BOOK REVIEWS

KURT GÖDEL COLLECTED WORKS, Volume III
(Unpublished Essays and Lectures)

Edited by Solomon Feferman et al.: 532 pp., £45.00, ISBN 0 19 507255 3
(Oxford University Press, 1995).

KURT GÖDEL: UNPUBLISHED PHILOSOPHICAL ESSAYS

Edited by Francisco A. Rodríguez-Consuegra: 235 pp., DM.88.–,
isbn 3 7643 5310 4 (Birkhäuser, 1995).

In the years 1929–40 Kurt Gödel proved most of the revolutionary results which have made him a symbol of twentieth-century thought. Then he went to Princeton and turned inwards. Up to his death in 1978, the oracle would throw out occasional statements, each of which made a deep impact. But Gödel increasingly used his hypochondria to avoid any engagement with current work in logic. Two possible exceptions were his discussions with Georg Kreisel on the interpretation of arithmetic, and his conversations with Hao Wang on philosophical foundations. Kreisel and Wang became Gödel’s leading philosophical interpreters.

Rumour said that Gödel’s unpublished papers contained important material. I think logicians and philosophers were hoping that three things in particular would emerge: first his unpublished anticipation of Paul Cohen’s proof of the independence of the axiom of choice, second his thoughts on his hero Leibniz, and third some further indication of the philosophical secret weapons that (according to hints in his conversations with Wang) enabled him to reach his best theorems. What would we pay to have such weapons!

Now the box is open, and Volume III of his Collected works contains essentially all of Gödel’s surviving unpublished papers and lectures. (Volume IV will include a selection of his correspondence.) There is one brief mention of Leibniz, and nothing on the independence of the axiom of choice. As for the secret weapons, some of the early material allows us to see Gödel preparing his lectures and letting his hair down for discussion groups. As we should have suspected all along, there are no secrets—just careful analysis and an amazingly clear head.

Later papers in this volume fill out the philosophical picture. His abiding project throughout his time at Princeton seems to have been to clarify the foundational consequences of his early work on the unprovability of consistency. The project includes six unpublished attempts to write a critique of Carnap’s view of mathematics as syntax (or perhaps many more than six if one takes into account the scratchings out and scribbled notes on these scripts; witness the photographs in the book edited by Rodríguez-Consuegra). Gödel sought the same kind of precision in philosophical arguments that we expect in mathematical ones; he came extraordinarily close, but never to his own satisfaction. The unpublished papers also throw a very welcome light
on Gödel’s notion that we perceive mathematical objects. His brief published statements on this had led people to accuse him of ‘postulating’ a ‘faculty’ of mathematical intuition. To my eye, the fuller unpublished accounts read like a painstaking and logically informed description of psychological facts familiar to every mathematical researcher. How far they answer any real question about our knowledge of mathematical truths is a moot point, but there is plenty here to discuss.

Among other things, the *Collected works* volume contains a lecture from 1949 in which Gödel explains his rotating solutions of the equations of general relativity, and how he reached them. Each item in the volume carries an introduction written by a leading expert, and there are detailed textual notes at the end of the book. The editorial standards are extremely high throughout.

Rodríguez-Consuegra’s elegantly produced book is independent of the *Collected works* volume. It gives us transcripts of versions 2 and 6 of Gödel’s commentary on Carnap (to compare with 3 and 5 in the *Collected works*), together with his Gibbs Lecture of 1951 defending a Platonist position. There is a long philosophical introduction by the editor, which aims to set both Carnap and Gödel within the logicist tradition running from Frege through Russell to Quine. I think that this is the wrong context for Gödel; one should go first to Kant, Brouwer, Hilbert, Husserl and the experiences of a working mathematician. The reader can judge.

**WILFRID HODGES**

**POLYNOMIAL INVARIANTS OF FINITE GROUPS**

(Research Notes in Mathematics 6)


Having survived several near-death experiences over the past 150 years, invariant theory approaches the end of the 20th century in strapping good health. Much of its rude energy stems from its close associations with algebraic geometry and algebraic group actions, but even in the case of finite group actions there is considerable current activity, as is witnessed by the fact that this is the second book with the same title to appear in the past 4 years, the other being [I]. And most of the concerns of the general case are present already in the finite case, which can therefore serve as an introduction to the subject needing a minimum of machinery while still affording many deep and beautiful theorems and examples, as well as many difficult open problems.

The basic set-up is easy to describe. The ingredients are a field $K$, a finite group $G$ and a finite-dimensional $K$-vector space $V$ on which $G$ acts linearly. Thus $G$ acts also on the linear dual $V^* = \sum_{i=1}^n Kx_i$ of $V$, and this action extends to the algebra $K[V] = K[x_1, \ldots, x_n]$ of polynomial functions on $V$: one has $g \cdot f(v) = f(g^{-1}v)$ for $g \in G$, $f \in K[V]$ and $v \in V$. The set of polynomials which are invariant under this action forms a subalgebra of $K[V]$, which is denoted by $K[V]^G$ and which is the principal object of study.

For example, if $G$ is the symmetric group on $n$ symbols acting naturally on $V$ by permuting the basis dual to $\{x_1, \ldots, x_n\}$, then $K[V]^G$ is the polynomial algebra on the $n$ elementary symmetric polynomials in $\{x_1, \ldots, x_n\}$. The first great period in the history of invariant theory culminated with Hilbert’s proof in 1890 that $\langle V \rangle^{\text{Sym}}$ is finitely generated when $V$ is a symmetric power of the natural representation.
Essentially the same proof works for any linearly reductive group $G$, and in particular for finite groups with order coprime to $\text{char } K$. However, not all invariant rings are finitely generated, as was first shown by Nagata in 1959.

Nagata’s example notwithstanding, $K[V]^G$ is always finitely generated when $G$ is finite. This 1926 result of E. Noether is proved in Chapter 2 of Smith’s book; Noether’s proof is non-constructive, and so a constructive proof (also due to Noether), which requires $\text{char } K > |G|$, is also included in Chapter 2. This is exploited to calculate many examples in this and the succeeding chapter.

The above is typical of the pattern of the book: development of theoretical aspects—including an account of the commutative algebra needed—is interwoven with the calculation of many interesting and beautiful examples, some motivated by topology. Theoretical highlights include the Shephard–Todd–Chevalley theorem characterising when $K[V]^G$ is a polynomial ring (in coprime characteristic), the Eagon–Hochster theorem that $K[V]^G$ is a Cohen–Macaulay ring in coprime characteristic, and a discussion of the failure of this result in the modular case (where much remains to be discovered).

