Supplementary Information for
“The dynamics of injunctive social norms”
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Contents

Below I provide more details on analytical derivations as well as additional figures discussed in the main text.

2 Model variables, functions, parameters and statistics

Table S1 lists model variables, functions, parameters and statistics.

3 Uniform distribution of v

Assume that v has a uniform distribution between $v_{\min}$ and $v_{\max}$. Then

$$F(z) = \begin{cases} 0, \text{if } z < v_{\min}, \\ \frac{z - v_{\min}}{v_{\max} - v_{\min}}, \text{if } v_{\min} < z < v_{\max} \\ 1, \text{if } z > v_{\max} \end{cases}$$

Assume also that $b > v_{\max}$ (so that nobody complies if $p = 0$) and that $\kappa > b - v_{\min}$ (so that everybody complies if $p = 1$). Then there is a threshold initial frequency

$$p^* = \frac{b - v_{\max}}{\kappa - (v_{\max} - v_{\min})}, \tag{S1}$$

and the population will lose the norm (i.e., evolve to the state with $p = 0$) if the initial frequency of compliance $p_0 < p^*$, but will “‘fix” it (i.e. evolve to the state where everybody complies) if $p_0 > p^*$. [One can see this by using a graphical cob-webbing method, i.e. plotting the graph of

Table S1: Model variables, functions and parameters.

	Symbols	Their meaning
Variables	x	complying or not with the norm ($x = 1$ or 0)
	y	punishing/admonishing or not norm violators ($y = 1$ or 0)
	p	frequency of people complying with the norm
	q	frequency of people punishing norm violators
Parameters	b	material benefit of norm violation
	v	normative value of following the norm
	v_a	normative value of being approved passively
	v_c	normative value of conformity
	κ	normative cost of being disapproved passively
	c	cost of being punished/admonished
	c_f	frequency-dependent cost
	δ	cost of punishing/admonishing others
Functions	$F(z)$	cumulative distribution function of v in the population;
		$\overline{v}$ and σ^2 are the corresponding mean and variance

$p' = 1 - F(z(p))$.] Decreasing the normative cost of disapproval κ, increasing material benefit of not complying b, decreasing the maximum normative benefit of complying $v_{\max}$, and increasing the range $v_{\max} - v_{\min}$ of variation in v increase p^*. Note that, as it should be, equation (S1) reduces to the value $\tilde{p}$ in the main text if $v_{\min} = 0$.

4 Four unimodal distributions

Normal distribution. The cumulative distribution function of the normal distribution is

$$F(z) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{z - \overline{v}}{\sqrt{2}\sigma}\right)\right), \tag{S2a}$$

where $\overline{v}$ and σ are the mean and standard deviation.

Logistic distribution. The cumulative distribution function of the logistic distribution is

$$F(z) = \frac{1}{1 + \exp\left(-\frac{z - \overline{v}}{s}\right)}, \tag{S2b}$$

where $s = \sqrt{3}\sigma/\pi$.

Log-normal distribution. The cumulative distribution function of the log-normal distribution is

$$F(z) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{\ln z - \mu}{\sqrt{2}s}\right)\right), \tag{S2c}$$

where the location and scale parameters μ and ν can be expressed in terms of the mean $\overline{v}$ and variance σ^2 of v as

$$\mu = \ln\left(\frac{\overline{v}}{\sqrt{1+\frac{\sigma^2}{\overline{v}^2}}}\right),\ s = \sqrt{\ln\left(1+\frac{\sigma^2}{\overline{v}^2}\right)}.$$

Laplace distribution. The cumulative distribution function of the log-normal distribution is

$$F(z) = \begin{cases} 1-\frac{1}{2}\exp\left(-\frac{z-\overline{v}}{s}\right), & \text{for} z \geq \overline{v} \\ \frac{1}{2}\exp\left(\frac{z-\overline{v}}{s}\right), & \text{for} z < \overline{v}. \end{cases} \tag{S2d}$$

where $s = \sigma/\sqrt{2}$.

Figure S1 illustrates these distributions.

