Online Appendix. Proofs

Proof of Proposition 1. Taking as given \( n^* \) and \( p^* \), the 1st-order condition to expression (3) leads to expression (14). This implies that the equilibrium level of cash flow to a firm \( i \) is

\[
X_{i}^{T*} = X^{T*} = (p_i^* - c_i) q_i^* = \left( \frac{\alpha}{n^*} \right)^2.
\]

Substituting the constraints (10), (11), and (12) into equation (9), we obtain that equation (9) can be written as

\[
(A-2) \quad \max_{B_i} \mathbb{E}_0 \left[ X_i^T(p^*, \tau_i(B_i)) - H_{i} - \beta(1 - \mu) \max \left\{ X_i^T(p^*, \tau_i(B_i)) - B_i; 0 \right\} \right],
\]

s.t. \( \tau_i(B_i) = \arg \max_{\tau_i \in \{H,L\}} \mathbb{E}_1 X_i^E(p^*, \tau_i, \kappa_i) \).

Since low-quality technology is not sustainable, in equilibrium only firms that are expected (and have the incentive) to choose high-quality technology enter the market. This leads to the incentive-compatibility condition (20). From expression (A-2) it is easy to see that entrepreneurs first issue debt up to debt capacity \( \bar{D} \), after which they will issue equity. Given expression (21), the maximum amount of equity that the marginal entrepreneur with cash flow \( X^{T*} \) can issue is \( S_{n^*} = (1 - \beta) \eta \). This implies that \( n^* \) is determined by

\[
(A-3) \quad \bar{D} + S_{n^*} = \left( \frac{\alpha}{n^*} \right)^2 - \beta \eta = F_{H,n^*} = F_H + \theta n^*,
\]

giving equation (13). Inframarginal entrepreneurs will issue an amount of equity that is just sufficient to cover the fixed cost \( F_{H,i} \), giving equation (15). Thus, the fraction of equity sold to outside investors, \( \kappa_i \), is \( S_i^*/(1 - \beta) \eta \), giving equation (17). The payoff to the marginal entrepreneur, who given expression (A-3) sells all his shares to obtain entry, is \( \mu \beta \eta \). The payoff to inframarginal entrepreneurs is thus equation (18). Finally, from equation (17), it is easy to see that \( 1 - \kappa_i < \mu \) for all \( i < n^* \) if

\[
(A-4) \quad \mu \geq \mu_c \equiv \frac{\theta n^*}{(1 - \beta) \eta}.
\]

In addition, note that no additional entrepreneur with \( i > n^* \) can enter when \( \phi(\frac{n^*}{n^*}) < F_L + \theta n^* \),
that is, when

\[ \phi \leq \phi_c \equiv \frac{F_L + \theta n^*}{(\alpha n^*)^2}. \]  

(A-5)

The proof is concluded by noting that expression (A-5) implies that

\[ V_i = \mu \beta \eta + \theta (n^* - i) > \phi \left( \frac{\alpha}{n^*} \right)^* - F_L - \theta i \]  

(A-6)

and, thus, all entrepreneurs that enter the market prefer to adopt high-quality technology rather than low-quality technology.

Proof of Proposition 2. The 1st result follows immediately from Proposition 1 and implicit function differentiation of equation (13), obtaining

\[ \frac{\partial n^*}{\partial \beta} = -\frac{\eta}{2 \alpha^2 n^* + \theta} < 0. \]  

(A-7)

The sign of \( \frac{\partial \tilde{D}}{\partial \beta} \) follows from direct differentiation of \( \tilde{D} \) in expression (21) and from expression (A-7).

The sign of \( \frac{\partial S_t^*}{\partial \beta} \) follows from the 1st equality in expression (15) and the previous result that \( \frac{\partial \tilde{D}}{\partial \beta} > 0 \).