How does Smith’s book compare with Benson’s volume [1]? Benson’s is the more succinct, while Smith includes many more examples; Smith includes more coverage of the modular case than does Benson; Benson goes deeper into some of the homological aspects than does Smith.

Presentation, style and layout in the book under review are all first-rate. A word of warning is necessary, however, about the sometimes rather shaky treatment of commutative algebra. (To give one example from several possible, Macaulay’s Unmixedness Theorem (6.7.5) is badly mis-stated.) Despite this caveat, I can strongly recommend this book to anyone looking for a clear introduction to the subject, especially to a reader who enjoys the study of concrete examples. I myself read it in the early summer of 1996 while sitting on a terrace overlooking Villeneuve-sur-Lot, fortified by the occasional glass of dark red Cahors—I advise similar preparation to anyone faced with the rigours of dealing with Research Assessment Exercise 2000.

Reference


KEN A. BROWN

RIGHT-ORDERED GROUPS
(Siberian School of Algebra and Logic)


This book provides a good up-to-date account of right-ordered groups, and includes a number of results on groups admitting other order relations that are needed in the discussion of the central theme. The class of right-orderable groups coincides with the class of subgroups of order-preserving permutations of ordered sets. The study of this class of groups is well developed, with many interesting open problems to occupy our attention.
The book has eight chapters. Basic notions of partially-ordered groups, totally-ordered groups and lattice-ordered groups, including a number of generic examples, are presented in the first chapter. This is followed, in the second chapter, by the central role played by convex subgroups in ordered and right-ordered groups. Abstract group theoretical characterization of such groups together with conditions for extensions of partial orders are given in Chapter 3. The next chapter deals with groups of order-preserving permutations of totally-ordered sets, and includes detailed constructions of simple ordered groups due to Higman, Chehata and Dlab. Further properties of right-orderable groups admitting a finite number of orders and other specialized results are presented in Chapter 5, followed by a discussion of orderability of free products of ordered groups. The last two chapters deal with quasivarieties and semilinearly ordered groups. Recent results of Bergman and Tararin on right-ordered groups that are not locally indicable are presented here. Also included is Tararin’s proof that right-ordered groups which are periodic extensions of radical groups are locally indicable.

On the whole, the book is well written but contains numerous errors. Most of these are minor misprints, but there are a few places where the reader may have to refer to the original papers. The authors have done an excellent job of giving references to all the recent results. This book is indispensable for anyone wishing to work in this area.

A. H. RHEMTULLA

SPINORS IN HILBERT SPACE
(Cambridge Tracts in Mathematics 114)

(Cambridge University Press, 1994).

Clifford algebras, which enter significantly into such areas of mathematics as quadratic forms, orthogonal groups and $K$-theory, are also influential in quantum statistical mechanics and quantum field theory in the guise of their $C^*$-algebra variants.

This attractive book gives a stylish account of complex Clifford $C^*$-algebras and distinguished aspects of their representations and automorphism groups.

A gentle foundational opening chapter provides a clear treatment of the basic theory and constructions of the complex Clifford algebra of a real inner product space, and introduces the complex Clifford $C^*$-algebra, $C[V]$, of a real Hilbert space $V$. (Real Clifford algebras are not discussed.) The algebraic simplicity of $C[V]$ is established for infinite dimensional $V$ and its double commutant in the GNS representation induced by the tracial state is termed the Clifford von Neumann algebra, $A[V]$, of $V$. For $V$ infinite dimensional and separable, $C[V]$ can be realised as the Fermion algebra and $A[V]$ as the Type $\text{II}_1$ hyperfinite factor. The important class of (irreducible) Fock representations is treated in depth and with revealing detail in Chapters 2 and 3. Unitary implementation of Bogolubov automorphisms in Fock representations is characterised, as is equivalence of Fock representations. The final part of the book discusses spin groups and establishes valuable results on the innerness of Bogolubov automorphisms. Each chapter concludes with useful historical remarks and helpful guidance on alternative approaches.
There are surprisingly few prerequisites. Apart from a swallowable concession to operator algebras, substantially redeemed in the Appendix, the book is self-contained.

The authors have succeeded in achieving a declared objective of producing an interesting and substantial treatment of their subject at a level suitable for graduate students of mathematics and theoretical physics. The exposition is very good indeed. As well as being accessible to its intended audience, the book will be a valuable reference for everyone with an interest in Clifford C*-algebras.

L. J. Bunce

FUNCTION SPACES AND POTENTIAL THEORY
(Grundlehren der mathematischen Wissenschaften 314)


This book is one of a number of excellent recent accounts (Heinonen, Kilpeläinen and Martio [4], Maz'ya [6], Meyer [7], Ziemer [14]) of various different aspects of the theory of function spaces and potential theory. These books expose minimally-overlapping developments that have flowed from the ideas presented a generation ago in the enormously influential books of Carleson [1], Stein [12], Morrey [8], Federer [2], Peetre [9], Triebel [13] and Hörmander [5], and further developed in the interim in other books of Triebel, Stein, Tarkhanov, Rubio de Francia and Garcia-Cuerva, and among the schools of hard harmonic analysis, quasi-regular mappings, Cauchy integral estimates and PDEs. They also make available in book form the results of the classic papers of the same period, such as the work of Serrin, John, Nirenberg, Fefferman, Fuglede, Moser, Meyers, Havin, Jones and Wolff, besides those mentioned above, and many others. No library should be without them.

Do you need to buy this particular one? If you are interested in capacities and thinness concepts associated with Sobolev spaces, or traces (that is, 'restrictions') of Sobolev functions, or in $L^p$-norm approximation of solutions to elliptic partial differential equations, then it is an essential reference. One, not quite accurate, way to describe the book is as a leisurely proof of the quite comprehensive results now available about $L^p$ approximation by solutions of constant-coefficient elliptic equations (including approximation by holomorphic functions in one variable). Another way to describe it is as an introduction to potentials and capacities associated to Sobolev, Besov and Triebel–Lizorkin spaces, with applications to trace problems and to approximation theory.

The account assumes familiarity with the material of a good graduate course in real analysis, and the elements of functional analysis and complex analysis; for instance, the contents of Rudin’s books [10, 11]. Also, the reader needs to be familiar with much of Stein’s classic [12] on singular integrals. Also assumed at various points are some items from Ziemer’s book [14], the full strength of the Fefferman–Stein maximal theorem [3], the elements of Suslin sets and Choquet’s capacitability theorem. In view of the quite central role of the Fefferman–Stein maximal theorem in the atomic-decomposition theory, and hence in the whole book, one feels that it would have been more appropriate to have included a proof, instead of sending the reader to the American Journal of Mathematics. In one or two other cases, such as the
fact that truncation works on $W^{1,p}$, it would have taken relatively little space to include a proof.

