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RESEARCH ARTICLE

# Co-Evolutionary strategies at the collective level for improved generalism - Appendix

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## Abstract
In many complex practical optimisation cases the dominant characteristics of the problem are often not known prior. Therefore, there is a need to develop general solvers as it is not always possible to tailor a specialised approach to each application. The previously developed Multi-Level Selection Genetic Algorithm already shows good performance on a range of problems due to its diversity-first approach, which is rare among Evolutionary Algorithms. To increase the generality of its performance this paper proposes utilisation of multiple distinct evolutionary strategies simultaneously, similarly to algorithm selection, but with co-evolutionary mechanisms between the sub-populations. This distinctive approach to co-evolution provides less regular communication between sub-populations with competition between collectives rather than individuals. This encourages the collectives to act more independently creating a unique sub-regional search, leading to the development of co-evolutionary MLSGA (cMLSGA). To test this methodology nine genetic algorithms are selected to generate several variants of cMLSGA, which incorporates these approaches at the individual level. The mechanisms are tested on 100 different functions and benchmarked against the 9 state-of-the-art competitors to evaluate the generality of each approach. The results show that the diversity divergence in the principles of working of the selected co-evolutionary approaches is more important than their individual performances. The proposed methodology has the most uniform performance on the divergent problem types, from across the tested state-of-the-art, leading to an algorithm more likely to solve complex problems with limited knowledge about the search space, but is outperformed by more specialised solvers on simpler benchmarking studies.

### Impact Statement
It is proposed in this paper that the uptake of many Genetic Algorithms is low as they are evaluated over a narrow range of problems. This means they have similar characteristics that do not properly reflect the complexity of real-world problems. The results show that those that perform across a range of problems are more likely to perform well on real applications. This explains how the leading algorithm presented in this benchmarking, cMLSGA, is now being implemented into a variety of different applications.

## 1. Additional tables

Table 7: Summary of the utilised two-objective test set

| Category | Problem | d | Additional properties |
|---|---|---|---|
| **Unconstrained** | | | |
| **I. Simple** | ZDT1 | 30 | Convex |
| | ZDT2 | 30 | Concave |
| | ZDT3 | 30 | Discontinuous |
| | ZDT4 | 10 | Multimodal, Convex |
| | ZDT6 | 10 | Multimodal, Biased, Concave |
| **II. Convex** | UF1 | 30 | Complex PS |
| | UF2 | 30 | Complex PS |
| | UF3 | 30 | Complex PS |
| **III. Concave** | UF4 | 30 | Complex PS |
| | WFG4 | 22 | Multimodal |
| | WFG5 | 22 | Deceptive |
| | WFG6 | 22 | Non-separable |
| | WFG7 | 22 | Biased |
| | WFG8 | 22 | Biased, Non-separable |
| | WFG9 | 22 | Biased, Non-separable, Deceptive |
| **IV. Linear/Mixed** | UF7 | 30 | Complex PS, Linear |
| | WFG1 | 22 | Biased, Mixed |
| | WFG3 | 22 | Non-separable, Degenerated, Linear |
| **V. Discontinuous** | UF5 | 30 | Linear, Distinct points, Complex PS |
| | UF6 | 30 | Complex PS |
| | WFG2 | 22 | Convex, Non-Separable |
| | MOP4 | 10 | Discontinuous |
| **VI. Imbalanced** | MOP1 | 10 | Convex |
| | MOP2 | 10 | Convex |
| | MOP3 | 10 | Concave |
| | MOP5 | 10 | Convex |
| | IMB1 | 10 | Convex |
| | IMB2 | 10 | Linear |
| | IMB3 | 10 | Concave |
| | IMB7 | 10 | Convex, Non-separable |
| | IMB8 | 10 | Linear, Non-separable |
| | IMB9 | 10 | Concave, Non-separable |

Table 7: Summary of the utilised two-objective test set (continued)

