A corporate-crime perspective on fisheries: liability rules and non-compliance

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ONLINE APPENDIX
Appendix A: The basic model

A.1. Reaction functions

In this section, we derive and characterize the reaction functions presented and analyzed in the paper.

From the main text, the cost function satisfies the following properties:

\[
\begin{align*}
\frac{\partial c}{\partial h_{it}} & > 0, \quad \frac{\partial^2 c}{\partial h_{it}^2} > 0, \quad \frac{\partial c}{\partial h_{tt}} > 0, \quad \frac{\partial^2 c}{\partial h_{tt}^2} > 0, \quad \frac{\partial^2 c}{\partial h_{il} \partial h_{tt}} > 0, \quad \frac{\partial c}{\partial x_i} < 0, \\
\frac{\partial^2 c}{\partial h_{it} \partial x_i} & < 0, \quad \frac{\partial^2 c}{\partial h_{il}^2} < 0.
\end{align*}
\]

(A1)

In addition, the properties of the penalty functions are:

\[
\begin{align*}
G'(h_{it}) & > 0, \quad G''(h_{it}) > 0, \quad F'(h_{it}) > 0, \quad F''(h_{it}) > 0.
\end{align*}
\]

(A2)

From section 2.1, we have the following first-order conditions for the private optimum:

\[
p - \frac{\partial c}{\partial h_{it}} - \varepsilon_i = 0
\]

(A3)

\[
p - \frac{\partial c}{\partial h_{it}} - \gamma \left[ F'(h_{it}) + G'(h_{it}) \right] = 0
\]

(A4)

\[
h_{it} = Q_i.
\]

(A5)

We can express the equation system (A3)-(A5) as the reaction function presented in the main text:

\[
h_{it} = h_{it}(Q_i, x_i, \gamma).
\]

(A6)

Total differentiating (A3)-(A5) yields the following:

\[
\frac{\partial^2 c}{\partial h_{il}^2} dh_{it} + \frac{\partial^2 c}{\partial h_{il} \partial h_{tt}} dh_{tt} + d\varepsilon = - \frac{\partial^2 c}{\partial h_{il} \partial x_i} dx_i
\]

(A7)
\[ \frac{\partial^2 c}{\partial h_{r} \partial h_{r}} dh_{r} + \left[ \frac{\partial^2 c}{\partial h_{r}^2} + \gamma (F''(h_{r}) + G''(h_{r})) \right] dh_{r} = \]

\[ - \frac{\partial^2 c}{\partial h_{r} \partial x_{r}} dx_{r} - [F'(h_{r}) + G'(h_{r})]d\gamma \]

\[ dh_{r} = dQ_{r} . \]  

(A8)

(A9)

Inserting equation (A9) into (A7) and (A8) yields:

\[ \frac{\partial^2 c}{\partial h_{r} \partial h_{r}} dh_{r} + d\epsilon = - \frac{\partial^2 c}{\partial h_{r} \partial x_{r}} dx_{r} - \frac{\partial^2 c}{\partial h_{r}^2} dQ_{r} \]

\[ \left[ \frac{\partial^2 c}{\partial h_{r}^2} + \gamma (F''(h_{r}) + G''(h_{r})) \right] dh_{r} = - \frac{\partial^2 c}{\partial h_{r} \partial x_{r}} dx_{r} - \]

\[ [F'(h_{r}) + G'(h_{r})]d\gamma - \frac{\partial^2 c}{\partial h_{r} \partial h_{r}} dQ_{r} . \]

(A10)

(A11)

Note that equation (A11) only depends on \( dh_{r} \). Using this equation, we can now find \( \frac{dh_{r}}{dx_{r}} \) by setting \( d\gamma = dQ_{r} = 0 \):

\[ \frac{dh_{r}}{dx_{r}} = - \frac{\frac{\partial^2 c}{\partial h_{r} \partial x_{r}}}{\frac{\partial^2 c}{\partial h_{r}^2} + \gamma (F''(h_{r}) + G''(h_{r}))} . \]

(A12)

In (A12), the denominator is positive because \( 0 < \gamma < 1 \), \( \frac{\partial^2 c}{\partial h_{r}^2} > 0 \), \( F''(h_{r}) > 0 \), and \( G''(h_{r}) \)

