equivalence on X corresponded to algebraic equivalence of the intersections with S. Although true in certain dimensions, its general validity is doubtful, especially since the corresponding statement for homology is definitely false.

Meanwhile, and quite independently, Eger published an announcement ⟨**1**⟩ with an outline of an alternative approach. Instead of using a linear system of dimension $h+1$, Eger used $h+1$ different pencils. Although the geometrical content is essentially similar, there are technical advantages in Eger's approach. Todd recognized this and, using Eger's starting point, he was able to present in [**34**] a complete and rigorous treatment. Eger's own detailed account, delayed no doubt because of the war, did not appear until 1943. Todd and Eger were meanwhile in correspondence with each other, and fully acknowledged their respective contributions. An account of these developments is contained in the later paper [**73**] by Todd.

Successive use of the adjunction formula in (1) implies that $J_h(X)$ can be expressed in terms of appropriate intersections of $K_j(X)$ and S^i. Moreover, these equations are invertible, so we can write an explicit formula for the canonical classes in terms of the Jacobians:

$$K_h(X) = \sum_{i=0}^{d-h} (-1)^i \binom{h+1+i}{i} J_h(S^i). \tag{2}$$

These canonical classes have been called the Todd–Eger classes, but the terminology has been overtaken by events, since they have now been replaced by the more general Chern classes. With the rapid development of topological ideas, characteristic classes of vector bundles became the standard tool of geometers. For complex vector bundles these were introduced by Chern. It was soon recognized that, when applied to the tangent vector bundle of a complex algebraic variety, they are a cohomological version of the Todd–Eger classes. More precisely, the homology class of $K_h(X)$ is dual to $(-1)^{d-h} c_{d-h}(X)$, where c_j is the jth Chern class. The verification of this identification is a simple matter, and the following account can be found in Nakano ⟨**5**⟩.

From a modern viewpoint, the Jacobians can be described as follows. Let E be the $(d+1)$-dimensional vector bundle that lifts the projective tangent bundle of X embedded in P_N. Then J_h is dual to $c_{d-h}(E^*)$. On the other hand, we have an exact sequence of bundles

$$0 \longrightarrow 1 \longrightarrow E(1) \longrightarrow T \longrightarrow 0,$$

where 1 is the trivial line bundle, $E(1) = E \otimes H$ with H the hyperplane line bundle, and T is the tangent vector bundle of X. Thus the Chern classes of T are the same as those of $E(1)$. If, as usual, one writes Chern classes as elementary symmetric functions of two-dimensional classes

$$c(E) = 1 + \gamma_1 + \cdots + \gamma_d = \prod_{j=1}^{d+1} (1 + y_j), \tag{3}$$

then

$$c(E(1)) = \prod_{j=1}^{d+1} (1 + x + y_j),$$

where $x = c_1(H)$ is dual to the hyperplane section S. Thus the total Chern class of X, that is, of T, is

$$1 + c_1 + \cdots + c_d = \sum_{i=0}^{d} (1+x)^{d+1-i} \gamma_i.$$

Noting that $\gamma_i = (-1)^i c_i(E^*)$ is dual to $(-1)^i J_{d-i}$ and $K_h(X)$ is dual to $(-1)^{d-h} c_{d-h}(X)$, equating terms of cohomological dimension $2(d-h)$ in (3) we recover (2).

Although the above calculations are given in cohomological terms, they essentially take place in a grassmannian, and so there is little difference between the topological and algebraic versions.

This work by Todd and Eger was a major step in the development of modern algebraic geometry. Whereas most earlier work in this direction was restricted to low dimensions, the Todd–Eger classes dealt with the most general case. Although the subsequent development of Chern classes, in a modern topological context, in due course superseded the work of Todd and Eger, the essential ingredients remain the same. Given the knowledge and techniques of the time, the Todd–Eger definitions of canonical classes are the natural ones and, as the formula above shows, are easily recognizable in modern notation.

Already in his first paper [**26**] on canonical classes, Todd appreciated their relevance for producing numerical invariants by taking appropriate intersections. These invariants we now refer to as Chern numbers, and they are given by all the monomials in the Chern classes of total dimension $2d$. Thus for $d = 2$ we have c_1^2 and c_2. In general, the Chern numbers are labelled by the partitions of d, and the list grows rapidly with d.

Just a few months after [**26**], Todd produced [**27**], which set out to derive a formula for the arithmetic genus in terms of Chern numbers. In fact, there are several different definitions of the arithmetic genus, and part of the problem consists in reconciling these definitions. With the introduction of sheaf theory methods in the 1950s, the relation between the various definitions was clarified (see Kodaira and Spencer ⟨**4**⟩) and, for a modern reader, it is perhaps easier to explain the story in modern notation.

If L is a holomorphic line bundle over X, and H is the line bundle of a projective embedding, then the space of sections $H^0(X, L \otimes H^n)$ was known, for large n, to be a polynomial in n of degree d. This followed from Hilbert's work on ideals and related algebra. The coefficient of the constant term was called the 'virtual dimension' of L, and was known to be independent of H. Note that in the modern theory the virtual dimension is just the sheaf theory Euler characteristic, because the higher cohomology groups $H^q(X, L \otimes H^n)$ vanish for large n (and $q \geqslant 1$). Taking the two cases $L = 1$ and $L = K$ then gives two numerical invariants of X. Up to the sign $(-1)^d$, these were essentially two of the definitions of the arithmetic genus (except that the classical definitions used projective dimensions and so differed by 1).