| Category | Problem | d | Additional properties |
|---|---|---|---|
| **Constrained** | | | |
| **VII. Discontinuous** | CF1 | 10 | Linear, Complex PS, Distinct points |
| | CF2 | 10 | Convex, Complex PS |
| | CF3 | 10 | Concave, Complex PS |
| **VIII. Continuous** | CF4 | 10 | Linear, Complex PS |
| | CF5 | 10 | Linear, Complex PS |
| | CF6 | 10 | Mixed, Complex PS |
| | CF7 | 10 | Mixed, Complex PS |
| **IX. Imbalanced** | IMB11 | 10 | Convex |
| | IMB12 | 10 | Linear |
| | IMB13 | 10 | Concave |
| **X. Diversity-hard** | DAS-CMOP1(5) | 30 | Concave, Discontinuous |
| | DAS-CMOP2(5) | 30 | Mixed, Continuous |
| | DAS-CMOP3(5) | 30 | Linear, Discontinuous, Multimodal |
| | DAS-CMOP4(5) | 30 | Concave, Discontinuous |
| | DAS-CMOP5(5) | 30 | Mixed, Discontinuous |
| | DAS-CMOP6(5) | 30 | Distinct points, Degenerated |
| **XI. Feasibility-hard** | DAS-CMOP1(6) | 30 | Concave, Discontinuous |
| | DAS-CMOP2(6) | 30 | Mixed, Continuous |
| | DAS-CMOP3(6) | 30 | Linear, Discontinuous, Multimodal |
| | DAS-CMOP4(6) | 30 | Concave, Discontinuous |
| | DAS-CMOP5(6)) | 30 | Mixed, Discontinuous |
| | DAS-CMOP6(6) | 30 | Distinct points, Degenerated |
| **XII.Convergence-hard** | DAS-CMOP1(7) | 30 | Concave, Discontinuous |
| | DAS-CMOP2(7) | 30 | Mixed, Continuous |
| | DAS-CMOP3(7)) | 30 | Linear, Discontinuous, Multimodal |
| | DAS-CMOP4(7) | 30 | Concave, Discontinuous |
| | DAS-CMOP5(7) | 30 | Mixed, Discontinuous |
| | DAS-CMOP6(7) | 30 | Distinct points, Degenerated |

*d denotes the number of decision variables.*

Table 8: Summary of the utilised three-objective test set

| Category | Problem | d | Additional properties |
|---|---|---|---|
| **Unconstrained** | | | |
| **I. Concave** | DTLZ2 | 12 | |
| | DTLZ3 | 12 | Multimodal |
| | DTLZ4 | 12 | Biased |
| | DTLZ5 | 12 | Degenerated |
| | DTLZ6 | 12 | Degenerated, Biased |
| | UF8 | 30 | Complex PS |
| | UF10 | 30 | Complex PS |
| | WFG4 | 24 | Multimodal |
| | WFG5 | 24 | Deceptive |
| | WFG6 | 24 | Non-separable |
| | WFG7 | 24 | Biased |
| | WFG8 | 24 | Biased, Non-separable |
| | WFG9 | 24 | Biased, Non-separable, Deceptive |
| **IV. Linear/Mixed** | DTLZ1 | 7 | Linear, Multimodal |
| | WFG1 | 24 | Biased, Mixed |
| | WFG3 | 24 | Non-separable, Degenerated, Linear |
| **V. Discontinuous** | DTLZ7 | 22 | Mixed, Multimodal |
| | UF9 | 30 | Complex PS |
| | WFG2 | 24 | Convex, Non-Separable |
| **VI. Imbalanced** | MOP6 | 10 | Linear |
| | MOP7 | 10 | Concave |
| | IMB4 | 10 | Linear |
| | IMB5 | 10 | Concave |
| | IMB6 | 10 | Linear |
| | IMB10 | 10 | Linear |
| **Constrained** | | | |
| **VII. Discontinuous** | DTLZ8 | 30 | Mixed, Degenerated, Biased |
| | DTLZ9 | 30 | Concave, Degenerated |
| | CF8 | 10 | Concave, Degenerated, Complex PS |
| | CF9 | 10 | Concave, Complex PS |
| | CF10 | 10 | Concave, Complex PS |
| **IX. Imbalanced** | IMB14 | 10 | Linear |
| **X. Diversity-hard** | DAS-CMOP7(5) | 30 | Linear, Degenerated, Discontinuous |
| | DAS-CMOP8(5) | 30 | Concave, Discontinuous |
| | DAS-CMOP9(5) | 30 | Concave, Discontinuous, Biased |
| **XI. Feasibility-hard** | DAS-CMOP7(6) | 30 | Linear, Degenerated, Discontinuous |
| | DAS-CMOP8(6) | 30 | Concave, Discontinuous |
| | DAS-CMOP9(6) | 30 | Concave, Discontinuous, Biased |
| **XII.Convergence-hard** | DAS-CMOP7(7) | 30 | Linear, Degenerated, Discontinuous |
| | DAS-CMOP8(7) | 30 | Concave, Discontinuous |
| | DAS-CMOP9(7) | 30 | Concave, Discontinuous, Biased |

*d denotes the number of decision variables. The same categories are utilised as for the two-objective cases.*