The sign of \( \frac{\partial E_{M^*}}{\partial \beta} \) follows from direct differentiation of \( E_{M^*} = (1 - \beta) \eta \). By differentiation of

\[ \frac{\partial \omega_i^*}{\partial \beta} = \theta \left[ \frac{2 \alpha^2}{n^*} + \theta \right] \frac{(n^* - i) - (1 - \beta) \eta}{(2 \alpha^2 + \theta)(1 - \beta)^2 \eta} \]  

(A-8)

using expression (A-7), we obtain that

\[ \frac{\partial \omega_i^*}{\partial \beta} = \theta \left[ \frac{2 \alpha^2}{n^*} + \theta \right] \frac{(n^* - i) - (1 - \beta) \eta}{(2 \alpha^2 + \theta)(1 - \beta)^2 \eta} > 0 \]  

(A-9)

iff \( i < i_c(\beta, \eta) \equiv n^* - \frac{(1 - \beta) \eta}{2 \alpha^2 + \theta} \). The inefficiency of low-quality technology implies that \( n^* > i_c(\beta, \eta) > 0 \). To see this, note that \( \phi F_H < F_L \) implies

\[ \frac{2 \alpha^2}{n^*^2} = 2 \left( F_H + \theta n^* + \eta \beta \right) > F_L > \frac{\phi (F_H - F_L)}{(1 - \phi)} = \eta. \]  

(A-10)

Finally, expression (24) is obtained by substituting expression (A-7) into \( \varepsilon = \left| \frac{\beta}{n^*} \frac{\partial n^*}{\partial \beta} \right| \), giving

\[ \varepsilon = \frac{\eta \beta}{2 \alpha^2 + \theta n^*} = \frac{\eta \beta}{2 \left( F_H + \theta n^* + \eta \beta \right) + \theta n^*} = \frac{1}{\frac{2 F_H + 3 \theta n^*}{\eta \beta} + 2}, \]  

(A-11)

which is increasing in \( \eta \) (since, in the Proof of Proposition 3, we will show that \( n^* \) is decreasing in \( \eta \)).
Proof of Proposition 3. The 1st result that $\frac{\partial n^*}{\partial \eta} < 0$ follows immediately from Proposition 1 and implicit function differentiation of equation (13). The sign of $\frac{\partial S^*}{\partial \eta}$ follows from direct differentiation of $S^*$ in expression (15) and the result that $\frac{\partial n^*}{\partial \eta} < 0$. The sign of $\frac{\partial \bar{D}}{\partial \eta}$ then follows from the 1st equality in expression (15). The sign of $\frac{\partial E^M_i}{\partial \eta}$ follows from direct differentiation of $E^M_i = (1 - \beta)\eta$. The result that $\frac{\partial \omega_i}{\partial \eta} < 0$ follows from expression (A-8) and $\frac{\partial n^*}{\partial \eta} < 0$.

Proof of Proposition 4. Entrepreneurs maximize their expected profits, that is,  
\[
\max_{B_i,\tau_i,e_i} \mathbb{E}_0 \left[ X_t^*(\tau_i) - F_{H,i} - (1 - e_i)\beta(1 - \mu) \max \{ X_t^*(\tau_i) - B_i; 0 \} \right] - C(k, e_i),
\]
subject to  
\[
\tau_i = \arg \max_{\tau_i \in \{H,L\}} \mathbb{E}_1[\mu \beta + (1 - \kappa_i)(1 - \beta)] \max \{ X_t^*(\tau_i) - B_i; 0 \}.
\]

With the given cost function for effort, assuming that Assumptions 1 and 2 hold, we can rewrite the entrepreneurs’ objective function, (A-12), using our previous results, regarding $B_i^*$, as  
\[
\max_{e_i} \mathbb{E}_0 \left[ \left( \frac{\alpha}{n^*} \right)^2 - F_H - \theta_i - (1 - e_i)\beta(1 - \mu)\eta - ke_i(1 - e_i)^{-1} \right].
\]

Let  
\[
k_1 = \frac{(1 - 2\mu)^2}{1 - \mu} \beta \eta.
\]

Under our assumption that $k \leq k_1$, the 1st-order condition with respect to $e_i$ gives the optimal level of effort for all entrepreneurs $i$:  
\[
e_i^{**} = 1 - \sqrt{\frac{k}{\beta(1 - \mu)\eta}}.
\]

