Appendix B

Measures for linear displacement can be converted into estimates of $\mu$, but this conversion may introduce bias. Suppose an experiment was terminated after 1 day. Then a biased estimated of $\mu$ could be calculated using equation 2:

$$\dot{\mu} \approx \frac{1}{4} \bar{d}^2$$

eqn B.1

Where $\bar{d}$ is the average movement distance. This estimate of $\mu$ from $\bar{d}$ underestimates the true value of motility because of the variability in the distance covered around the mean and the convex (upward curving) relationship between $\mu$ and $\bar{d}$ (Jensen’s inequality, e.g. Hilborn & Mangel, 1997, p. 58). A better estimate of $\mu$ is achieved by including the effect of the non-linear relationship between $\bar{d}$ and $\mu$ by using the Delta method (Hilborn & Mangel, 1997, p. 58):

$$E(g(d)) = g(\bar{d}) + \frac{1}{2} g''(\bar{d}) \text{var}(d)$$

eqn B.2

Where $E$ denotes the mathematical expectation, $g$ is a non-linear function linking motility and dispersal distance of individual beetles, i.c.

$$\mu_i = g(d_i, t_i) = \frac{1}{4} \frac{d_i^2}{t_i}$$

eqn B.3

$g''$ is the second derivative of $g$ with respect to $d$, and var($d$) denotes the variance of dispersal distance. The second derivative of $g$ is

$$g''(d_i, t_i) = \frac{1}{2t_i}$$

eqn B.4

At chosen $t_i$, the Delta method then yields:

$$\dot{\mu} \approx \frac{1}{4} \bar{d}^2 + \frac{1}{4} \text{var}(d)$$

eqn B.5