(cf. equations (A1) and (A2)). From (A1) we also have that \( \frac{\partial^2 c}{\partial h_{r} \partial x_{r}} < 0 \), which implies that

\[ \frac{dh_{r}}{dx_{r}} > 0 . \]

Turning to \( \frac{dh_{r}}{d\gamma} \), we set \( dx_{r} = dQ_{r} = 0 \) in (A11) and obtain:
\[
\frac{dh}{d\gamma} = -\frac{[F'(h) + G'(h)]}{\frac{\partial^2 c}{\partial h^2} + \gamma(F''(h) + G''(h))}.
\] 

(A13)

The denominator is identical to the one in equation (A12), and is thus positive, and from (A2) we have that \(G'(h) > 0\) and \(F'(h) > 0\). Consequently, we find that \(\frac{dh}{d\gamma} < 0\).

Let us finally determine the effect of quota on illegal harvest. We let \(d\gamma = dx = 0\) in (A11), and arrive at:

\[
\frac{dh}{dQ} = -\frac{\frac{\partial^2 c}{\partial h_i \partial h_i}}{\frac{\partial^2 c}{\partial h_i^2} + \gamma(F''(h) + G''(h))}.
\] 

(A14)

From before, we know that both the denominator and the numerator are positive, since \(\frac{\partial^2 c}{\partial h_i \partial h_i} > 0\). This implies that \(\frac{dh}{dQ} < 0\).

**A.2 Enforcement costs**

In this section, we derive and characterize the enforcement cost function used in the paper.

We start out by inverting the reaction function in (A6), which yields:

\[
\gamma = \gamma(Q, x, h).
\] 

(A15)

Total differentiating (A15) produces:

\[
\frac{\partial\gamma}{\partial Q} dQ + \frac{\partial\gamma}{\partial x_i} dx_i + \frac{\partial\gamma}{\partial h_i} dh_i = 0.
\] 

(A16)

Next, we define the probability of being detected as a function of enforcement effort, \(\gamma(e_i)\), and we assume that:

\[
\frac{\partial\gamma}{\partial e_i} > 0.
\] 

(A17)

Note that we can invert \(\gamma(e_i)\) to yield \(e_i(\gamma)\), and because of (A17) we obtain the following:
\[ \frac{\partial e_i}{\partial \gamma} = \frac{1}{\frac{\partial e_i}{\partial \gamma}} > 0 \]. \quad (A18) \\

Substituting the inverted reaction function into \( e_i(\gamma) \) gives \( e_i = e_i(\gamma(Q_x, x, h)) = \alpha(Q_x, x, h) \). Now, we want to find the sign of the derivatives of \( \alpha(Q_x, x, h) \). First, we investigate the sign of \( \frac{\partial \alpha}{\partial h_r} \) by using:

\[ \frac{\partial \alpha}{\partial h_r} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial h_r}. \] \quad (A19)

From (A18), we have that \( \frac{\partial e_i}{\partial \gamma} > 0 \) and we note that:

\[ \frac{\partial \gamma}{\partial h_r} = \frac{1}{\frac{\partial h_r}{\partial \gamma}}. \] \quad (A20)

From (A13) we have that \( \frac{\partial h_r}{\partial \gamma} < 0 \), and consequently, from (A20) we get that \( \frac{\partial \gamma}{\partial h_r} < 0 \). Using (A19) now implies that \( \frac{\partial \alpha}{\partial h_r} < 0 \).

Concerning the sign of \( \frac{\partial \alpha}{\partial x_i} \) we have that:

\[ \frac{\partial \alpha}{\partial x_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial x_i}. \] \quad (A21)

As above \( \frac{\partial e_i}{\partial \gamma} > 0 \). By setting \( dQ = 0 \) in (A16) and solving for \( \frac{\partial \gamma}{\partial x_i} \) we reach:

\[ \frac{\partial \gamma}{\partial x_i} = \frac{-\frac{\partial h_r}{\partial x_i}}{\frac{\partial h_r}{\partial \gamma}}. \] \quad (A22)
From (A12), we know that $\frac{\partial h}{\partial x_t} > 0$, and from (A20) that $\frac{\partial h}{\partial \gamma} < 0$. Combining this with (A22) gives us that $\frac{\partial \gamma}{\partial x_t} > 0$, which in turn implies that $\frac{\partial \alpha}{\partial x_t} > 0$.