One regularly finds people who try to prove mathematical theorems with one hand tied behind their backs, metaphorically speaking. This can be regarded as a kind of virtuosity, but is probably just foolish. In function space problems, there is a tendency among old hands to try to work around the use of the Besov spaces $B^p_{p,q}$ and the Triebel–Lizorkin spaces $F^p_{p,q}$ (also known as atomic-decomposition spaces), if possible. In this book, several major results are proven twice. Typically, the non-atomic proof is longer and gives a less comprehensive or less precise result. This convinces me that if you are interested in Sobolev spaces, and you have not digested the elements of the atomic-decomposition technology, then it is high time you did. A major feature of the book is the exposition of penetrating atomic-based results of Netrusov, appearing here for the first time.

In general, this is a very well-written and meticulously proof-read account. The authors achieve a very high standard of expository writing. Proofs are broken down to expose the key ideas. Major results are proved by a deliberate process of gradual generalisation. This is usually helpful, although in one case (Theorem (3.3.3)) the result is a proof that is perhaps too scattered; this proof is spread over Chapters 3 and 4, and the details of the most general case are not actually given. Perhaps the organisation here is due to the distribution of tasks between the two authors, or to a relatively late decision to include Section 4.8 (due to Netrusov). The layout, careful cross-referencing and excellent indices make it very easy to find things. The book provides encyclopaedic coverage of its area, has substantial new results, and will be an essential reference for some time to come. It is a very fine book.

References


A. G. O’FARRELL
GEOMETRY OF HARMONIC MAPS
(Progress in Nonlinear Differential Equations and Their Applications 23)

By Yuanlong Xin: 241 pp., DM 138.–, ISBN 0 8176 3820 2
(Birkhäuser, 1996).

A Riemannian manifold is a smooth (that is, infinitely differentiable) manifold on which we can measure the length of tangent vectors and hence of curves; a smooth map $\phi: M \to N$ between two Riemannian manifolds is called harmonic if it is an extremal, in the sense of the calculus of variations, of a certain natural ‘energy integral’ which generalizes the Dirichlet integral and measures the total stretching of the map in an $L^2$ sense. Harmonic maps include geodesics (paths of locally shortest distance), minimal surfaces (soap films), harmonic functions and holomorphic (complex analytic) maps between suitable complex manifolds. In general, harmonic maps satisfy a semi-linear system of second-order partial differential equations, the non-linearity caused by the curvature of the target manifold. Since the celebrated paper [3] of Eells and Sampson, the subject has developed tremendously with over 2000 papers [1]. One reason for this is the large number of applications, for example the strong rigidity theorem of Siu that the homotopy class of a compact Kähler manifold of ‘strongly’ negative curvature determines its type as a complex manifold.

A graduate student wishing to learn the subject faces a difficult choice of textbook. Because the subject is now so large, any textbook must necessarily contain a selection of material. There have been about 10 textbooks, excluding conference proceedings. These include the book of Eells and Ratto [2], which concentrates on constructing symmetric harmonic maps and minimal immersions by reducing the harmonic equation to an ordinary differential equation (see the review in this Bulletin 27 (1995) 406); that of Urakawa [7], which gives an introduction to the relevant part of the calculus of variations and Morse theory, and a rather slow introduction to harmonic maps including twistorial methods; two books by Toth [5, 6] (reviewed in this Bulletin 17 (1985) 617 and 23 (1991) 509); and several books by Jost, for example [4], which concentrates on more analytical aspects (see this Bulletin 17 (1985) 294). The book under review is less specialised than [2, 5, 6] and covers more ground than [7]. The choice of topics is rather biased towards the author’s work in the subject, which is frequently quite technical—that is, of the nature of pushing known techniques to their limits to obtain rather difficult-to-state generalizations of known results. However, on the way, the author introduces many useful techniques. Basic facts from Riemannian geometry are assumed, but otherwise, the book is self-contained.

The topics covered are as follows. I. Introduction. Basic facts on connections on vector bundles and harmonic maps, including Bochner techniques and second variation of the energy. II. Conservation law. By Noether’s famous theorem, there is a conservation law for harmonic maps—the form of this was found by Baird and Eells. III. Harmonic maps and Gauss maps. The (generalized) Gauss map of a submanifold immersed with constant mean curvature in a Euclidean space, or of a minimally immersed submanifold of a sphere, is a harmonic map; a novel feature of the treatment is an examination of the Gauss map of a space-like hypersurface in Minkowski space giving a Bernstein-type theorem. IV. Harmonic maps and holomorphic maps. Harmonicity of holomorphic maps and some partial converses, leading to the strong rigidity theorem of Siu. V. Existence, nonexistence and regularity. The theory of singular sets of Schoen and Uhlenbeck together with some
related results of the author. VI. Equivariant harmonic maps. This takes some aspects of [2] further, in particular reducing the harmonic equation to a partial differential equation, and includes a proof of the author’s theorem that the odd classes of the top homotopy groups $\pi_{2m+1}(S^{2m+1})$ of odd-dimensional spheres can be represented by harmonic maps.

The book is nicely written except for some minor errors of English and one or two unclear points. Occasionally, the citations in the text do not correspond to the bibliography, for example it is not clear what reference [Mi] on page 79 is supposed to be. The book should certainly be in the library of every institution where there is some interest in harmonic maps, and in the personal collection of anyone working in the geometric aspects of harmonic maps. On omitting some of the more technical sections, it could form the basis of a graduate course in the geometric aspects of harmonic maps, and is, in the reviewer’s opinion, the most suitable book so far written for such a course.

References


J. C. Wood

QUASI-PROJECTIVE MODULI FOR POLARIZED MANIFOLDS
(Ergebnisse der Mathematik und ihrer Grenzgebiete (3) 30)

By Eckart Viehweg: 320 pp., DM.158.--., ISBN 3 540 59255 5
(Springer, 1995).