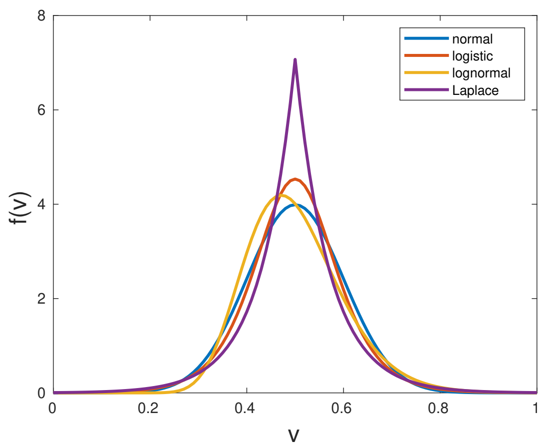

Figure S1: Probability density functions $f(x)$ for four distributions with $\overline{v} = 0.5, \sigma = 0.1$.

Figures S2-S7 illustrate equilibria in different models of unimodal and bimodal distributions of p as discussed in the main text.

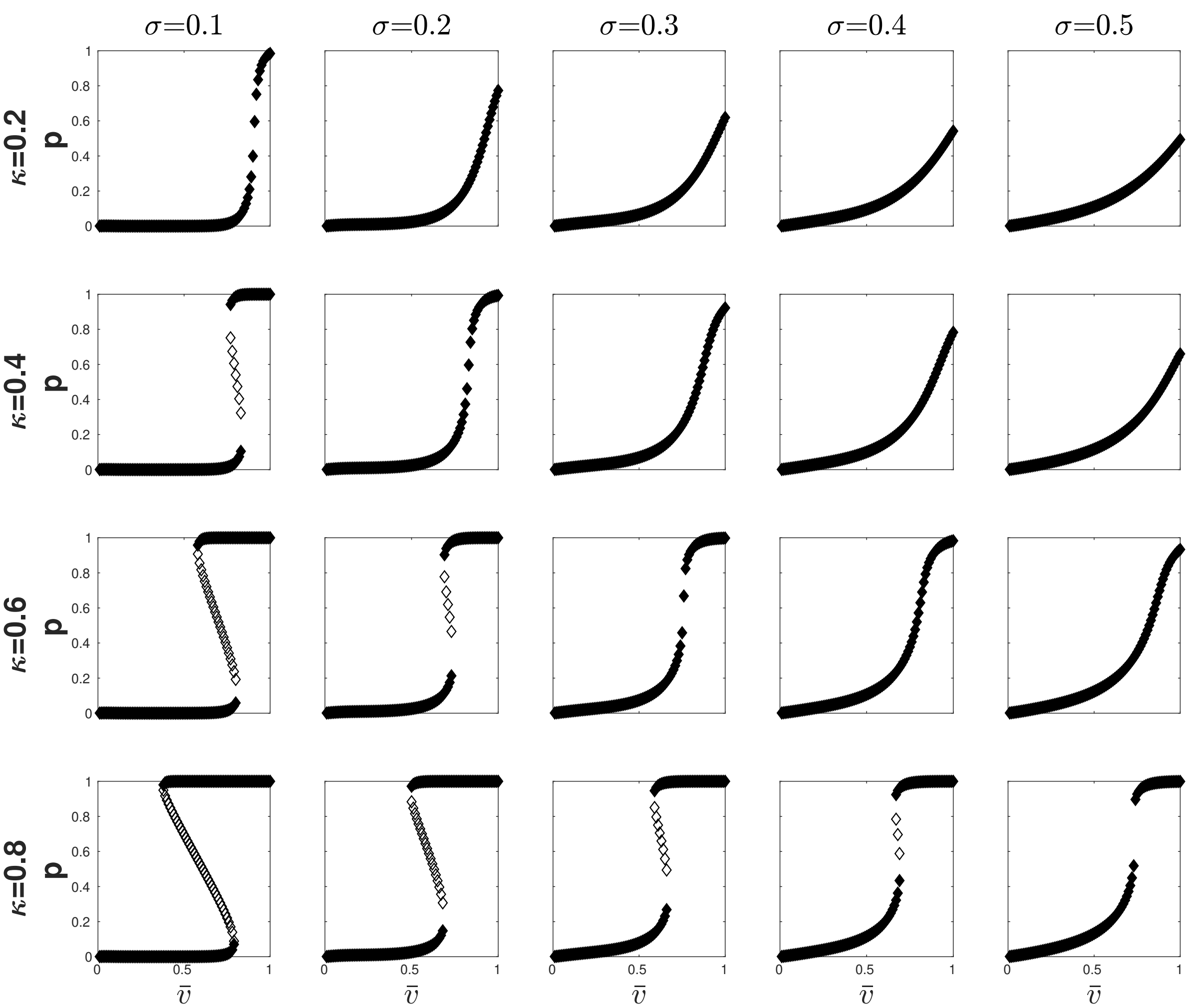

Figure S2: Equilibrium values of p in model (2) with $\overline{v}$ as the bifurcation parameter. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality. Lognormal distribution of v.