Finally, we find the sign of $\frac{\partial \alpha}{\partial Q_t}$ by using that:

$$\frac{\partial \alpha}{\partial Q_t} = \frac{\partial e}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t}. \quad (A23)$$

Setting $dx_t = 0$ in (A16) and solving for $\frac{\partial \gamma}{\partial Q_t}$ we get:

$$\frac{\partial \gamma}{\partial Q_t} = -\frac{\partial h}{\partial Q_t} \frac{1}{\frac{\partial h}{\partial \gamma}}. \quad (A24)$$

We have established that $\frac{\partial h}{\partial \gamma} < 0$, and from (A14) we learned that $\frac{\partial h}{\partial Q_t} < 0$. Therefore, (A24) implies that $\frac{\partial \gamma}{\partial Q_t} < 0$, and using this in (A23) gives us that $\frac{\partial \alpha}{\partial Q_t} < 0$.

Let us next turn to the enforcement cost function, $K(e_t)$. We assume that:

$$\frac{\partial K}{\partial e_t} > 0 \text{ and } \frac{\partial^2 K}{\partial e_t^2} > 0. \quad (A25)$$

From above, $e_t = e_t(\gamma(Q_t, x_t, h_t)) = \alpha(Q_t, x_t, h_t)$ and inserting this into the enforcement cost function gives $K(e_t(\gamma(Q_t, x_t, h_t))) = F(\alpha(Q_t, x_t, h_t)) = E(Q_t, x_t, h_t)$. We now want to determine the signs of the derivatives of the enforcement cost function, and we start by considering $\frac{\partial E}{\partial Q_t}$:

$$\frac{\partial E}{\partial Q_t} = \frac{\partial F}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t} = \frac{\partial K}{\partial e_t} \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t}. \quad (A26)$$
From (A23), \( \frac{\partial \alpha}{\partial Q_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial Q_i} < 0 \), and from (A25) we have that \( \frac{\partial K}{\partial e_i} > 0 \), which implies that \( \frac{\partial E}{\partial Q_i} < 0 \).

Next, for \( \frac{\partial E}{\partial x_i} \) we have that:

\[
\frac{\partial E}{\partial x_i} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial x_i} = \frac{\partial K}{\partial e_i} \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial x_i}.
\]  \hspace{1cm} (A27)

Using (A21), we have that \( \frac{\partial \alpha}{\partial x_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial x_i} > 0 \), and according to (A25), \( \frac{\partial K}{\partial e_i} > 0 \), from which it follows that \( \frac{\partial E}{\partial x_i} > 0 \).

Finally, for \( \frac{\partial E}{\partial h_i} \) we get:

\[
\frac{\partial E}{\partial h_i} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial h_i} = \frac{\partial K}{\partial e_i} \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial h_i}.
\]  \hspace{1cm} (A28)

From (A19) we know that \( \frac{\partial \alpha}{\partial h_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial h_i} < 0 \), and using (A23) we have that \( \frac{\partial K}{\partial e_i} > 0 \). This implies that \( \frac{\partial E}{\partial h_i} < 0 \).
Appendix B: Share of profit

B.1. Reaction functions

From section 3 we have the following first-order conditions:

\[ \frac{\partial W}{\partial h_{L,t}} - \alpha \frac{\partial c}{\partial h_{L,t}} - u_t = 0 \]  
(B1)

\[ \frac{\partial W}{\partial h_{R}} - \alpha \frac{\partial c}{\partial h_{R}} - \gamma G'(h_R) = 0 \]  
(B2)

\[ h_{L,t} = Q_t. \]  
(B3)

We also have the following wage scheme from section 3:

\[ W(h_{L,t}, h_R) = \beta \left[ p_t (h_{L,t} + h_R) - (1 - \alpha) c(h_{L,t}, h_R, x_t) - \gamma F(h_R) \right]. \]  
(B4)

From the wage scheme in (B4) we may obtain:

\[ \frac{\partial W}{\partial h_{L,t}} = \beta (p - (1 - \alpha) \frac{\partial c}{\partial h_{L,t}}) \]  
(B5)

\[ \frac{\partial W}{\partial h_{R}} = \beta (p - (1 - \alpha) \frac{\partial c}{\partial h_{R}} - \gamma F'(h_R)). \]  
(B6)

