1 Cascade Simulations

At each point in time $t = 1, 2, \ldots$ there is a court with $k$ judges, with all courts deciding on the same question, and no judges serving in more than one time period. The state space is $\Theta = \{0, 1\}$ so that the world can either be in state $\theta = 1$ or $\theta = 0$, and a judge $i$ at time $t$ receives an independent symmetric binary signal $S^t_i$ with instantiations 1 or 0, signifying the two possible states. All judges have the same signal quality $p$ (which we call ‘competence’ in the main text), such that for all $i = 1, \ldots, k$ and $t = 1, \ldots$:

$$\Pr(S^t_i = 1|\theta = 1) = \Pr(S^t_i = 0|\theta = 0) = p > 0.5 \quad (1)$$

Each judge has a judgement function with the parameters $w$ for the weight the judge attaches to his own private signal, and $m$ for the memory the judge has. The output of the judgement function is a vote $v^t_i$ for a judge $i$ at the current time $t$. The judgement function takes as input the sequence of previous judgements:

1
\[ h_{t-1} = (v_1^1, v_1^2, \ldots, v_k^1, v_2^1, v_2^2, \ldots, v_k^2, \ldots, v_1^{t-1}, v_1^{t-1}, \ldots, v_k^{t-1}) . \]

To vote, each judge considers all votes within his reach of memory from time \( t - 1 \) back to \( t - m \) (or back to time 1 if \( t - m < 1 \)), and adds his own vote according to his private signal \( S_t^i \) weighted by \( w \). Let the pooled voting result be

\[ g_t^i = wS_t^i + \sum_{j:x \geq t-m} v_j^t . \]

The judge then votes for the majority winner among the previous votes pooled by comparing the result to the majority threshold \( r = (\min(t-1, m)k + w)/2 \), and breaks the tie by the own signal. Thus, the judgement function is:

\[
v_t^i = \begin{cases} 
1 & \text{if } g_t^i > r \\
S_t^i & \text{if } g_t^i = r \\
0 & \text{otherwise.}
\end{cases}
\] (2)

To estimate the competence levels presented in figures 1 to 3, we run simulations and determine the average rate of correctness for each point of time, given parameters \( k, p, m, \) and \( w \). Here we provide pseudo code for the crucial routine (one round of voting) to show how these simulations are conducted:

Set \( k, p, m, w \)

For judge = 1 to \( k \)

Set private_signal s.t. signal is 1 with probability \( p \),
otherwise 0

Set \texttt{votesfor1} = \text{Count votes } v=1 \text{ in history down to } t-m,

or down to 1 if greater

If \texttt{votesfor1} + \text{private_signal} * w > (\text{Min} [(t-1),m] * k + w) / 2

Then vote for 1

If \texttt{votesfor1} + \text{private_signal} * w = (\text{Min} [(t-1),m] * k + w) / 2

Then vote for \text{private_signal}

Otherwise vote for 0

Next judge

If \text{correct votes} > k/2

Then Return \text{court majority is correct}.

In the simulations where judges are heterogeneous and distinguish between informative signals and cascade signals (Figure 3), a separate look-up table records for each judgement in the history whether the judge’s own signal was pivotal in his vote, or whether he followed the past majority no matter what his own signal was (or would have been). Discriminating judges consider only previous informative votes (where the private signal of the judge was pivotal) in their judgement according to this look-up table.

2 One Opinion Leader

We largely follow the methods (though not necessarily the notation) as laid out in Bolandt et al.\textsuperscript{1}, but with one important caveat. Boland et al. consider

only those cases where the competence of the opinion leader and all voters is identical. In this special case, if all voters follow the opinion leader with probability $\pi$ this results in a correlation coefficient of $\pi$ between leader and voters. However, if the competence of opinion leader and followers differ, this equivalence result no longer holds. Unlike other authors who prefer to use the correlation coefficient to describe dependence, we prefer to talk in terms of the probability-of-following. To capture opinion leadership as the probability of others following the opinion leader is more plausible because this is closer to the causal nature of the process.