Entry to an industry occurs until the marginal entrepreneur’s payoff equals 0. Hence, $n^{**}$ satisfies  
\[
\left( \frac{\alpha}{n^{**}} \right)^2 - F_H - \theta n^{**} - (1 - e_i^{**})\beta(1 - \mu)\eta - ke_i^{**}(1 - e_i^{**})^{-1} =  
\left( \frac{\alpha}{n^{**}} \right)^2 - F_H - \theta n^{**} - 2\sqrt{\frac{k}{\beta(1 - \mu)\eta}} + k = 0,
\]

implying that $n^{**}$ is implicitly determined by  
\[
n^{**} = \frac{\alpha}{\sqrt{F_H + \theta n^{**} + 2\sqrt{k\beta(1 - \mu)\eta}} - k} > n^*.
\]
To see that $n^{**} > n^*$, note that

\[(A-19) \quad \beta \eta > 2\sqrt{k \beta \eta} - k > 2\sqrt{k \beta (1 - \mu) \eta} - k,\]

since

\[(A-20) \quad \beta \eta - 2\sqrt{k \beta \eta} + k = (\sqrt{k} - \sqrt{\beta \eta})^2 > 0.\]

We now need to show that, by exerting effort $e^{**}$, the marginal entrepreneur is able to raise financing, that is

\[(A-21) \quad (\frac{\alpha}{n^{**}})^2 - F_H - \theta n^{**} - (1 - e^{**})\beta \eta \geq 0.\]

Using expression (A-17), it is easy to check that expression (A-21) is verified when

\[(A-22) \quad ke^{**}(1 - e^{**})^{-1} \geq (1 - e^{**})\beta \mu \eta,\]

that is, from equation (A-16), when

\[(A-23) \quad k \leq k_1 \equiv \frac{(1 - 2\mu)^2}{1 - \mu} \beta \eta \leq (1 - \mu)\beta \eta.\]

The proof is concluded by noting that Assumption 1 holds with the previous definition of $\phi_c$ and redefining $\mu_c$ as $\mu_c = \frac{\theta n^{**}}{(\eta - \sqrt{\frac{k \beta \eta}{1 - \mu}})}$.

**Proof of Proposition 5.** In this case, the financing constraint (A-21) fails with $n^{**}$ firms in the market. Hence, fewer firms enter, and at the effort level $e^{**}$ all entering firms would have strictly positive payoffs. This implies that for some marginal firms (which otherwise would be left out), it pays to exert an amount of effort $\hat{e}_i > e^{**}$ in order to obtain entry. For these firms, $\hat{e}_i$ is set sufficiently high to raise the necessary funds to successfully enter the market, that is,

\[(A-24) \quad \left(\frac{\alpha}{\hat{n}}\right)^2 - F_H - \theta i - (1 - \hat{e}_i)\beta \eta = 0.\]

The number of firms in this equilibrium, $\hat{n}$, is again determined by the condition that the marginal entrepreneur earns zero expected profits. That is, by

\[(A-25) \quad \left(\frac{\alpha}{\hat{n}}\right)^2 - F_H - \theta \hat{n} - (1 - \hat{e}_\hat{n})(1 - \mu)\beta \eta - k \hat{e}_\hat{n}(1 - \hat{e}_\hat{n})^{-1} = 0.\]

Substituting equation (A-24) to equation (A-25) gives

\[(A-26) \quad (1 - \hat{e}_\hat{n})^2 \mu \beta \eta - k \hat{e}_\hat{n} = 0.\]
From equation (A-24) and the 1st-order condition for effort (A-16), it is easy to see that for other firms,

\( \hat{e}_i = \max \left\{ \hat{e}_n - \frac{\theta (\hat{n} - i)}{\beta \eta}, e^{**} \right\} \).

