North-South Trade and Standards: What Can General Equilibrium Analysis Tell Us?

Appendix

Production function:

The output $y$ of any good $z$ in the continuum is a function of combining both effective labor $l$ and the bad $b$ via the following constant returns to scale Cobb-Douglas technology:

$$y(l, b; z) = \begin{cases} l^{1 - \alpha(z)} b^{\alpha(z)} & \text{if } b \leq \lambda l, \\ 0 & \text{if } b > \lambda l, \end{cases}$$

(A1)

where $\lambda > 0$, $\alpha(z)$ varies across goods, and $\alpha(z) \in [\alpha, \hat{\alpha}]$, with $0 < \alpha < \hat{\alpha} < 1$. Differentiating (A1) with respect to $l$ and $b$, allows derivation of the marginal rate of substitution, $\frac{1 - \alpha(z) b}{\alpha(z) l}$.

Consumption function:

Assuming consumption goods $z$ and the public bad $b$ are separable in utility, the indirect utility function of a representative consumer is:

$$V = \int_0^1 f(z) \ln[x(z)] dz - \int_0^1 f(z) \ln[p(z)] dz + \ln i - \frac{BD}{\gamma},$$

(A2)

where $x(z)$ is consumption of $z$, $f(z)$ is the budget share for each good in the continuum, and the sum of budget shares is $\int_0^1 f(z) dz = 1$; $p(z)$ is the continuum of prices for the consumption goods $z$; $i = II/L$ is income per capita of a representative consumer, where $I$ is national income and $L$ is the total number of workers in the economy; $D$ is aggregate production of the public bad; $\beta$ measures the representative consumer’s disutility associated with the public bad; and $\gamma \geq 1$, implying consumers’ willingness to pay for a reduction in the level of the public bad is non-decreasing in its aggregate level.
\(T(\bar{z})\) function:

By minimizing total costs subject to the production function, the unit cost function for a good \(z\) in the continuum is:

\[
a(w_e, c_b; h, z) = \Omega(z)c_b^{a(\bar{z})}[w/A(h)]^{1-a(\bar{z})},
\]

(A3)

where \(\Omega(z) = \alpha^a(1-\alpha)^{-(1-a)}\) is a good-specific constant, and \(w\) is the wage rate for raw labor. For given wages and compliance costs, a good \(z\) in the continuum will be produced in the North if

\[
a(w_e, c_b; h, z) \leq a^*(w_e^*, c_b^*, h^*, z),
\]

such that:

\[
\bar{\omega} \equiv \frac{w}{w^*} \leq \frac{A}{A^*} \left( \frac{c_b^*}{c_b} \right)^{a(\bar{z})/(1-a(\bar{z}))} \equiv T(\bar{z}).
\]

(A4)

Given (A4) the optimal level of compliance costs \(c_b\) is derived by maximizing (A2) with respect to the public bad:

\[
V_p dp / dD + V_i di / dD + V_D = 0,
\]

and assuming \(dp/dD = 0\), (A5) can be re-arranged as:

\[
di / dD = -(V_D / V_i).
\]

(A6)

From differentiation of (A2):

\[
-(V_D / V_i) = (\beta D^{i-1}) / (1 / i) = \beta D^{i-1}i,
\]

(A7)

which given the definition of \(i\) can be re-written as:

\[
-L(V_D / V_i) = \beta D^{i-1}I = c_b,
\]

(A8)

Balanced trade requires that \(I = \psi(\bar{z})(I + I^*)\) and \(I^* = \psi^*(\bar{z})(I^* + I)\), where \(\psi(\bar{z}) \equiv \int_0^{\bar{z}} f(z)dz\) and \(1 - \psi(\bar{z}) = \psi^*(\bar{z}) \equiv \int_{\bar{z}}^{1} f(z)dz\) are the shares of world spending on Northern and Southern goods.
respectively. Solving for \( I(I^*) \) and \( D(D^*) \) in terms of \( \bar{z} \), an expression for relative compliance costs as a function of \( \bar{z} \) can be derived as:

\[
\frac{c^*_b}{c_b} = \left( \frac{\psi^*(\bar{z})}{\psi(\bar{z})} \right)^{\frac{1}{\gamma}} \left( \frac{\phi^*(\bar{z})}{\phi(\bar{z})} \right)^{\frac{\gamma - 1}{\gamma}} \equiv C(\bar{z}),
\]

(A9)

where \( \phi(\bar{z}) = \int_{0}^{\bar{z}} a(z) f(z) dz \) and \( \phi^*(\bar{z}) = \int_{0}^{\bar{z}} a(z) f(z) dz \) are the portions of the shares of world spending on Northern (Southern) goods that contribute to Northern (Southern) compliance costs, \( C(\bar{z}) < 1 \) if compliance costs are higher in the North than the South. Substituting (A9) into expression (A4) gives:

\[
\bar{\omega} = \frac{w}{w^*} \leq \frac{A}{A^*} \left[ C(\bar{z}) \right]^{\alpha(z)/(1-\alpha(z))} \equiv T(\bar{z}),
\]

(A10)

where \( d \ln T(\bar{z}) / d\bar{z} < 0 \), and \( T(1) = 0 \).

If the cost of an aid transfer is \( \tau \), then compliance costs in the North become \( c_b = \beta D^{\gamma - 1}(I - \tau) \), (A9) being re-written as:

\[
\frac{c^*_b}{c_b} = \left( \frac{\psi^*(\bar{z})}{\psi(\bar{z})} + h(\tau, \bar{z}) \right)^{\frac{1}{\gamma}} \left( \frac{\phi^*(\bar{z})}{\phi(\bar{z})} \right)^{\frac{\gamma - 1}{\gamma}} \equiv C(\bar{z}, \tau),
\]

(A11)

where \( h(\tau, \bar{z}) = D / \psi(\bar{z})(I - \tau) \). After substitution, (A10) becomes:

\[
\bar{\omega} = \frac{w}{w^*} \leq \frac{A}{A^*} \left[ C(\bar{z}, \tau) \right]^{\alpha(z)/(1-\alpha(z))} \equiv T(\bar{z}, \tau).
\]

(A12)