On the Strategic Use of Border Tax Adjustments as a Second-Best Climate Policy Measure∗

by

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Appendix A. Details of the Linear-Quadratic Example

In this appendix we provide calculations leading to various equations in the text. We start with the description of payoffs. Putting aside export payments and any damages from the carbon stock for the moment, each country $k$’s gross payoffs $V_k$ at any point in time are the sum of utility (area under its inverse demand curve) less production costs. With linear demand the amount purchased when the price in D is $p$ and the border tax is $\tau$ equals $Q_d^k = a_k - bP$, where $P = p$ in D and $P = p - \tau$ in U, and so we can write

$$\omega_k = \left(\frac{a_k}{b} + P\right)Q_d^k/2 = \frac{a_k^2}{2b} - \frac{b}{2}p^2.$$  

It is then easy to see that

$$\omega_u = \frac{a_u^2}{2b} - \frac{b}{2}(p - \tau)^2,$$

$$\omega_d = \frac{a_d^2}{2b} - \frac{b}{2}p^2.$$  

With linear marginal costs, costs in country $k$ are $\frac{Q_k^2}{2c}$. We saw in the main body of the paper that the output produced in country $k$ is $Q_k = c(p - T)$ if a per-unit charge $T_k$ is levied on firms in $k$, where $T_u = \tau$ and $T_d = F_d$. It follows that costs in country $k$ are $\frac{c^2}{2}(p - T_k)^2$. In the socially optimal problem, we have $F_d = \tau$, and so

$$V_u = \frac{a_u^2}{2b} - \left(\frac{b + c}{2}\right)(p - \tau)^2,$$  

$$V_d = \frac{a_d^2}{2b} - \frac{b}{2}p^2 - \frac{c}{2}(p - F_d)^2$$

$$= \frac{a_d^2}{2b} - \left(\frac{b + c}{2}\right)p^2 + cp\tau - \frac{c}{2}\tau^2.$$  

(1)
Upon adding export payments \((p - \tau)Y\) to \(V_u\) we get the flow payoffs for \(U\); after subtracting export payments \((p - \tau)Y\) and damages \(\frac{\delta}{2}Z^2\) from \(V_d\) we get the flow payoffs for \(D\). To express these forms in terms of \(p^0\), we apply eq. (24) from the main text; this yields \(p = p^0 + \frac{c}{b+\tau} \tau\) and so \(p - \tau = p^0 - \frac{b}{b+\tau} \tau\). Combining these elements, straightforward algebraic manipulation yields eqs. (35) and (36) in the main text.

To obtain the combined welfare flow in the social optimum problem we sum the payoffs in eqs. (1) and (2) from this Appendix and then substitute \(Q_k = Q/2\). To obtain each country’s steady-state payoffs, we first sum demand curves to get \(Q = a_d + a_u - 2bp + b\tau\), use this form to substitute for \(p\), and then sum the two countries’ welfare flows to get eq. (28) in the main text.

The form for \(U\)'s gross payoff function in \(D\)'s non-cooperative problem is the same as above. To obtain \(D\)'s gross payoff function, we note that \(F_d = \tau - \frac{Y}{c}\) so that \(p = p^0 + \frac{c}{b+\tau} \tau - \frac{Y}{2(b+\tau)}\) and so \(p - \tau = p^0 - \frac{b}{b+\tau} \tau - \frac{Y}{2(b+\tau)}\). We therefore have

\[
V_d = \frac{a_d^2}{2b} - \frac{b}{2} p^2 - \frac{c}{2} (p - F_d)^2 \\
= \frac{a_d^2}{2b} - \left(\frac{b + c}{2}\right) p^2 + cpF_d - \frac{c}{2} F_d^2 \\
= \frac{a_d^2}{2b} - \left(\frac{b + c}{2}\right) p^2 + cp\tau - (p - \tau)Y - \frac{c}{2\tau^2} - \frac{1}{2c} \tau^2 - \frac{1}{2c} Y^2.
\]

As above, after subtracting export payments \((p - \tau)Y\) and damages \(\frac{\delta}{2}Z^2\) from \(V_d\) we get the flow payoffs for \(D\):

\[
W_d = \frac{a_d^2}{2b} - \left(\frac{b + c}{2}\right) p^2 + cp\tau - 2(p - \tau)Y - \frac{c}{2} \tau^2 - \frac{1}{2c} Y^2 - \frac{\delta}{2} Z^2.
\]
As in the socially optimal program, the steady-state payoffs can be related to $p^0$; unlike the socially optimal program, the steady-state payoffs are also related to $Y^0(=3Y/2)$. That fact explains the ambiguity in comparing D’s steady-state payoffs between the two programs.

**Figure 1.** Phase diagram of the simple border-tax adjustment analysis