(B5) can be substituted into (B1) and (B6) into (B2). This gives the following rewritten first-order conditions:

\[ \beta (p - \frac{\partial c}{\partial h_{L,t}}) - (1 - \beta) \alpha \frac{\partial c}{\partial h_{L,t}} - u_t = 0 \]  
(B7)

\[ \beta (p - \frac{\partial c}{\partial h_{R}}) - (1 - \beta) \alpha \frac{\partial c}{\partial h_{R}} - \gamma (\beta F'(h_R) + G'(h_R)) = 0 \]  
(B8)

\[ h_{L,t} = Q_t. \]  
(B9)

(B7) - (B9) may be total differentiated which gives:
\[
[\beta \frac{\partial^2 c}{\partial h_{1s}^2} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{1s}^2} ]dh_{ts} + \beta \frac{\partial^2 c}{\partial h_{ts}\partial h_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}
\]

\[(1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dh_{ts} + du_{t} = -[\beta \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dx_{t}.
\]

\[
[\beta \frac{\partial^2 c}{\partial h_{ts}\partial h_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dh_{ts} + [\beta \frac{\partial^2 c}{\partial h_{ts}^2} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}^2} + \gamma(\beta F''(h_{ts}) + G''(h_{ts}))]dh_{ts} = -(\beta F'(h_{ts}) + G'(h_{ts}))d\gamma
\]

\[
[\beta \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dx_{t}
\]

\[
dh_{ts} = dQ_{t}.
\]

(B12) can be substituted into (B10) and (B11) which gives:

\[
+ \beta \frac{\partial^2 c}{\partial h_{ts}\partial h_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dh_{ts} + du_{t} = \]

\[-[\beta \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dx_{t} - [\beta \frac{\partial^2 c}{\partial h_{ts}^2} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}^2}]dQ_{t}
\]

\[
[\beta \frac{\partial^2 c}{\partial h_{ts}\partial h_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}} + \gamma(\beta F''(h_{ts}) + G''(h_{ts}))]dh_{ts} = \]

\[-(\beta F'(h_{ts}) + G'(h_{ts}))d\gamma - [\beta \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}} + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial x_{ts}}]dx_{t} - \]

\[dQ_{t},
\]

\[
dh_{ts} \text{ is the only variable that enters in (B14) and, therefore, (B14) can be used to characterize the reaction function.}
\]

In (B14) we may set \(d\gamma = dx_{t} = 0\) and reach:

\[
\frac{dh_{ts}}{dQ_{t}} = -\frac{(\beta + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}\partial h_{ts}})}{(\beta + (1 - \beta)\alpha \frac{\partial^2 c}{\partial h_{ts}^2} + \gamma(\beta F''(h_{ts}) + G''(h_{ts}))}
\]

(B15)
We have that $0 < \beta < 1$, $0 < \alpha < 1, 0 < \gamma < 1$, $\frac{\partial^2 c}{\partial h_i^2} > 0$, $F''(h_i) > 0$ and $G''(h_i) > 0$ and this imply that the denominator in (B15) is positive. With respect to the nominator $\frac{\partial^2 c}{\partial h_i \partial h_{ij}} > 0$ so the nominator is also positive. In total, we, therefore, reach the conclusion that $\frac{dh_i}{dQ_i} < 0$.

Concerning $\frac{dh_i}{d\gamma}$ we set $dQ_i = dx_i = 0$ in (B14) and arrive at:

$$
\frac{dh_i}{d\gamma} = - \frac{\beta F'(h_i) + G'(h_i)}{(\beta + (1 - \beta)\alpha) \frac{\partial^2 c}{\partial h_i^2} + \gamma (\beta F''(h_i) + G''(h_i))}.
$$

(B16)

From (B15) we have that the denominator is positive and, in addition, the nominator in (B16) is positive because $G'(h_i) > 0$ and $F'(h_i) > 0$. Therefore, we obtain that $\frac{dh_i}{d\gamma} < 0$.