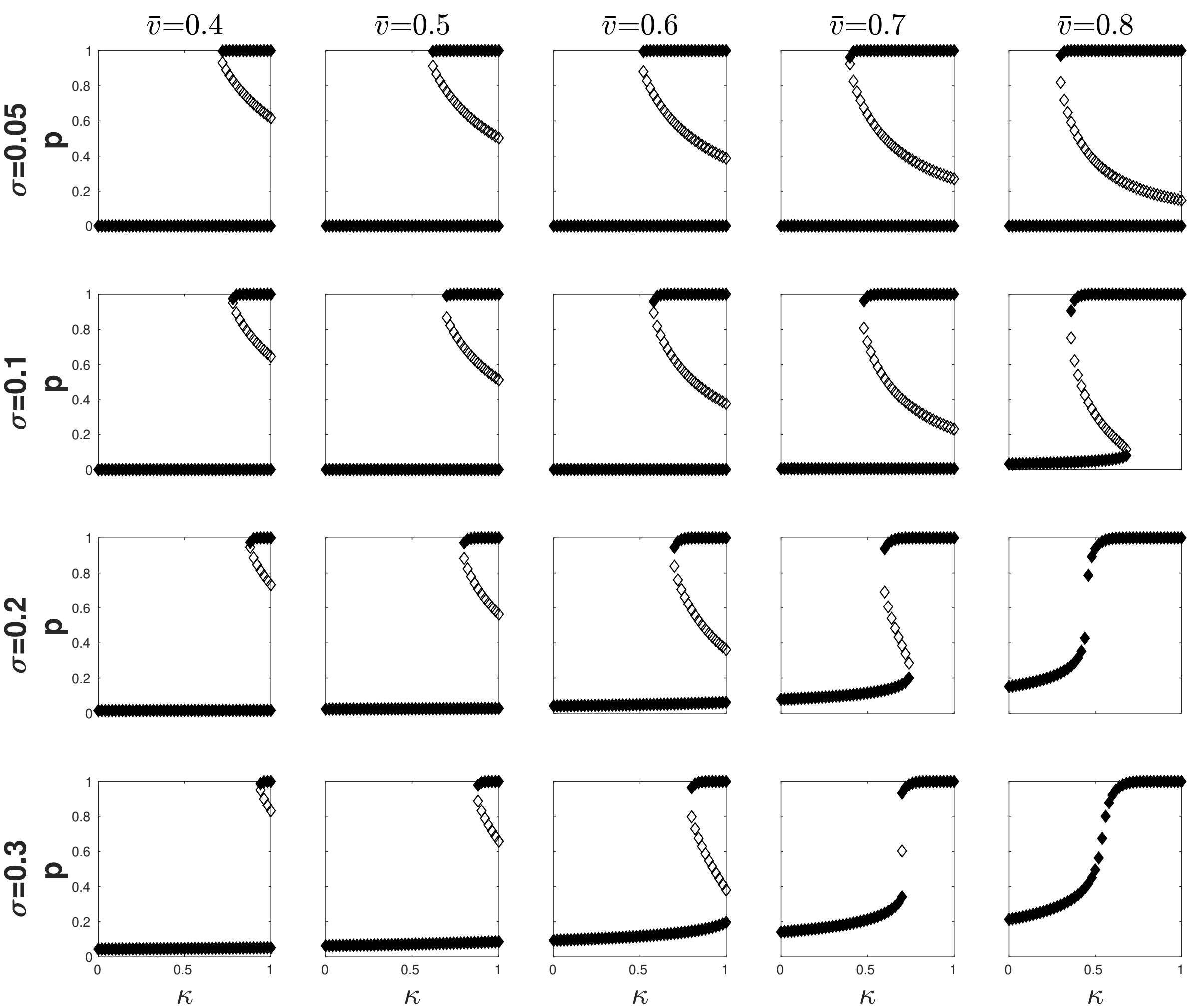

Figure S3: Equilibrium values of p in model (2) with κ as the bifurcation parameter. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality. Lognormal distribution of v.