Taking the derivatives with respect to \( \beta \) and \( \eta \) gives

\[
\frac{\partial \hat{e}_n}{\partial \beta} = \left( \sqrt{\frac{1}{\frac{k}{\mu \beta \eta} + \left( \frac{k}{2 \mu \beta \eta} \right)^2} - 1 \right) \frac{k}{2 \mu \beta \eta^2} > 0, \tag{A-30}
\]

\[
\frac{\partial \hat{e}_n}{\partial \eta} = \left( \sqrt{\frac{1}{\frac{k}{\mu \beta \eta} + \left( \frac{k}{2 \mu \beta \eta} \right)^2} - 1 \right) \frac{k}{2 \mu \beta \eta^2} > 0, \tag{A-31}
\]

which implies, given our previous results for \( e^{**} \), and the fact that \( \frac{\partial n}{\partial \beta} < 0 \) and \( \frac{\partial n}{\partial \eta} < 0 \), as can be verified using equation (A-25), that these derivatives are positive also for other firms.

**Proof of Proposition 6.** Low-quality technology is sustainable in equilibrium if

\[
\phi > \phi_c \equiv \frac{F_L + \theta n^*}{\left( \frac{\alpha}{n'} \right)^2} \iff \phi \left( \frac{\alpha}{n'} \right)^2 - F_L - \theta n^* > 0. \tag{A-32}
\]

When expression (A-32) holds, if the first \( n^* \) firms choose high-quality technology, some additional marginal firms can enter the market by adopting low-quality technology. Let \( \{n', n''\} \) be a candidate equilibrium in which \( n' \) is the total number of firms in the industry and \( n'' \in [0, n') \) is the number of firms that choose high-quality technology. Note first that, in the candidate equilibrium, firms with high-quality technology produce \( \hat{q}_i^x = \frac{\alpha}{n'} + \phi(n' - n'') \), and sell their production at a price \( \hat{p}_i^x = c + \frac{\alpha}{n'' + \phi(n' - n'')} \). This results in cash flow

\[
X_i^T = \left( \frac{\alpha}{n'' + \phi(n' - n'')} \right)^2. \tag{A-33}
\]
Thus, debt capacity for firms selecting high-quality technology is now equal to

\[(A-34) \quad \mathcal{D} = \left( \frac{\alpha}{n'' + \phi(n' - n'')} \right)^2 - \eta. \]

In equilibrium, firms selecting high-quality technology finance \( \mathcal{D} \) with debt and \( F_{H,i} - \mathcal{D} \) with equity. The remaining \( n' - n'' > 0 \) entrepreneurs who enter the market produce with low-quality technology, and with probability \( \phi \) can produce superior quality goods in the quantity \( \tilde{q}_i^* \). Furthermore, these firms can be financed entirely with debt; thus, they borrow \( D_i^* = F_L + \theta n' \) of debt with a face value \( B_i = \frac{F_{L,i} + \theta n'}{\phi} \), and repurchase shares for \( D_i^* - F_{L,i} \).

Equilibrium is determined by 3 conditions: (31), (32), and the entry condition for the \( n' \):th low-quality producer

\[(A-35) \quad \phi \left( \frac{\alpha}{n'' + \phi(n' - n'')} \right)^2 = F_L + \theta n'. \]

Furthermore, 2 of the 3 conditions bind, equation (A-35) and either expression (31) or (32). Consider 2 cases: First, if \( \mu \geq \phi \), it is easy to verify that expression (31) implies expression (32) for all \( i \geq 0 \) if

\[(A-36) \quad (1 - \phi)\theta n'' + \beta \eta (\mu - \phi) + F_L - \phi F_H \geq 0, \]

which holds for all \( \beta \). In this case, using equation (A-35) and expression (31) as equalities gives

\[(A-37) \quad n'' = \frac{n'}{\phi} - \frac{\phi F_H - F_L + \phi \beta \eta}{\theta \phi}. \]