Last, by setting $dQ_i = d\gamma = 0$ we reach:

$$
\frac{\partial h_i}{\partial x_i} = - \frac{(\beta + (1 - \beta)\alpha) \frac{\partial^2 c}{\partial h_i \partial x_i}}{(\beta + (1 - \beta)\alpha) \frac{\partial^2 c}{\partial h_i^2} + \gamma (\beta F''(h_i) + G''(h_i))}.
$$

(B17)

From above the denominator is positive. In addition, we have that $\frac{\partial^2 c}{\partial h_i \partial x_i} < 0$ so the nominator is negative. In total, this implies that $\frac{\partial h_i}{\partial x_i} > 0$.

**B.2. Enforcement costs**

The inverted reaction function is:

$$
\gamma = \gamma(Q_i, x_i, h_i).
$$

(B18)

From (B18) we get:
\[
\frac{\partial \gamma}{\partial Q} dQ_i + \frac{\partial \gamma}{\partial x_i} dx_i + \frac{\partial \gamma}{\partial h_i} dh_i = 0. \quad (B19)
\]

Now \(\gamma(e_i)\) is the probability of being detected as a function of enforcement effort and we have:

\[
\frac{\partial \gamma}{\partial e_i} > 0. \quad (B20)
\]

We invert \(\gamma(e_i)\) to get \(e_i(\gamma)\) and due to (B20) we obtain:

\[
\frac{\partial e_i}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e_i}} > 0. \quad (B21)
\]

\(\gamma = \gamma(Q_i, x_i, h_i)\) can be used in \(e_i(\gamma)\) and this gives \(e_i = e_i(\gamma(Q_i, x_i, h_i)) = \alpha(Q_i, x_i, h_i)\). Now we can find the sign of \(\frac{\partial \alpha}{\partial h_i}\) by using:

\[
\frac{\partial \alpha}{\partial h_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial h_i}. \quad (B22)
\]

From (B21) \(\frac{\partial e_i}{\partial \gamma} > 0\) and furthermore we have that:

\[
\frac{\partial \gamma}{\partial h_i} = \frac{1}{\frac{\partial h_i}{\partial \gamma}}. \quad (B23)
\]

From (B16) \(\frac{\partial h_i}{\partial \gamma} < 0\) and by using this in (B23) we obtain \(\frac{\partial \gamma}{\partial h_i} < 0\). Now (B22) now imply that \(\frac{\partial \alpha}{\partial h_i} < 0\).

For the sign of \(\frac{\partial \alpha}{\partial x_i}\) we have:

\[
\frac{\partial \alpha}{\partial x_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial x_i}. \quad (B24)
\]
In (B21) it was stated that \( \frac{\partial e_i}{\partial \gamma} > 0 \) and using by \( dQ_i = 0 \) in (B19) it is obtained that:

\[
\frac{\partial \gamma}{\partial x_i} = -\frac{\partial h_{hi}}{\partial h_{hi}}. 
\]  

(B25)

From (B17) \( \frac{\partial h_{hi}}{\partial x_i} > 0 \), and in (B16) we reached that \( \frac{\partial h_{hi}}{\partial \gamma} < 0 \). Combining this in (B25)

\[
\frac{\partial \gamma}{\partial x_i} > 0,
\]

which by using (B24) gives \( \frac{\partial \alpha}{\partial x_i} > 0 \).

Lastly, we turn attention to the sign of \( \frac{\partial \alpha}{\partial Q_i} \) where we have:

\[
\frac{\partial \alpha}{\partial Q_i} = \frac{\partial e_i}{\partial Q_i} \frac{\partial \gamma}{\partial Q_i}. 
\]  

(B26)

Using \( dx_i = 0 \) in (B19) and solving for \( \frac{\partial \gamma}{\partial Q_i} \) we get:

\[
\frac{\partial \gamma}{\partial Q_i} = -\frac{\partial h_{hi}}{\partial h_{hi}}. 
\]  

(B27)

From (B16) \( \frac{\partial h_{hi}}{\partial \gamma} < 0 \) and using (B15) implies that \( \frac{\partial h_{hi}}{\partial Q_i} < 0 \). Therefore, \( \frac{\partial \gamma}{\partial Q_i} < 0 \) and by using this in (B26) it follows that \( \frac{\partial \alpha}{\partial Q_i} < 0 \).