The classification of complex projective varieties of a given kind usually resolves itself into two parts: finding discrete invariants and the description of a coarse moduli space naturally parametrizing varieties with given discrete invariants. The most classical example of this is given by compact Riemann surfaces, where the only discrete invariant is the genus. Riemann surfaces of a given genus $g$ are then parametrized by a space $\mathcal{M}_g$. Thus $\mathcal{M}_1$ is a single point, $\mathcal{M}_1$ is the affine line (which parametrizes isomorphism classes of elliptic curves via the $j$-invariant), and $\mathcal{M}_g$ for $g > 1$ has dimension $3g - 3$ (as shown by Riemann). Mumford applied his Geometric Invariant Theory (GIT) in this case to show that $\mathcal{M}_g$ has the structure of a quasi-projective variety (that is, an open subset of a projective variety) and to produce natural ample line bundles on $\mathcal{M}_g$.

A polarization on a projective variety $X$ is given by an ample invertible sheaf on $X$. A smooth projective variety $X$ of dimension $n$ has a canonical invertible sheaf $\mathcal{O}_X$, whose sections are the holomorphic $n$-forms. We say that $X$ is canonically polarized.
if \( \omega_X \) is ample—in the above case of compact Riemann surfaces, this is equivalent to the condition \( g > 1 \). The monograph under review concerns itself with the existence of quasi-projective moduli spaces for other classes of polarized manifolds.

In higher dimensions \( n \), the discrete invariants one needs to specify are encoded in the Hilbert polynomial \( h \). For canonically polarized smooth projective varieties, this is a polynomial of degree \( n \) in one variable with the property that, for \( m > 1 \), \( h(m) \) is the dimension of the space of sections of \( \omega_X^m \). One of the main results described in this book says that the coarse moduli space of canonically polarized complex projective manifolds of dimension \( n \) and with given Hilbert polynomial \( h \) has the structure of a quasi-projective scheme. For smooth complex projective surfaces (or even surfaces with rational double point singularities), this was known from work of Gieseker, whose proof followed a similar path to that of Mumford for curves. The idea is to use the sections of \( \omega^r_X \) for some \( r \) (independent of \( X \)) to embed all \( X \) with the given Hilbert polynomial \( h \) into a fixed projective space \( \mathbb{P}^{N-1} \). The subvarieties of \( \mathbb{P}^{N-1} \) obtained in this way are parametrized by a quasi-projective Hilbert scheme \( H \), where two closed points on \( H \) represent the same variety if they differ by an element of \( G = \text{SL}(h(r), \mathbb{C}) \).

Since \( G \) is a reductive group, one can use GIT to produce a quasi-projective quotient if a certain Hilbert–Mumford criterion (stated in terms of the action of one-parameter subgroups) is satisfied. In dimensions greater than two, this criterion no longer works. It is here that the second main theme of the book appears—the positivity of direct images of certain sheaves on flat families.

This idea appeared a number of years ago in the works of Fujita, Kawamata, Kollár, Viehweg and others, in the study of Iitaka’s conjectures concerning Kodaira dimensions of fibre spaces. In particular, if \( g: X \to Y \) is a smooth family of canonically polarized complex projective manifolds of dimension \( n \) (with \( Y \) quasi-projective), the direct image sheaves \( g_* (\omega_{X/Y}) \) (when non-zero) have a property known as weak positivity—here \( \omega_{X/Y} \) denotes the relative canonical sheaf of the family. Viehweg also demonstrates weak positivity for the direct images of certain related sheaves. Applying these results to the universal family over the quasi-projective Hilbert scheme \( H \) (and to the family over a partial compactification), Viehweg is able to prove quasi-projectivity of the coarse moduli space \( H/G \) by means of a new criterion he produces to replace that of Hilbert–Mumford.

By similar methods, Viehweg constructs a quasi-projective coarse moduli space for smooth polarized projective varieties (that is, pairs \((X, \mathcal{L})\) with \( \mathcal{L} \) an ample invertible sheaf on \( X \)) for which the canonical sheaf \( \omega_X \) is semi-ample (namely, for some \( r > 0 \), the sections of \( \omega^r_X \) have no simultaneous zero). Here one needs to specify a Hilbert polynomial \( h \) in two variables, where \( h(p, q) > 0 \) gives the dimension of the space of sections of \( \mathcal{L}^p \otimes \omega^q \). The arguments are more complicated because one has to consider ‘double polarizations’, but the result itself is of considerable interest. In particular, when \( \omega_X \) is trivial, we recover previously known results that there are quasi-projective coarse moduli spaces for polarized K3 surfaces (proved before via periods) or polarized abelian varieties, and the new result that the same is true for Calabi–Yau varieties of any dimension; in all these cases, the moduli space will in fact be a quasi-projective variety.

Having described the main methods, I should add that the book attempts to be as encyclopaedic as possible. Thus other approaches to constructing the quotient of the Hilbert scheme are described, although they do not yield the very general results outlined above. The problems encountered if one wishes to allow certain singularities on the varieties are discussed in some detail, and the question of non-zero
characteristic is also touched upon. Accordingly, the theory is developed in a form which may be useful beyond the proofs of the main theorems. This makes the book even more valuable as a work of reference, but does unfortunately make it more difficult to read. The author tries to alleviate this with a Leitfaden suggesting a simplified route through the text, and he also includes helpful prefaces to each chapter. It is nonetheless still quite a hard book to get into—the survey article by the author and Esnault in the Proceedings of the Tokyo ICM-90 Satellite Conference is, for instance, a rather easier initial read. The book under review is, however, an impressive and very thorough treatise on a significant topic, and represents an important contribution to scholarship.

Reference


P. M. H. Wilson

FUNDAMENTAL GROUPS OF COMPACT KÄHLER MANIFOLDS
(Mathematical Surveys and Monographs 44)


Recall that a complex manifold \((X,J)\) is Kähler if it admits a riemannian metric \(g\) such that its associated Kähler 2-form \(\omega(\cdot, \cdot) := g(\cdot, J\cdot)\) is \(d\)-closed. Projective manifolds and their small deformations are examples of compact Kähler manifolds; conversely, all known compact Kähler manifolds are deformations of—and thus diffeomorphic to—some projective manifold. Hodge theory and its generalisations give various restrictions on the topology of compact Kähler manifolds.

This book, based on talks at the 1995 Borel Seminar, surveys the main known obstructions for a (finitely presented) group \(\Gamma\) to be Kähler (that is, of the form \(\pi_1(X)\) for some compact Kähler \(X\)), and also describes the construction of some non-trivial Kähler groups. It may serve as a concise, but clear and serious introduction to various ideas and techniques (group theory, \(L^2\)-cohomology, harmonic maps, rational homotopy theory, Hodge and Morse theories) used in the study of this presently very active topic.

Chapter 1 is introductory, and surveys among other things the construction of closed manifolds with partial Kähler structure (symplectic, almost-complex or even complex and symplectic) but arbitrary finitely presented fundamental groups. The special case of complex compact surfaces is also studied.