This can be used in equation (A-35) or expression (31) to substitute for either \( n' \) or \( n'' \) to verify that \( n'' \) is decreasing in \( \beta \), while \( n' \) is increasing in \( \beta \). Substituting for \( n' \) from equation (A-37) into expression (31) and setting \( n'' = 0 \) gives that \( n'' \geq 0 \) if and only if \( \beta \leq \beta_1 \), where \( \beta_1 \) is defined implicitly by

\[(A-38) \quad \left( \frac{\alpha \theta}{\phi (\phi F_H - F_L + \phi \beta_1 \eta)} \right)^2 = F_H + \beta_1 \eta. \]

Second, if \( \mu < \phi \), expression (A-36) holds for \( \beta \leq \beta_2 \), where \( \beta_2 \) is defined by

\[(A-39) \quad \beta_2 = \frac{F_L - \phi F_H}{\eta (\phi - \mu)}. \]

Let \( \bar{\beta} = I_{\mu > \phi} \beta_1 + I_{\mu < \phi} \min(\beta_1, \beta_2) \). Note that our assumption that \( F_L > \phi F_H \) implies that \( \bar{\beta} > 0 \).

**Proof of Proposition 7.** When \( \mu \geq \phi \), or when \( \mu < \phi \), but \( \beta_1 \leq \beta_2 \), let \( \beta' = \bar{\beta} \). When \( \mu < \phi \), but \( \beta_1 > \beta_2 \), equation (A-35) and expression (32) hold as an equality for small enough \( n'' \). Solving
for \( n'' \) using equation (A-35) and expression (32), we can verify that \( n'' \) is decreasing in \( \beta \). Thus, \( n'' = 0 \) whenever \( \beta > \beta_3 \), where \( \beta_3 \) solves

\[
(1 - \phi) \left( \frac{\alpha \theta}{\phi \left( \frac{\phi F_H - F_L + (1 - \mu) \phi \eta}{(1 - \phi)} \right)} \right)^2 - (F_H - F_L) - (1 - \mu) \beta \eta = 0.
\]

Let \( \beta' = I_{\mu > \phi} \beta_1 + I_{\mu < \phi} (I_{\beta_1 < \beta_2} \beta_1 + I_{\beta_1 > \beta_2} \beta_3) \). The result regarding the limit when \( \theta \to 0 \) follows from (A-38), since in the limit \( \beta_1 < \beta_2 \) when \( \mu < \phi \).

**Proof of Proposition 8.** The proof is similar to the Proof of Proposition 1, and is only sketched. Taking again \( n^o \) and \( \bar{p} \) as given, entrepreneurs choosing high-quality technology set \( p_i = \frac{\alpha'}{\mu n^o} \), which gives \( p^o_i = \sqrt{\frac{\alpha'}{n^o}} \) and \( q^o_i = \sqrt{\frac{\alpha'}{n^o}} \); thus, firm profits are now equal to \( X^{T^o} = \frac{\alpha'}{n^o} \). This implies that debt capacity now is \( \bar{D}^o = \frac{\alpha'}{n^o} - \eta \), where \( \eta \) is defined as before. Given that the marginal entrepreneur now issues \( S_{n^o}^o = (1 - \delta)(1 - \beta) \eta \) of equity, using a similar line of reasoning as the one in the Proof of Proposition 1, we obtain that \( n^o \) firms producing all with high-quality technology can enter the market, where \( n^o \) is the positive root of

\[
\theta n^2 + (F_H + \eta \xi)n - \alpha' = 0,
\]

giving (34). Defining \( \phi^c_o \equiv \frac{F_L + \theta n^o}{\alpha'} \), it is easy to show (along the lines in the Proof of Proposition 1) that all incumbents prefer to use high-quality technology, and that there cannot be any entry of firms that use low-quality technology when \( \phi \leq \phi^c_o \). Similarly, \( 1 - \kappa_i \leq \mu \) for all firms when \( \mu \geq \mu^c \equiv \frac{\theta n^o}{(1 - \delta)(1 - \beta) \eta} \). Direct calculation now gives that

\[
\frac{\partial \varepsilon(n^o, \delta)}{\partial \alpha'} = \frac{\eta (1 - \beta) \delta}{F_H + 2 \theta n^o + \eta \xi} = \frac{(1 - \beta) \delta}{F_H + 2 \theta n^o + \eta \xi}.
\]