Now the enforcement cost function is given as \( K(e_i) \) and we assume that:

\[
\frac{\partial K}{\partial e_i} > 0 \quad \text{and} \quad \frac{\partial^2 K}{\partial e_i^2} > 0. 
\]  

(B28)
From before \( e_t = e_t(\gamma(Q_t, x_t, h_t)) = \alpha(Q_t, x_t, h_t) \) and inserting this in the enforcement cost function gives \( K(e_t(\gamma(Q_t, x_t, h_t))) = F(\alpha(Q_t, x_t, h_t)) = E(Q_t, x_t, h_t) \). Now we can find the sign of the derivatives and we start by \( \frac{\partial E}{\partial Q_t} \) where we have:

\[
\frac{\partial E}{\partial Q_t} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial Q_t} = \frac{\partial K}{\partial e_t} \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t}. \tag{B29}
\]

In (B26) \( \frac{\partial \alpha}{\partial Q_t} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial Q_t} < 0 \) and from (B28) \( \frac{\partial K}{\partial e_t} > 0 \), implying that \( \frac{\partial E}{\partial Q_t} < 0 \).

Next for the sign of \( \frac{\partial E}{\partial x_i} \) we have that:

\[
\frac{\partial E}{\partial x_i} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial x_i} = \frac{\partial K}{\partial e_t} \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial x_i}. \tag{B30}
\]

Using (B24) we have that \( \frac{\partial \alpha}{\partial x_i} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial x_i} > 0 \) and from (B28) \( \frac{\partial K}{\partial e_t} > 0 \) which implies that \( \frac{\partial E}{\partial x_i} > 0 \).

Last for the sign of \( \frac{\partial E}{\partial h_t} \) we get:

\[
\frac{\partial E}{\partial h_t} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial h_t} = \frac{\partial K}{\partial e_t} \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial h_t}. \tag{B31}
\]

From (B22) \( \frac{\partial \alpha}{\partial h_t} = \frac{\partial e_t}{\partial \gamma} \frac{\partial \gamma}{\partial h_t} < 0 \) and using (B28) \( \frac{\partial K}{\partial e_t} > 0 \). In total, this implies that \( \frac{\partial E}{\partial h_t} < 0 \).
Appendix C: Share of revenue

C.1. Reaction functions

With the share of revenue rule the wage function is:

\[ W(h_{t_1}, h_{t_2}) = \beta \left( p \left( h_{t_1} + h_{t_2} \right) \right). \]  (C1)

The general first-order conditions for the employee are given by (B1)-(B3) in appendix B.1.

Inserting the derivatives of (C1) in the first-order conditions gives:

\[ p - \alpha \frac{\partial c}{\partial h_{t_1}} - u_t = 0 \]  (C2)
\[ p - \alpha \frac{\partial c}{\partial h_{t_2}} - \gamma G'(h_{t_2}) = 0 \]  (C3)
\[ h_{t_1} = Q_t. \]  (C4)

By total differentiating (C2) - (C4) we get that:

\[ \alpha \frac{\partial^2 c}{\partial h_{t_1}^2} dh_{t_1} + \alpha \frac{\partial^2 c}{\partial h_{t_1} \partial h_{t_2}} dh_{t_2} + du_t = -\alpha \frac{\partial^2 c}{\partial h_{t_1} \partial x_t} dx_t \]  (C5)
\[ \alpha \frac{\partial^2 c}{\partial h_{t_2} \partial h_{t_1}} dh_{t_1} + [\alpha \frac{\partial^2 c}{\partial h_{t_2}^2} + \gamma G''(h_{t_2})] dh_{t_2} = -\alpha \frac{\partial^2 c}{\partial h_{t_2} \partial x_t} dx_t - G'(h_{t_2}) d\gamma \]  (C6)
\[ dh_{t_1} = dQ_t. \]  (C7)

(C7) can be inserted into (C5) and (C6) which yields:

\[ \alpha \frac{\partial^2 c}{\partial h_{t_1} \partial h_{t_2}} dh_{t_2} + du_t = -\alpha \frac{\partial^2 c}{\partial h_{t_1} \partial x_t} dx_t - \alpha \frac{\partial^2 c}{\partial h_{t_2}^2} dQ_t \]  (C8)
\[ [\alpha \frac{\partial^2 c}{\partial h_{t_2}^2} + \gamma G'(h_{t_2})] dh_{t_2} = -\alpha \frac{\partial^2 c}{\partial h_{t_2} \partial x_t} dx_t - G'(h_{t_2}) d\gamma - \alpha \frac{\partial^2 c}{\partial h_{t_2} \partial h_{t_1}} dQ_t. \]  (C9)
Since (C9) only depends on $dh_t$, this equation is the one we will consider to derive the properties of the reaction function.