Chapter 2 establishes Siu’s theorem that a surjective holomorphic map \(f: X \to C\) with \(X\) compact Kähler and \(C\) a (compact complex) curve of genus \(g \geq 2\) exists if (and only if) there is a surjective morphism of groups \(g: \pi_1(X) \to \pi_1(C)\). Catanese’s cohomological proof is given (Siu’s is exhibited in Chapter 6).

Chapter 3 expounds Sullivan’s rational (or real) homotopy theory, and its application to the formality of compact Kähler manifolds \(X\) (after Deligne et al., 1975), based on the \(dd^c\)-Lemma (incorrectly stated in Appendix B7, p. 131: the assumption \(d^*\alpha = 0\) has been forgotten!). If \(\pi_1(X) = 0\), formality means that \(\pi_\ast(X) \otimes \mathbb{Q}\)
can be computed from the ring $H_q(X, \mathbb{Q})$; for $\pi_1(X)$ itself, formality (essentially) means that the quotients of the lower central series of $\pi_1(X)$ (modulo torsion) are determined by the natural map $\bigwedge^2 H^4(X, \mathbb{Q}) \to H^2(X, \mathbb{Q})$. Among other things, the existence of a functorial mixed Hodge structure on the Mal'cev algebra of $\pi_1(X)$ is discussed. It gives further restrictions on $\pi_1(X)$, beyond formality.

Chapter 4 applies Gromov's approach, as developed by Arapura, Bressler and Ramachandran, to the result that a Kähler group $\Gamma$ has 0 or 1 end, and cannot thus split as an amalgamated product over a finite group. It rests on $L_2$-Hodge theory on suitable étale covers of compact Kähler manifolds. Detailed and elementary proofs are given.

Chapter 5 sketches the proof of the Eells–Sampson theorem that any continuous map $f: M \to N$ is homotopic to a harmonic map $f_0: M \to N$ when $M, N$ are closed riemannian manifolds with $N$ of non-positive (not non-negative, as stated in 5.8) sectional curvature. The (crucial in Chapters 6 and 7) extension to the reductive equivariant case is also discussed (after Corlette, Donaldson and Labourie).

Chapter 6 proves very clearly the Siu–Sampson theorem that any $f_0$ as above is pluriharmonic when $M$ is Kähler and $N$ of hermitian non-positive sectional curvature. Applications when $N$ is locally symmetric are given (Carlson–Toledo, Siu, Sampson). For example: no lattice in $\text{SO}(1, n)$ is Kähler if $n > 2$. The analogous theory of harmonic maps to trees (or, more generally, Bruhat–Tits buildings (Gromov–Schoen) and non-positively curved metric spaces (Korevaar–Schoen)) is also discussed. It is explained how to use it to show, for example, that the Higman 4-group is not Kähler.

Chapter 7 expounds non-abelian Hodge theory of Betti moduli spaces of real Higgs bundles over a compact Kähler manifold $X$, and their natural $U(1)$-action. It establishes Simpson’s results that a fixed point of this action arises from a real variation of Hodge structure, and how this implies (together with rigidity results) that if some lattice $\Gamma$ in a simple Lie group $G$ is Kähler, then $G$ is of Hodge type (that is, has a compact Cartan subgroup). Among other things (Yang–Mills equations, hyperkahler structure, for instance) it also explains the twisted version of the formality of the De Rham complex with coefficients in a harmonic flat bundle, and its application (Goldman–Milsson) that singularities of the representation variety are at worst quadratic at reductive points.

Chapter 8 gives some recent constructions of non-trivial Kähler groups (lattices in Heisenberg groups, non-residually finite groups). The computations of the fundamental groups are based on Lefschetz-type theorems.

This book, presently the only one dealing with this subject, should be of interest to geometers (algebraic, complex and differential), and be accessible to graduate students interested in these topics as well.

F. Campana
The origins of quantum groups lie in the work of the Leningrad school on ‘quantum integrable systems’. They found that many of the properties of such a system were encoded in a ‘quantum $R$-matrix’ which, in its simplest form, is a linear map $R: V \otimes V \rightarrow V \otimes V$ (where $V$ is a finite-dimensional vector space). The ‘integrability’ of the system is implied by the fact that $R$ is a solution of the quantum Yang–Baxter equation (QYBE):

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}.$$  

Here, $R_{ij} = R \otimes 1$ identity $\in \text{End}(V \otimes V \otimes V)$, etc. By the end of the 1970s, large catalogues of solutions of the QYBE had been compiled, but their proper mathematical setting was lacking. Finally, around 1985, V. Drinfel’d and M. Jimbo realised that the missing framework was provided by the theory of Hopf algebras; Drinfel’d called the Hopf algebras which arise in this way ‘quantum groups’.

We recall that a Hopf algebra over a field (or even a commutative ring) $k$ is an algebra $A$ over $k$ equipped with some extra structure, in particular an algebra homomorphism $\Delta_A: A \rightarrow A \otimes A$ called the comultiplication (because the multiplication of $A$ can be regarded as a map $\mu_A: A \otimes A \rightarrow A$). The crucial fact about the comultiplication is that it allows one to make the tensor product of two representations of $A$ into another representation of $A$: if $\rho_1: A \rightarrow \text{End}(V_1)$ and $\rho_2: A \rightarrow \text{End}(V_2)$ are representations of $A$ on vector spaces $V_1$ and $V_2$, then $(\rho_1 \otimes \rho_2) \circ \Delta_A$ is a representation of $A$ on $V_1 \otimes V_2$. Another important feature of the definition of a Hopf algebra is that it is ‘self-dual’: if $A$ is a Hopf algebra, so is the vector space dual $A^*$, by taking the multiplication and comultiplication maps of $A^*$ to be the dual maps of the comultiplication and multiplication maps of $A$, respectively. (If $A$ is infinite-dimensional, this does not quite work because $\mu_A^*$ will not generally take values in the proper subspace $A^* \otimes A^*$ of $(A \otimes A)^*$, but there are well-understood ways of circumventing this difficulty.)