Severi had established the equivalence of these two definitions of the arithmetic genus for $d = 3$. Todd extended these results to all dimensions, and simultaneously showed that the arithmetic genus is given by a universal formula in terms of Chern numbers.

Todd's arguments rest on earlier work of Severi claiming that the first definition of the arithmetic genus (taking $L = 1$) can be expressed in terms of 'elementary projective characters'. These are the intersection numbers obtained from Jacobians and linear sections. As explained above, these are essentially equivalent to the intersection numbers obtained from the Chern classes and the class $x = c_1(H)$ of the hyperplane bundle.

If one assumes Severi's assertion, then Todd's task was to show that the expression for the arithmetic genus does not involve x (so verifying, in particular, that it is an intrinsic invariant independent of the embedding). Since one is looking for a universal formula, it is enough to check things for a 'universal family' of varieties, that is, a family depending on parameters whose elementary projective characters are linearly independent. Such a family is given by complete intersections of d hypersurfaces of degrees $n_1, n_2, \ldots, n_d$ in P_{2d}. By regarding the degrees n_i as parameters, it is easy to see that the elementary projective characters form a linear basis for the symmetric polynomials in the n_i of degree $\leqslant d$.

Todd calculated the Chern classes of complete intersections by repeated use of the adjunction formula. This reduced the problem to an explicit calculation with symmetric functions. He solved it by complicated, if elementary, formal arguments. In appropriate variables, he had to show that the highest degree terms of a certain symmetric polynomial determine the lower degrees. Interestingly, these calculations involved the Bernoulli numbers, which subsequently emerged as a natural output from Hirzebruch's formalism.

Todd's procedure produced, in principle (and subject to Severi's assertion), a method for calculating the polynomial $T_d(c_1, \ldots, c_d)$ in the Chern classes c_i which give the arithmetic genus. The algebra involved consists essentially of inverting an $N \times N$ matrix, where N is the number of partitions of d. For $d \leqslant 6$, Todd made these calculations explicitly, thus identifying $T_1, \ldots, T_6$. With these partial results, Todd also verified (for $d \leqslant 6$) the multiplicative property of the arithmetic genus, and conjectured that it should hold more generally.

Todd's calculations were all laboriously done by hand. It is interesting to speculate how much further he might have got had powerful computers been available in his time. Clearly, the evidence for the multiplicative property would have become much stronger, and the structure of the Todd polynomials T_d might have become more transparent. As it was, this had to wait for nearly 20 years.

The development of sheaf theory and the use of powerful cohomological methods in the 1950s led ultimately to the famous Hirzebruch–Riemann–Roch theorem ⟨**3**⟩.

This put on a firm footing all previous work; it identified easily all the definitions of the arithmetic genus and, in particular, it produced a generating function for the Todd polynomials. Expressing the Chern classes c_i as elementary symmetric functions of symbolic two-dimensional classes x_j, Herzebruch found the elegant formula

$$\sum_{j=0}^{d} T_j = \prod_{i=1}^{d}\left(\frac{x_i}{1-e^{-x_i}}\right).$$

Todd's determination of the T_j for $j \leqslant 6$ was an important piece of evidence for Hirzebruch, and the appearance of Bernoulli numbers may have helped to suggest the right generating function. As noted by Hirzebruch ⟨**3**⟩, the Todd polynomials, as formal pieces of algebra, occurred already in work by Nörlund and were called 'Bernoulli polynomials of higher order'.

A master craftsman at algebraic formulae like Todd was naturally impressed with the technical skill exhibited by Hirzebruch. In reply to a letter from Hirzebruch, he concedes: 'Incidentally, I have had to revise a long-held opinion that the Princeton School of Mathematicians despises anything in the nature of algorithmic ingenuity.'

In the hands of Hirzebruch and others, the Todd polynomials have gone on to play an important role in geometry. By attaching Todd's name firmly to the polynomials he initiated, Hirzebruch fittingly paid tribute to Todd's pioneering work.

In the postwar years, Todd attempted to follow modern developments in algebraic geometry, but his own research reverted to more classical topics. An exception was the joint paper [**77**] which we wrote on the K-theory of complex projective spaces. This collaboration arose from a formal algebraic problem that emerged out of the topology. Looking for help, I had proposed this as a challenge problem to colleagues. The problem was to find an explicit formula for the least integer m_k for which the first k coefficients of

$$(1+x/2+x^2/3+\cdots)^m$$

are all integers. Although several younger mathematicians took up the challenge, it was Todd who demonstrated his technical powers by finally producing a complete solution. If $v_p(x)$ denotes the largest power of the prime number p that divides an integer x, then Todd's formula is

$$v_p(m_k) = \begin{cases} \sup[r+v_p(r)],\ 1 \leqslant r \leqslant [(k-1)/(p-1)] & \text{if } p \geqslant k, \\ 0 \quad \text{if } p > k. \end{cases}$$

Todd had a wry sense of humour, and he took great delight in a paper [**42**] which he wrote about the peculiar properties of the symmetric group S_6. He entitled it 'The "odd" number six'.

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