First, we investigate the sign of $\frac{\partial h_t}{\partial Q_t}$ and by setting $d\gamma = dx_i = 0$ in (C9) we reach:

$$\frac{\partial h_t}{\partial Q_t} = -\frac{\alpha \frac{\partial^2 c}{\partial h_t \partial h_t} - \frac{\partial^2 c}{\partial h_t^2} + \gamma G''(h_t)}{\alpha \frac{\partial^2 c}{\partial h_t^2} + \gamma G''(h_t)}.$$  \hspace{1cm} (C10)

Concerning (C10) $0 < \alpha < 1$ $0 < \gamma < 1$ $\alpha \frac{\partial^2 c}{\partial h_t^2} > 0$ and $G''(h_t) > 0$ so the denominator is positive. The nominator is also positive because $\frac{\partial^2 c}{\partial h_t \partial h_t} > 0$. In total, (C10) therefore imply that $\frac{\partial h_t}{\partial Q_t} < 0$.

Setting $d\gamma = dQ_t = 0$ in (C9) gives:

$$\frac{\partial h_t}{\partial x_i} = -\frac{\alpha \frac{\partial^2 c}{\partial h_t \partial x_i} - \frac{\partial^2 c}{\partial h_t^2} + \gamma G''(h_t)}{\alpha \frac{\partial^2 c}{\partial h_t^2} + \gamma G''(h_t)}.$$  \hspace{1cm} (C11)

As in (C10) the denominator is positive. However, now $\frac{\partial^2 c}{\partial h_t \partial x_i} < 0$ so the nominator is negative and this imply that $\frac{\partial h_t}{\partial x_i} > 0$.

Last, we evaluate the sign of $\frac{\partial h_t}{d\gamma}$ by setting $dx_i = dQ_t = 0$ in (C9). This gives:

$$\frac{\partial h_t}{d\gamma} = -\frac{G'(h_t)}{\alpha \frac{\partial^2 c}{\partial h_t^2} + \gamma G''(h_t)}.$$  \hspace{1cm} (C12)
The denominator is positive from (C10) and the nominator is also positive because
\[ G'(h_t) > 0. \] This implies that \( \frac{\partial h_t}{\partial \gamma} < 0. \)

**C.2. Enforcement costs**

As before we have an inverted the reaction function given by:
\[ \gamma = \gamma(Q, x, h_t). \] (C13)

(C13) can be total differentiating:
\[
\frac{\partial \gamma}{\partial Q} dQ + \frac{\partial \gamma}{\partial x_t} dx_t + \frac{\partial \gamma}{\partial h_t} dh_t = 0. \] (C14)

Now the probability of being detected is defined as \( \gamma(e) \) and we have that:
\[
\frac{\partial \gamma}{\partial e} > 0. \] (C15)

From \( \gamma(e) \) we get \( e(\gamma) \) and because of (C15) we have that:
\[
\frac{\partial e}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e}} > 0. \] (C16)

(C13) can be substituted into \( e(\gamma) \) to obtain \( e = e(\gamma(Q, x, h_t)) = \alpha(Q, x, h_t) \). Now we want to find the sign of the derivatives of \( \alpha(Q, x, h_t) \). First, we consider the sign of \( \frac{\partial \alpha}{\partial h_t} \):
\[
\frac{\partial \alpha}{\partial h_t} = \frac{\partial e}{\partial \gamma} \frac{\partial \gamma}{\partial h_t}. \] (C17)

From (C16) it is obtained that \( \frac{\partial e}{\partial \gamma} > 0 \). Furthermore, we have:
\[
\frac{\partial \gamma}{\partial h_t} = \frac{1}{\frac{\partial h_t}{\partial \gamma}}. \] (C18)
(C12) imply that \( \frac{\partial h_h}{\partial \gamma} < 0 \), and therefore we have that \( \frac{\partial \gamma}{\partial h_h} < 0 \) by using (C18). Now (C17) implies that \( \frac{\partial \alpha}{\partial h_h} < 0 \). Concerning the sign of \( \frac{\partial \alpha}{\partial x_i} \) we get that:

\[
\frac{\partial \alpha}{\partial x_i} = \frac{\partial e_i}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial x_i}.
\]  