The most familiar examples of Hopf algebras are those associated to groups and Lie algebras. If $G$ is an affine algebraic group, for example, such as the group $\text{SL}_n(\mathbb{C})$ of $n \times n$ complex matrices of determinant one, then the algebra $\mathcal{F}(G)$ of regular functions on $G$ is a Hopf algebra: multiplication in $\mathcal{F}(G)$ is pointwise, and the group multiplication $G \times G \rightarrow G$ induces a map $\mathcal{F}(G) \rightarrow \mathcal{F}(G \times G) = \mathcal{F}(G) \otimes \mathcal{F}(G)$ which serves as the comultiplication. On the other hand, to any Lie algebra $\mathfrak{g}$ one associates its universal enveloping algebra $U(\mathfrak{g})$. This is the associative algebra generated by the elements of $\mathfrak{g}$ with a multiplicative relation $XY - YX = Z$ for all $X, Y, Z \in \mathfrak{g}$ such that the Lie bracket $[X, Y] = Z$ (and such that $U(\mathfrak{g})$ is universal with respect to this relation).
property). The universal enveloping algebra is a Hopf algebra with comultiplication given by \( \Delta(X) = X \otimes 1 + 1 \otimes X \) for all \( X \in \mathfrak{g} \). Note that \( U(\mathfrak{g}) \) is cocommutative, in the sense that the image of \( \Delta \) is contained in the symmetric part of \( U(\mathfrak{g}) \otimes U(\mathfrak{g}) \). These two constructions are actually closely related: if \( G \) is semisimple and the ground field \( k \) is algebraically closed and of characteristic zero, and if \( \mathfrak{g} \) is the Lie algebra of \( G \), then \( \mathcal{F}(G) \) is the Hopf dual of \( U(\mathfrak{g}) \) (the dual being defined with due care, as noted above).

Drinfel’d and Jimbo made the crucial observations that the Hopf algebras arising from quantum integrable systems are deformations of those of the form \( \mathcal{F}(G) \) or \( U(\mathfrak{g}) \). A (formal) deformation of a Hopf algebra \( A \) over \( k \) is a Hopf algebra \( A_h \) over the ring \( k[[h]] \) of formal power series in an indeterminate \( h \) such that \( A_h/hA_h \), provided with its natural Hopf algebra structure over \( k[[h]]/hk[[h]] = k \), is isomorphic to \( A \). Roughly, one thinks of \( A_h \) as a Hopf algebra ‘depending on a parameter \( h \’) which ‘tends to \( A \) as \( h \) tends to zero’.

The deformations \( \mathcal{F}_h(G) \) which arise are not commutative, and so are not algebras of functions on anything (except possibly a ‘non-commutative space’, whatever that might be). In fact, \( \mathcal{F}_h(G) \) is defined abstractly in terms of generators and relations which are themselves expressed in terms of the \( R \)-matrix of the integrable system in question. The deformations \( U_h(\mathfrak{g}) \), correspondingly, fail to be cocommutative, although only in a mild way. More precisely, it turns out that there is an invertible element \( \mathcal{R} \in U_h(\mathfrak{g}) \otimes U_h(\mathfrak{g}) \), called the ‘universal \( R \)-matrix’ of \( U_h(\mathfrak{g}) \), such that

\[
\Delta^u(X) = \mathcal{R} \Delta(X) \mathcal{R}^{-1} \quad \text{for all } X \in U_h(\mathfrak{g}),
\]

where \( \Delta^u(X) \) is the result of interchanging the factors in \( \Delta(X) \in U_h(\mathfrak{g}) \otimes U_h(\mathfrak{g}) \). The relation with the QYBE is that if \( \rho: U_h(\mathfrak{g}) \to \text{End}(V) \) is a representation of \( U_h(\mathfrak{g}) \) on a \( k[[h]] \)-module \( V \), then \( R = (\rho \otimes \rho)(\mathcal{R}) \) is a quantum \( R \)-matrix. This is so because \( \mathcal{R} \) itself satisfies the relation

\[
\mathcal{R}_{12} \mathcal{R}_{13} \mathcal{R}_{23} = \mathcal{R}_{23} \mathcal{R}_{13} \mathcal{R}_{12},
\]

where \( \mathcal{R}_{12} = \mathcal{R} \otimes 1 \in U_h(\mathfrak{g}) \otimes U_h(\mathfrak{g}) \otimes U_h(\mathfrak{g}) \), etc.

At this point, it is reasonable to ask why mathematicians (other than Hopf algebraists) should be interested in \( \mathcal{F}_h(G) \) or \( U_h(\mathfrak{g}) \). The wide interest that quantum groups have generated is unquestionably due to their unexpected connections with apparently unrelated parts of mathematics and physics. The first of these emerged at about the same time as quantum groups themselves: it was V. F. R. Jones’ construction of his new invariant of knots. Although this did not make use of quantum groups explicitly, it did employ a quantum \( R \)-matrix, in fact the one associated to the simplest non-trivial representation of \( U_h(\mathfrak{sl}_2) \) (where \( \mathfrak{sl}_2 \) is the Lie algebra of \( 2 \times 2 \) matrices of trace zero). Once the notion of quantum groups had been absorbed by knot theorists, it was not long before Jones’ construction had been generalized to other quantum groups and other representations.

A little later, G. Lusztig conjectured that the representation theory of a semisimple algebraic group \( G \) over a field of finite characteristic \( p \) should be related to that of the characteristic zero object \( U_h(\mathfrak{g}) \), if \( \mathfrak{g} \) is the Lie algebra of \( G \). To explain this in a little more detail, we take \( k = \mathbb{C} \) from now on, so that \( U_h(\mathfrak{g}) \) is an algebra over \( \mathbb{C}[[h]] \). Although the defining relations of \( U_h(\mathfrak{g}) \) involve \( h \), it turns out that they are essentially rational functions of \( q = e^h \). One can therefore define a Hopf algebra \( U_q(\mathfrak{g}) \) over the field \( \mathbb{C}(q) \) of rational functions of an indeterminate \( q \). With a little more effort, one
can even define an ‘integral’ form $U_\epsilon(q)$, a Hopf algebra over $\mathbb{Z}[q, q^{-1}]$, from which $U_\epsilon(q)$ is obtained by extending scalars. The point is that in $U_\epsilon(q)$, $q$ can be specialised to (put equal to) any non-zero complex number $\epsilon$. The resulting Hopf algebra $U_\epsilon(q)$ (over $\mathbb{C}$) turns out to be most interesting when $\epsilon$ is a root of unity. This is partly because $U_\epsilon(q)$ then has a large centre: in fact, the quotient of $U_\epsilon(q)$ by its centre is a finite-dimensional Hopf algebra $U_\epsilon^{\text{fin}}(q)$. Lusztig’s conjectures say, essentially, that if $\epsilon = e^{2\pi i/p}$ and $p$ is not too small, then $U_\epsilon^{\text{fin}}(q)$ has the same representation theory over $\mathbb{C}$ as that of $G$ in characteristic $p$. These conjectures have now largely been proved.

