**Supporting Information**

Competing Effects of Buoyancy-Driven and Electrothermal Flows for Joule Heating-Induced Transport in Microchannels

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**S1. COMSOL solver**

In this paper, COMSOL Multiphysics was used for simulation purposes. This software employed finite element method (FEM) to find a numerical solution to the coupled PDEs for the momentum, energy, and electric field. COMSOL applies Newton-Raphson method to linearize these nonlinear equations, which creates a system of linear equations that are solved by MUMPS (MUltifrontal Massively Parallel Sparse Direct Solver). MUMPS perform factorization of the coefficient matrix into a product of matrices, using a method known as LU decomposition, where L is a lower triangular matrix and U is an upper triangular matrix. The LU decomposition allows the solution of the linear system of equations to be performed efficiently, by first solving the two triangular systems Ly = b and Ux = y.

In order to improve the accuracy of the solution, the nonlinear solver performs iterations. Then, the values of the variables at each iteration are updated based on the results of the previous iteration. The is solution assumed to be converged when the changes in the values of the variables are below the tolerance (convergence criterion). The default convergence criterion for heat transfer is set to 1e-6, meaning that the simulation will stop when the relative error falls below 1e-6. A smaller convergence criterion will require more computational time. On the other hand, a larger convergence criterion will result in faster simulations, but may not provide accurate results. Therefore, the default convergence criterion was used for modeling to avoid either high computational costs or low accurate results.

Beside the iterations, it is important to find proper number of nodes (mesh densities) to ensure that the results of the FEM analysis are mesh independent. Mesh independency test performed for the case of H=300 µ), σ=1.2 S/m, and φ=7 Vpp with triangular elements (**Fig. S-1A**) starting with a coarse mesh first and then refining the mesh to get a more accurate solution. According to the results, increasing number of elements more than 969,660 result in less than 4% change in the maximum velocity magnitude in the channel (**Fig. S-1B**). Also, **Fig. S-1C to F** demonstrates flow stabilization for different mesh sizes. This figure also shows the velocity distribution has no significant change in mesh sizes higher than 969,660. Besides, the computational time is 30 times more when the number of elements increase from 969,660 to 2,806,012 on a PC with 32 GB RAM and a Core i7 CPU. Therefore, about 969,660 elements were selected for all simulations in this paper to reduce the computational costs.

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***Figure S-1. A)*** *the triangular grid produced by COMSOL in the computational domain.* ***B)*** *Maximum velocity magnitude induced in the fluid when**H=300 µm, σ=1.2 S/m, and φ=7 Vpp for different number of grids. Speed contours, velocity vectors and the streamlines in the domain with* ***C)*** *17864,* ***D)*** *156592,* ***E)*** *969660,* ***F)*** *2806012 number of elements.*

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***Figure S-2.*** *Velocity vectors, speed contours and the streamlines for* ***A)*** *φ=1 Vpp, σ=1.2 S/m,* ***B)*** *φ=1 Vpp, σ=0.24 S/m,* ***C)*** *φ=7 Vpp, σ=1.2 S/m,* ***D)*** *φ=7 Vpp, σ=0.24 S/m.* ***E)*** *Maximum induced temperature in the channel for three different buffer conductivities.* ***F)*** *Maximum induced velocity in the channel for three different buffer conductivities.*

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***Figure S-3.*** *Velocity vectors, speed contours and the streamlines at φ=7 Vpp and for different channel heights of* ***A)*** *H=50 µm,* ***B)*** *H=100 µm,* ***C)*** *H=200 µm,* ***D)*** *H=300 µm,* ***E)*** *H=500 µm,* ***F)*** *H=800 µm.*

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***Figure S-4.*** *Temperature contours around the pair of electrodes within a microchannel with different heights for* ***A)*** *conduction only, and* ***B)*** *a combination of conduction and convection heat transfer mechanisms. The Rayleigh numbers are 182, 1200, 2850, and 5560 for column* ***B****,* ***i*** *to* ***iv****, respectively. The applied electric potential is φ=7 Vpp.*

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***Figure S-5.*** *Temperature contours and velocity vectors, speed contours and the streamlines around the pair of electrodes within a microchannel at φ=7 Vpp and H=300 µm for* ***A)*** *Constant temperature boundary condition, and* ***B)*** *Natural convection boundary condition.*