(C19)

(C16) express that \( \frac{\partial e}{\partial \gamma} > 0 \) and by using \( dQ_i = 0 \) in (C14) we reach:

\[
\frac{\partial \gamma}{\partial x_i} = -\frac{\partial x_i}{\partial h_h} \cdot \frac{\partial h_h}{\partial \gamma}.
\]  

(C20)

In (C11) we have that \( \frac{\partial h_h}{\partial x_i} > 0 \), and from (C18) we reached that \( \frac{\partial h_h}{\partial \gamma} < 0 \). Combining this information implies that \( \frac{\partial \gamma}{\partial x_i} > 0 \) and using (C19) gives \( \frac{\partial \alpha}{\partial x_i} > 0 \).

Lastly, we find the sign of \( \frac{\partial \alpha}{\partial Q_i} \) by using that:

\[
\frac{\partial \alpha}{\partial Q_i} = \frac{\partial e_i}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial Q_i}.
\]  

(C21)

By setting \( dx_i = 0 \) in (C14) we get:

\[
\frac{\partial \gamma}{\partial Q_i} = -\frac{\partial Q_i}{\partial h_h} \cdot \frac{\partial h_h}{\partial \gamma}.
\]  

(C22)

From (C18) \( \frac{\partial h_h}{\partial \gamma} < 0 \) and furthermore we have that \( \frac{\partial h_h}{\partial Q_i} < 0 \) in (C10). Therefore, (C22) implies that \( \frac{\partial \gamma}{\partial Q_i} < 0 \) and using this in (C21) gives \( \frac{\partial \alpha}{\partial Q_i} < 0 \).

Now the enforcement cost function is given as \( K(e_i) \) and we assume that:
\[ \frac{\partial K}{\partial e_i} > 0 \text{ and } \frac{\partial^2 K}{\partial e_i^2} > 0. \]  

(C23)

Now we have that \( e_i = e_i(\gamma(Q_i, x_i, h_i)) = \alpha(Q_i, x_i, h_i) \) and inserting this in the enforcement cost function gives \( K(e_i(\gamma(Q_i, x_i, h_i))) = F(\alpha(Q_i, x_i, h_i)) = E(Q_i, x_i, h_i) \). Now we can find the sign of the derivatives of the enforcement cost function and we start by the sign of \( \frac{\partial E}{\partial Q_i} \) where we have:

\[ \frac{\partial E}{\partial Q_i} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial Q_i} + \frac{\partial K}{\partial e_i} \frac{\partial e_i}{\partial Q_i} \frac{\partial \gamma}{\partial Q_i}. \]  

(C24)

From (C21) we get that \( \frac{\partial \alpha}{\partial Q_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial Q_i} < 0 \) and from (C23) \( \frac{\partial K}{\partial e_i} > 0 \), implying that \( \frac{\partial E}{\partial Q_i} < 0 \).

Next for the sign of \( \frac{\partial E}{\partial x_i} \) we have that:

\[ \frac{\partial E}{\partial x_i} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial x_i} + \frac{\partial K}{\partial e_i} \frac{\partial e_i}{\partial x_i} \frac{\partial \gamma}{\partial x_i}. \]  

(C25)

Using (C19) we have that \( \frac{\partial \alpha}{\partial x_i} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial x_i} > 0 \) and using that \( \frac{\partial K}{\partial e_i} > 0 \) in (C23) this implies that \( \frac{\partial E}{\partial x_i} > 0 \).

Last for \( \frac{\partial E}{\partial h_{it}} \) we get:

\[ \frac{\partial E}{\partial h_{it}} = \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial h_{it}} + \frac{\partial K}{\partial e_i} \frac{\partial e_i}{\partial h_{it}} \frac{\partial \gamma}{\partial h_{it}}. \]  

(C26)

(C17) gives \( \frac{\partial \alpha}{\partial h_{it}} = \frac{\partial e_i}{\partial \gamma} \frac{\partial \gamma}{\partial h_{it}} < 0 \) and using (C23) we have that \( \frac{\partial K}{\partial e_i} > 0 \). In total, this implies that \( \frac{\partial E}{\partial h_{it}} < 0 \).