Overview

We describe the methods used to find the seasonal transmission rates of pertussis in Thailand. The ESM is structured as follows:

1. Seasonality in Thailand
   - Time series SIR model (TSIR)
   - Susceptible reconstruction
Seasonality in Thailand

We applied the time series SIR (TSIR) framework to estimate monthly transmission parameters in Thailand (Finkenstadt et al., 2000; Finkenstadt et al., 2002). Here, we first describe the susceptible reconstruction which is required to provide estimates of particular parameters. Second, we describe the TSIR model.

Susceptible reconstruction

Susceptible reconstruction is used to estimate the reporting rate $\rho$ in addition to the deviations of the number of susceptibles from the mean at each time point in our data ($Z_t$), both of which are required in the TSIR model.

Following the methodology of (Finkenstadt et al., 2000), the number of susceptible individuals at time $t$ is assumed to be given by unvaccinated births and susceptible individuals from the last time point minus the number of new infections:

$$S_t = S_{t-1} + b_t(1 - p_t) - \frac{C_t}{\rho}$$  \hspace{1cm} (A1)

where $C_t$ is the number of reported cases, $\rho$ is the constant-valued reporting rate, $S_t$ is the number of susceptibles, and $b_t$ is the number of births reduced by a proportion $p_t$ vaccinated individuals. Susceptible individuals can be written as $S_t = Z_t + \overline{S}$ where $Z_t$ are the deviations from the mean number of susceptibles ($\overline{S}$) at each time.

$S_t = Z_t + \overline{S}$ is substituted into Equation A1 so that

$$Z_t = Z_{t-1} + b_t(1 - p_t) - \frac{C_t}{\rho},$$  \hspace{1cm} (A2)

Iterating Equation A2 for an initial condition $Z_0$ gives us

$$Z_t = Z_0 + \sum_{i=1}^{t} b_i(1 - p_i) - \sum_{i=1}^{t} \frac{C_i}{\rho}.$$  \hspace{1cm} (A3)

This equation can be rearranged as

$$\sum_{i=1}^{t} b_i(1 - p_i) = \frac{1}{\rho} \sum_{i=1}^{t} C_i + Z_t - Z_0.$$  \hspace{1cm} (A4)

Under the assumption that all unvaccinated individuals acquire infection within their lifetime, the slope of a linear regression between the cumulative births (discounted by vaccination) and the cumulative reported cases provides the inverse of the reporting rate ($1/\rho$) and $Z_t$ is estimated as the residuals from the regression (Finkenstadt et al., 2000; Metcalf et al., 2009). The resulting value of $\rho$ is 0.0063 (or 0.63% reporting).
Time series SIR model

Susceptible individuals are given by $S_t$ and the true number of infected individuals $I_t$ – case reports $C_t$ corrected for under reporting so that $I_t = C_t / \rho$ – are described by the equations:

$$E[I_t|I_{t-1}, S_{t-1}] = \lambda_t S_{t-1} = \frac{\beta_{t-1}(I_{t-1})^\alpha}{N_{t-1}} S_{t-1}$$  \hspace{1cm} (A5)

$$S_t = b_t(1 - p_t) + S_{t-1} - I_t.$$ \hspace{1cm} (A6)

Starting with $I_t$ infected individuals and assuming independence, this is a birth-death process following the negative binomial distribution such that

$$I_t \sim NB(\lambda_t S_{t-1}, I_{t-1})$$ \hspace{1cm} (A7)

where $\lambda_t S_{t-1}$ is the expectation and $I_{t-1}$ is the clumping parameter. Using our estimates of $I_t$ and replacing $S_t$ with $\bar{S} + Z_t$ (see previous section), and given that $I_t$ follows the negative binomial distribution, we used maximum likelihood to estimate $\beta_t$, $\alpha$, and $\bar{S}$.

References