Thus,

\[
\frac{\partial \varepsilon(n^o, \delta)}{\partial \alpha'} = -\frac{2 \eta \delta (1 - \beta) \frac{\partial n^o}{\partial \alpha'}}{(F_H + 2 \theta n^o + \eta \xi)^2} < 0,
\]

and

\[
\frac{\partial \varepsilon(n^o, \delta)}{\partial \eta} = \delta (1 - \beta) \left( \frac{F_H + 2 \theta n^o}{\eta} - \frac{2 \theta n^o}{\eta} \right) > 0,
\]

and

\[A7\]
\[
\frac{\partial \varepsilon(n^\circ, \delta)}{\partial \beta} = -\frac{\eta \delta}{F_H + 2\theta n^\circ + \eta \xi} - \frac{\eta (1 - \beta) \delta}{(F_H + 2\theta n^\circ + \eta \xi)^2} \left[ 2\theta \frac{\partial n^\circ}{\partial \beta} + \eta (1 - \delta) \right]
\]

\[
= -\frac{\eta \delta (F_H + 2\theta n^\circ + \eta \xi) + \eta (1 - \beta) \delta \left[ \eta(1 - \delta) - 2\theta \left( \frac{\eta \delta(1 - \delta)}{F_H + 2\theta n^\circ + \eta \xi} \right) \right]}{(F_H + 2\theta n^\circ + \eta \xi)^2}
\]

\[
= -\frac{\eta \delta (F_H + 2\theta n^\circ + \eta \xi) + \eta (1 - \beta) \delta \eta(1 - \delta) \left[ \frac{F_H + \eta \xi}{F_H + 2\theta n^\circ + \eta \xi} \right] < 0.
\]

Proof of Proposition 9. Low-quality technology is sustainable in equilibrium if

\[
\phi > \phi^\circ \equiv \frac{F_L + \theta n^\circ}{n^\circ} \iff \phi \frac{\alpha'}{n^\circ} - F_L - \theta n^\circ > 0.
\]

When expression (A-46) holds, as in the limiting case where \( \delta = 0 \), the 1st \( n^{o''} \) firms choose high-quality technology and \( n^{o'} - n^{o''} \) select low-quality technology. Equilibrium is determined by 3 conditions: The entry condition for the \( n^{o''} \):th entrepreneur,

\[
(1 - \phi) \left( \frac{\alpha'}{n^{o''} + \phi(n^{o'} - n^{o''})} \right) - (F_H + \theta n^{o''}) - \xi \eta \geq 0;
\]

the condition that entrepreneurs prefer to raise \( F_{H,n^{o''}} \), and select high-quality technology, rather than to raise \( F_{L,n^{o''}} \) and select low-quality technology, that is,

\[
(1 - \phi) \left( \frac{\alpha'}{n^{o''} + \phi(n^{o'} - n^{o''})} \right) - (F_H - F_L) - (\xi - \mu \beta) \eta \geq 0;
\]

and the entry condition for the \( n^{o'} \):th low-quality producer,

\[
\phi \left( \frac{\alpha'}{n^{o''} + \phi(n^{o'} - n^{o''})} \right) = F_L + \theta n^{o'}.
\]

Furthermore, 2 of the 3 conditions bind, equation (A-49) and either expression (A-47) or (A-48). Expression (A-48) is implied by expression (A-47) when

\[
(1 - \phi) \theta n^{o''} + (\mu \beta - \phi \xi) \eta + F_L - \phi F_H > 0.
\]

This is satisfied when

\[
\frac{(\mu - \phi) \beta \eta + F_L - \phi F_H}{\phi (1 - \beta) \eta} \equiv \bar{\delta} \geq \delta.
\]
Now $\delta^0 > 0$ when $(\mu - \phi)\beta \eta + F_L - \phi F_H > 0$. As $F_L - \phi F_H > 0$, there exists $\beta^0 > 0$ such that this holds for all $\beta < \beta^0$.