There are two alternatives, of which exactly one is correct or better. There are $n$ judges labelled from 1 to $n$. Let $X_i$ be a random variable for judge $i$ such that $X_i = 1$ if judge $i$ votes for the correct, $X_i = 0$ if $i$ votes for the incorrect alternative. All judges have competence $p$ to vote for the correct alternative, such that $\Pr(X_i = 1) = p > 0.5$ for each judge $i$, and let the converse probability be $q = 1 - p$. If we also assume that the votes of all judges are independent conditional on the true state of the world regarding the correct alternative, we arrive at the standard Condorcet jury theorem formula to calculate the probability that a majority of judges will be correct, which we write $C(n,p)$, is:

$$C(n,p) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k (1-p)^{n-k} \text{ with } n \text{ being odd.}$$

Let there be one opinion leader who does not himself vote. We denote all variables pertaining to opinion leaders with a hat. The opinion leader

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has competence $\hat{p}$ and the converse probability is $\hat{q} = 1 - \hat{p}$. The vote of the opinion leader is a random variable $\hat{X}$. Each voter defers to the opinion of the opinion leader with probability $\pi$ and follows his own private signal with probability $1 - \pi$. The probability of a correct vote depends on the view of the opinion leader:

$$\Pr(X = 1|\hat{X} = 1) = \pi + (1 - \pi)p$$  \hspace{1cm} (4)$$
$$\Pr(X = 1|\hat{X} = 0) = (1 - \pi)p$$  \hspace{1cm} (5)$$
$$\Pr(X = 0|\hat{X} = 1) = (1 - \pi)q$$  \hspace{1cm} (6)$$
$$\Pr(X = 0|\hat{X} = 0) = \pi + (1 - \pi)q.$$  \hspace{1cm} (7)$$

The resulting probability distribution can be displayed in a table:

<table>
<thead>
<tr>
<th>$\hat{X}$</th>
<th>$X$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\hat{q}\pi + \hat{q}(1 - \pi)q$</td>
<td>$\hat{q}(1 - \pi)p$</td>
</tr>
<tr>
<td>1</td>
<td>$\hat{p}(1 - \pi)q$</td>
<td>$\hat{p}\pi + \hat{p}(1 - \pi)p$</td>
</tr>
<tr>
<td></td>
<td>$1 - (p - \pi p + \hat{p}\pi)$</td>
<td>$p - \pi p + \hat{p}\pi$</td>
</tr>
</tbody>
</table>

The group competence in the presence of an external opinion leader can be calculated by distinguishing the case where the opinion leader is correct and incorrect, using the standard CJT formula (3) twice and (4) and (5) for the respective competences:

$$\Pr^{\text{OLLI}}(n, p, \hat{p}, \pi) = \hat{p} \cdot C[n, \pi + (1 - \pi)p] + \hat{q} \cdot C[n, (1 - \pi)p].$$  \hspace{1cm} (8)
We will only see a convergence to 1 if both $C$ terms receive a competence argument greater than 0.5. Consequently, we will see a convergence to the competence of the opinion leader $\hat{p}$ if $(1 - \pi)p < 0.5$, or, after reformulation

$$\pi > \frac{p - 0.5}{p}.$$  \hspace{1cm} (9)

If $\pi = (p - 0.5)/p$ the convergence is to $\hat{p} + \hat{q}/2$.