Finally, we have seen how the notion of a quantum group arose from the study of integrable quantum systems, via the quantum $R$-matrices associated to such systems. More recently, quantum groups have arisen in another guise, as ‘quantum symmetry groups’ of certain $1+1$ dimensional integrable quantum field theories. Roughly, one can say that each quantum particle of the theory is associated to an irreducible representation of the quantum symmetry group, and the possible interactions between the particles are reflected in the way that the corresponding representations behave under taking tensor products.

Turning now to the books under review, one is first struck by how little they have in common. Both books begin with a review of Hopf algebras (and in Kassel’s book even of modules, algebras and tensor products), but after this their contents diverge rapidly. The treatment in both books is based logically and pedagogically, rather than historically. In particular, neither book has any discussion of the physical origins of quantum groups.

Kassel’s book is divided into four parts, the first two of which are mainly devoted to $U_\epsilon(sl_2)$ and its function algebra counterpart $F(sl_2)$. The latter is ‘derived’ by first writing down the simplest conceivable ‘non-commutative’ plane $C^2$, and requiring that the natural action of $sl_2$ on $C^2$ should go over unchanged to the quantum case. (This approach is due to Yu. Kobyzev and Yu. Manin.) The definition of $U_\epsilon(sl_2)$, on the other hand, is ad hoc. The universal $R$-matrix is discussed (in Part II) only for $U_\epsilon^{\text{fin}}(sl_2)$ (and $\epsilon$ a root of unity). This approach is forced on the author because $U_\epsilon(sl_2)$ does not, strictly speaking, have a universal $R$-matrix, and $U_\epsilon^{\text{fin}}(sl_2)$ (which does) is not treated until much later in the book. Despite this peculiarity, the first two parts of Kassel’s book give a careful and attractive introduction to the subject.

Parts III and IV are written at a somewhat higher level, and treat the applications of quantum groups to knot invariants (see above) and to the Knizhnik–Zamolodchikov equation of conformal field theory (the latter application being due to Drinfel’d). The book concludes with a very nice discussion of M. Kontsevich’s universal knot invariant.

The prerequisites for Kassel’s book do not go much beyond undergraduate algebra, but the ‘mathematical sophistication’ required would, I think, test many undergraduates who may not be used to the Bourbaki style.

Kassel gives scant attention to $U_\epsilon(q)$ when $q$ is other than $sl_2$. One does not even see a definition until Chapter XVII, and then the discussion occupies less than five pages. Joseph’s book has a completely different agenda, and is clearly written for a reader already well versed in the theory of semisimple algebraic groups and Lie algebras. The treatment is dense and, as the author says in his Introduction, ‘sparing neither detail nor precision’. The aim is to study the structure and representation theory of $U_\epsilon(q)$ and $F(G)$ in maximum generality, although curiously only the case when $\epsilon$ is not a root of unity is considered. None of the applications of quantum groups to other fields is discussed. The main tools used by Joseph are the so-called
canonical bases of the finite-dimensional representations of $U_q$ introduced, independently and by quite different methods, by M. Kashiwara and G. Lusztig. Joseph’s treatment of these is self-contained, and follows the more elementary approach of Kashiwara. The book culminates in the determination of the primitive ideals of $U_q$ (the kernels of the finite-dimensional representations) and the prime ideals of $\mathcal{F}(G)$, generalising classical results to which the author himself has made very significant contributions. The use of canonical bases, and of methods from non-commutative ring theory, give this book a flavour different from that of other treatments in the literature, and lead to several new results. Anyone seriously interested in quantum groups will want to have access to this book.

ANDREW PRESSLEY

THE FLOER MEMORIAL VOLUME
(Progress in Mathematics 133)


The mathematical work of Andreas Floer was compressed into a tragically short period (1984–91) but had a resounding effect on a number of diverse fields, and opened up questions which will stimulate research for many years to come. Floer’s work lay in the rich territory between mathematical physics and geometry, involving a confluence of ideas from Hamiltonian mechanics, gauge theory, quantum field theory, global analysis, and differential and algebraic topology. We begin with Hamiltonian mechanics, the oldest of these subjects, and Floer’s own starting point. Recall that a Hamiltonian system is a dynamical system which has the shape, at least locally,

$$\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}, \quad \frac{dq_i}{dt} = -\frac{\partial H}{\partial p_i},$$

where $p_i, q_i (1 \leq i, j \leq n)$ are local coordinates and $H = H(p_i, q_i)$ is a Hamiltonian function. Classically, these equations arise when the equations of motion of a mechanical system are put into first-order form; at a more abstract level, the natural invariant setting involves a function $H$ on a symplectic manifold $(M, \omega)$, on which $p_i, q_i$ are local coordinates. One is interested in finding properties which are specific to Hamiltonian systems, as opposed to general dynamical systems. In particular, one can ask about global properties, relating to the topology of $M$. A simple example comes from considering the fixed points of the system. These correspond to critical points of the function $H$, and one knows from algebraic topology that these are constrained by the topology of $M$. If $M$ is compact and all the critical points are non-degenerate, then the Morse theory can be applied, and in particular the number of critical points cannot be less than the sum of the Betti numbers of $M$. By contrast, for a general dynamical system (that is, a vector field on $M$) one has only a bound in terms of the Euler characteristic, the alternating sum of the Betti numbers.