In this case, using expressions (A-47) and (A-49) as equalities gives

\[
(A-52) \quad n'' = \frac{n'}{\phi} - \frac{\phi F_H - F_L + \phi \xi \eta}{\theta \phi}.
\]

This can be used in expression (A-47) or (A-49) to substitute for either $n^0'$ or $n^{0''}$ to verify that $n^{0''}$ is decreasing in $\delta$, while $n^{0'}$ is increasing in $\delta$. The claim on total production can now be verified, as an increase in $\delta$ must lead to a decrease in total output $\alpha'/\bar{p}$ as $\bar{p} = \sqrt{\frac{\alpha'}{n^{0'} + \phi(n^{0'} - n^{0''})}}$, which increases by equation (A-49), given the result that $n^{0'}$ increases in $\delta$.

\textbf{Proof of the Claims Related to Table 1.}

The comparative statics results for $n^*$ follow from the results in Propositions 2 and 3 and equation (13). The comparative statics results in Table 1 related to the partial derivatives with respect to $\eta$ follow from the results in Propositions 2 and 3 given that the ratios are

\[
(A-53) \quad \left( \frac{D_i^*}{S_i^*} \right)^{\text{ind}} = \frac{\int D_i^* di}{\int S_i^* di} = \frac{\int \left( \frac{\alpha}{n^*} \right)^2 - \eta di}{\int (1 - \beta) \eta - \theta (n^* - i) di} = \frac{F_H + \theta n^* + \eta (\beta - 1)}{(1 - \beta) \eta - \frac{\eta n^*}{2}},
\]

\[
(A-54) \quad \left( \frac{S_i^*}{E_i^*} \right)^{\text{ind}} = \frac{\int S_i^* di}{\int E_i^* di} = \frac{\int (1 - \beta) \eta - \theta (n^* - i) di}{\int (1 - \beta) \eta di} = 1 - \frac{\theta n^*}{(1 - \beta) \eta},
\]

\[
(A-55) \quad (\omega_i)^{\text{ind}} = \frac{\int [E_i^* - S_i^*] di}{\int E_i^* di} = \frac{\int (1 - \beta) \eta - (1 - \beta) \eta + \theta (n^* - i) di}{\int (1 - \beta) \eta di} = \frac{\theta n^*}{(1 - \beta) \eta},
\]

\[
(A-56) \quad (\text{ROA}_i^*)^{\text{ind}} = \frac{\int X_i^{T} di}{\int F_i^* di} = \frac{\int (\frac{\alpha}{n^*})^2 di}{\int [F_H + \theta i] di} = \frac{(\frac{\alpha}{n^*})^2}{F_H + \frac{\theta n^*}{2}} - 1.
\]

The comparative statics results for the partial derivative with respect to $\beta$ also follow from the results in Propositions 2 and 3. First note that

\[
(A-57) \quad \left( \frac{D_i^*}{S_i^*} \right)^{\text{ind}} = \frac{\int D_i^* di}{\int S_i^* di} = \frac{\int \left( \frac{\alpha}{n^*} \right)^2 - \eta di}{\int (1 - \beta) \eta - \theta (n^* - i) di} = \frac{(\frac{\alpha}{n^*})^2 - \eta}{\eta - \beta \eta - \frac{\theta n^*}{2}}.
\]
increases in $\beta$. This result follows as the fact that $(\frac{\alpha}{n})^2 = F_H + \theta n^* + \eta\beta$ increases in $\beta$ implies that $\eta - \beta \eta - \frac{1}{2} \theta n^*$ decreases in $\beta$.