3 Several Opinion Leaders

Correlated

We consider first the special case where two opinion leaders are perfectly negatively correlated. The first opinion leader has competence $\hat{p}$ the second (due to the correlation) $\hat{q} = 1 - \hat{p}$. Let there be $n_1$ voters in group 1 following opinion leader 1 with probability $\pi_1$ and $n_2$ voters in group 2 following opinion leader 2 with probability $\pi_2$. If voters are not following an opinion leader they all have the same competence $p > 0.5$ to vote for the correct result. Let a winning tuple $(w_1, w_2)$ be the number of correct votes $w_1$ in group 1 and $w_2$ in group 2, such that $w_1 + w_2 > n/2$. Let $W$ be the set of all tuples where the majority is correct. For notational simplicity we substitute the binomial probability mass function by

$$B(n, k, p) = \binom{n}{k} p^k (1 - p)^{n-k}.$$ \hspace{1cm} (10)

The probability that a specific tuple $(w_1, w_2)$ obtains is
The probability that a correct majority arises is the sum of all probabilities of all tuples where the majority is correct:

\[
\Pr_{\text{maj}}^{\text{OL2c}}(n_1, n_2, \hat{p}, \pi_1, \pi_2) = \sum_{\forall (w_1, w_2) \in W} \Pr^{\text{OL2c}}((w_1, w_2), n_1, n_2, \hat{p}, \pi_1, \pi_2). \tag{12}
\]

**Uncorrelated**

We consider cases where there are several opinion leaders, but each voter is influenced by exactly one opinion leader only. This means that each opinion leader has a group of potential followers, and that the groups of followers are mutually exclusive and jointly exhaustive.

For two opinion leaders, the group competence can easily be calculated as a function of the competence of the two opinion leaders \( \hat{p}_1 \) and \( \hat{p}_2 \), the size of the groups of followers \( n_1 \) and \( n_2 \), the competence of the voters that are attached to opinion leader one \( p_1 \) and opinion leader two \( p_2 \) (the voter competence applies when they do not follow their opinion leader), and the probabilities \( \pi_1 \) and \( \pi_2 \) that the voters in the two groups follow their opinion
leader. A winning tuple \( \langle w_1, w_2 \rangle \) is defined as above.

The probability that one specific winning tuple arises is

\[
\Pr^{\text{OL2}}(\hat{\rho}_1, \hat{\rho}_2, n_1, n_2, p_1, p_2, \pi_1, \pi_2, \langle w_1, w_2 \rangle) = \\
[\hat{\rho}_1 \mathcal{B}(n_1, w_1, \pi_1 + (1 - \pi_1)p_1) + (1 - \hat{\rho}_1)\mathcal{B}(n_1, w_1, (1 - \pi_1)p_1)] \\
[\hat{\rho}_2 \mathcal{B}(n_2, w_2, \pi_2 + (1 - \pi_2)p_2) + (1 - \hat{\rho}_2)\mathcal{B}(n_2, w_2, (1 - \pi_2)p_2)].
\]  \hspace{1cm} (13)

The probability that the majority is correct is the sum of the probabilities of all correct tuples:

\[
\Pr^{\text{OL2}}_{\text{maj}}(\hat{\rho}_1, \hat{\rho}_2, n_1, n_2, p_1, p_2, \pi_1, \pi_2) = \\
\sum_{\forall \langle w_1, w_2 \rangle \in \mathcal{W}} \Pr^{\text{OL2}}(\hat{\rho}_1, \hat{\rho}_2, n_1, n_2, p_1, p_2, \pi_1, \pi_2, \langle w_1, w_2 \rangle).  \hspace{1cm} (14)
\]

Since the analytical results become increasingly unwieldy for situations with many opinion leaders, we rely on Monte Carlo simulations to arrive at estimated group competences. The routine for a Monte Carlo estimation is as follows:

\text{success} = 0

\hspace{1cm} \text{Do r times:}

\hspace{1cm} \text{Set votes of all opinion leaders by random draw}

\hspace{2cm} \text{according to their competence}

\hspace{1cm} \text{For each voter i:}
Determine by random draw according to probability of following if \( i \) follows

If voter \( i \) follows

Then \( \text{vote}(i) = \text{vote opinion leader}(i) \)

Otherwise draw \( \text{vote}(i) \) according to competence of \( i \)

Count correct votes

If correct votes \( \geq \) majority threshold Then increase success

Loop

Return success / \( r \)