Much more subtle problems emerge when one considers periodic orbits of
Hamiltonian systems. These are related to fixed points of symplectomorphisms: smooth maps $\phi: M \to M$ which preserve the symplectic form. In his book [1] Arnold suggested that the number of fixed points of such maps should obey constraints of a kind similar to those for Hamiltonian vector fields. More precisely, he conjectured that if $\phi$ is isotopic through symplectomorphisms to the identity, and has trivial ‘Calabi invariant’, then the number of fixed points (assuming non-degeneracy) should be at least the sum of the Betti numbers. This Arnold conjecture, which was motivated in part by the theorem of Poincaré and Birkhoff on area-preserving twist maps of the annulus (a result which in turn had its roots in applications to questions about periodic orbits in celestial mechanics), was one of the major foci of Floer’s work, and Floer’s proof of a version of the Arnold conjecture led him to introduce the idea of ‘Floer homology’. (Just at the time of writing, Floer’s ideas have been extended by a number of different authors to prove a very general form of the Arnold conjecture.) The fixed points of a symplectomorphism $\phi: M \to M$ can be identified with the critical points of a certain functional $\mathcal{A}$ on an infinite-dimensional space $\Omega_\phi$ of paths in $M$. Infinite-dimensional Morse theory is well-established in some contexts; for example, the ‘energy’ functional on paths whose critical points yield geodesics, but this symplectic case is definitely different, for example the Morse indices of all the critical points are infinite. Floer’s insight was that one could nevertheless develop a version of the Morse theory in this situation, using the point of view which had come into prominence in the paper of Witten [3], in which the homology groups of a manifold endowed with a Morse function are computed from a complex which encodes data about the critical points and the gradient flow-lines running between different critical points. The key thing is that these gradient lines in the infinite-dimensional space $\Omega_\phi$ correspond to rather concrete geometric objects in the original manifold $M$—they are variants of the ‘pseudo-holomorphic curves’ which had been introduced into symplectic geometry shortly before by Gromov [2]. The infinite Morse indices are regularised, so that one makes sense of the difference of the indices of any two critical points, and the upshot is that one has new Floer homology groups $HF^*_\phi$, which are formally the homology groups in approximately the ‘middle dimension’ of the infinite-dimensional space $\Omega_\phi$. In the case relevant to Arnold’s conjecture, when $\phi$ is isotopic to the identity, Floer showed that the groups $HF^*_\phi$ reproduce the ordinary homology of the manifold $M$, hence obtaining a proof of the conjecture.

In parallel with this attack on the Arnold conjecture, Floer had worked on the differential geometry of gauge theories, specifically the Bogomolny monopole equation on asymptotically Euclidean 3-manifolds. His second major achievement came about with his realisation that the ideas he had developed for the symplectic mapping problem could be taken over to 3-dimensional gauge theory. Here the space $\Omega_\phi$ is replaced by the space of connections on a bundle over a closed 3-manifold $Y$, and the functional $\mathcal{A}$ goes over to the Chern–Simons functional whose critical points are the flat connections. The gradient lines correspond to solutions of the Yang–Mills instanton equation on the cylinder $Y \times \mathbb{R}$. The resulting Floer homology groups are new invariants of the 3-manifold $Y$, which can be viewed as a refinement of the Casson invariant, and provided one of the main motivations for the concept of a ‘topological quantum field theory’ which was developed at about that time by Atiyah and others. In outline, the Floer homology groups of 3-manifolds are the companions to the instanton invariants of 4-manifolds, and provide the natural home for invariants of 4-manifolds with boundary. In the quantum field theory picture, the
Floer groups appear as the Hilbert space of the Hamiltonian formulation of the theory.

The volume under review brings together a collection of invited contributions in memory of Floer, spanning the whole range of topics touched on above.

On the one hand, there are papers by most of the leading exponents of the burgeoning field of symplectic geometry—a subject which has grown in the past twenty years from a specialist area to a major research field on a par with Riemannian geometry. Some of the contributions, such as those of Audin and Dell’Antonio, D’Onofrio and Ekeland, deal with the geometry and dynamics of Hamiltonian systems; others have to do with the development of Floer’s theory, and related subjects such as pseudo-holomorphic curves and Hofer’s symplectic capacities. There are articles by Eliashberg and McDuff on different facets of symplectic topology, which illustrate inter alia the applications of pseudo-holomorphic curve techniques in this direction.

On the other hand there are a range of substantial papers on gauge theory. The landscape here has changed dramatically in the last two years (after these contributions were gathered together) by the introduction of the Seiberg–Witten equations, but this development does not really detract from the interest of the articles: indeed, the construction of the version of the Floer theory corresponding to the Seiberg–Witten equations is a very active research area at the time of writing, and to a large extent the ideas developed in other contexts can be taken over to this new case—indeed, the main idea of Floer homology, ‘semi-infinite’ dimension Morse theory, has a rather universal character, and it is quite possible that yet further geometric problems will emerge where the ideas can be usefully applied. Several of the papers in the volume bear very directly on work of Floer: his manuscripts from his early work on monopoles, previously unpublished, are included, along with two papers of Ernst which develop the subject further. Some rather deep work of Floer concerns an exact sequence for the Floer homology groups of 3-manifolds which are related by Dehn surgeries: Floer’s manuscript is included in the volume, together with an exposition of Floer’s ideas by Braam and the author of this review.

Among other notable contributions, there is a characteristically wide-ranging article by Witten which shows how the Chern–Simons topological field theory on 3-manifolds can be obtained as a limiting form of the Floer/Gromov theory of pseudo-holomorphic curves, where the relevant symplectic manifold is the cotangent bundle of the 3-manifold. An interesting paper of Cohen, Jones and Segal discusses the algebro-topological basis for Floer’s homology theory. They propose a general setting involving ‘polarised manifolds’—the data which allow one to make sense of a semi-infinite dimensional cycle—and suggest a definition of Floer homotopy type.

In sum, this is a substantial volume which contains many articles of current research interest, but which also has a certain historical significance. It is a book which any geometer would want to have access to, and the Editors deserve thanks for their work in putting together this memorial.

References


S. K. Donaldson
INDEX OF BOOK REVIEWS

SOLOMON FEERMAN et al. (eds), Kurt Gödel collected works, Volume III
[reviewed by Wilfrid Hodges] 622
FRANCISCO A. RODRÍGUEZ-CONSUEGRA (ed.), Kurt Gödel: unpublished philo-
sophical essays [Wilfrid Hodges] 622
LARRY SMITH, Polynomial invariants of finite groups [Ken A. Brown] 623
VALERII M. KOPYTOV and NIKOLAI YA. MEDVEDEV, Right-ordered groups
[A. H. Rhemtulla] 624
R. J. PLYMEN and P. L. ROBINSON, Spinors in Hilbert space [L. J. Bunce] 625
DAVID R. ADAMS and LARS INGE HEDBERG, Function spaces and potential
theory [A. G. O’Farrell] 626
YUANLONG XIN, Geometry of harmonic maps [J. C. Wood] 628
ECKART VIEHWEG, Quasi-projective moduli for polarized manifolds
[P. M. H. Wilson] 629
J. AMORÓS, M. BURGER, K. CORLETTE, D. KOTSCHICK and D. TOLEDO,
Fundamental groups of compact Kähler manifolds [F. Campana] 631
CHRISTIAN KASSEL, Quantum groups [Andrew Pressley] 633
ANTHONY JOSEPH, Quantum groups and their primitive ideals [Andrew Pressley] 633
HELMUT HOfer, CLIFFORD H. TAUBES, et al. (eds), The Floer memorial volume
[S. K. Donaldson] 636