Next note that

$$(A-58) \quad \left( \frac{S_i^*}{E_i^{M*}} \right)^{\text{ind}} = \frac{\int S_i^* d\theta}{\int E_i^{M*} d\theta} = \frac{\int [(1 - \beta)\eta - \theta(n^* - i)] d\theta}{\int [1 - (1 - \beta)\eta] d\theta} = 1 - \frac{\theta n^*}{(1 - \beta)\eta}$$

decreases and

$$(A-59) \quad (\omega_i)^{\text{ind}} = \frac{\int [E_i^{M*} - S_i^*] d\theta}{\int E_i^{M*} d\theta} = \frac{\int [(1 - \beta)\eta - (1 - \beta)\eta + \theta(n^* - i)] d\theta}{\int (1 - \beta)\eta} = \frac{\theta n^*}{(1 - \beta)\eta}$$

increases in $\beta$ as

$$(A-60) \quad \frac{\theta n^*}{(1 - \beta)\eta} = \frac{\theta}{2} \frac{\alpha}{\sqrt{(F_H + \theta n^* + \eta\beta)(1 - \beta)^2\eta^2}}$$

increases in $\beta$ when $(F_H + \eta\beta)(1 - \beta)^2\eta^2$ decreases in $\beta$. This, in turn, occurs as taking derivatives

$$(A-61) \quad \frac{\partial (F_H + \eta\beta)(1 - \beta)^2\eta^2}{\partial \beta} = \eta(1 - \beta)^2\eta^2 - 2(F_H + \eta\beta)(1 - \beta)\eta^2$$

$$= [\eta(1 - \beta) - 2(F_H + \eta\beta)]\eta^2(1 - \beta) < 0$$

under the assumption that $F_H > \eta$, as is implied by our assumption that $F_L > \phi F_H$. Also,

$$(A-62) \quad (\text{ROA}_i^*)^{\text{ind}} = \frac{\int X_i^T d\theta}{\int F_i^* d\theta} = \frac{\int (\frac{\alpha}{n})^2 d\theta}{\int [F_H + \theta i] d\theta} = \frac{(\frac{\alpha}{n})^2}{F_H + \frac{\theta n^*}{2}}$$

increases in $\beta$.

The comparative statics results for the partial derivatives with respect to $\theta$ follow from the results in Propositions 2 and 3 and the fact that $\theta n^*$ is increasing in while $n^*$ is decreasing in $\theta$ given equation(13), as the relevant ratios can be written as

$$(A-63) \quad \left( \frac{D_i^*}{S_i^*} \right)^{\text{ind}} = \frac{\int D_i^* d\theta}{\int S_i^* d\theta} = \frac{\int \left[ (\frac{\alpha}{n})^2 - \eta \right] d\theta}{\int [(1 - \beta)\eta - \theta(n^* - i)] d\theta} = \frac{(\frac{\alpha}{n})^2 - \eta}{(1 - \beta)\eta - \frac{\theta n^*}{2}}$$

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\[(A-64) \quad \left( \frac{S_i^*}{E_i^M*} \right)_{\text{ind}} = \int_i \frac{S_i^* \text{di}}{E_i^M* \text{di}} = \int_i \frac{[(1 - \beta)\eta - \theta(n^*-i)] \text{di}}{[(1 - \beta)\eta] \text{di}} = 1 - \frac{\theta n^*}{(1 - \beta)\eta},\]

\[(A-65) \quad (\omega_i)_{\text{ind}} = \int_i \frac{E_i^M* - S_i^* \text{di}}{E_i^M* \text{di}} = \int_i \frac{[(1 - \beta)\eta - (1 - \beta)\eta + \theta(n^*-i)] \text{di}}{(1 - \beta)\eta \text{di}} = \frac{\theta n^*}{(1 - \beta)\eta},\]

\[(A-66) \quad (\text{ROA}_i^*)_{\text{ind}} = \int_i \frac{X_i^T \text{di}}{F_i^\text{di}} = \frac{\int_i \left( \frac{\alpha}{n^*} \right)^2 \text{di}}{\int_i [F_H + \theta i] \text{di}} - 1 = \frac{\left( \frac{\alpha}{n^*} \right)^2}{F_H + \frac{\theta n^*}{2}} - 1 = \frac{\beta \eta + \frac{\theta n^*}{2}}{F_H + \frac{\theta n^*}{2}}.\]

The last result follows as $\beta \eta < F_H$ by our assumption that $F_L > \phi F_H$. 

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