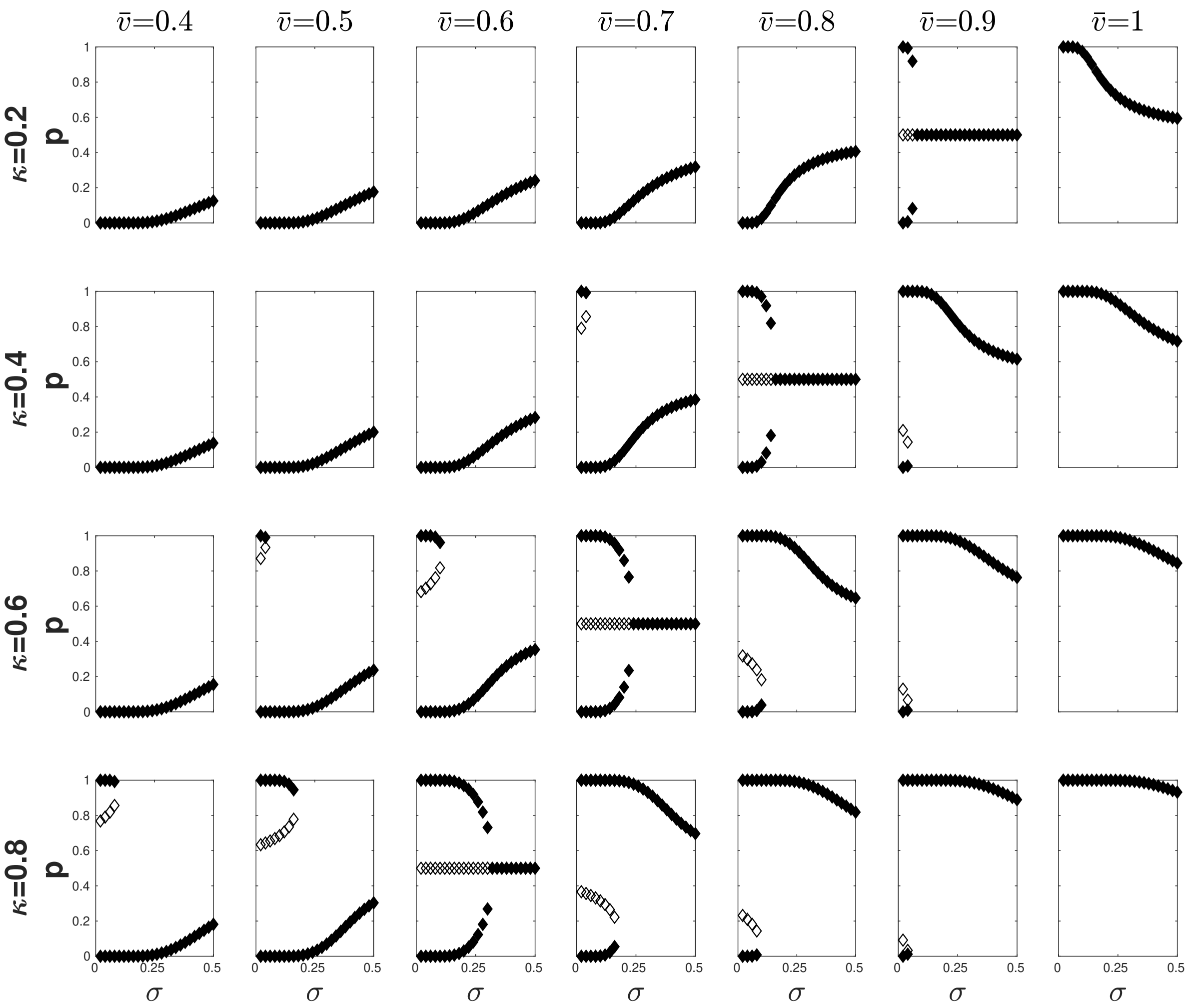

Figure S4: Bifurcation diagrams for the case of normal distribution of v. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality.

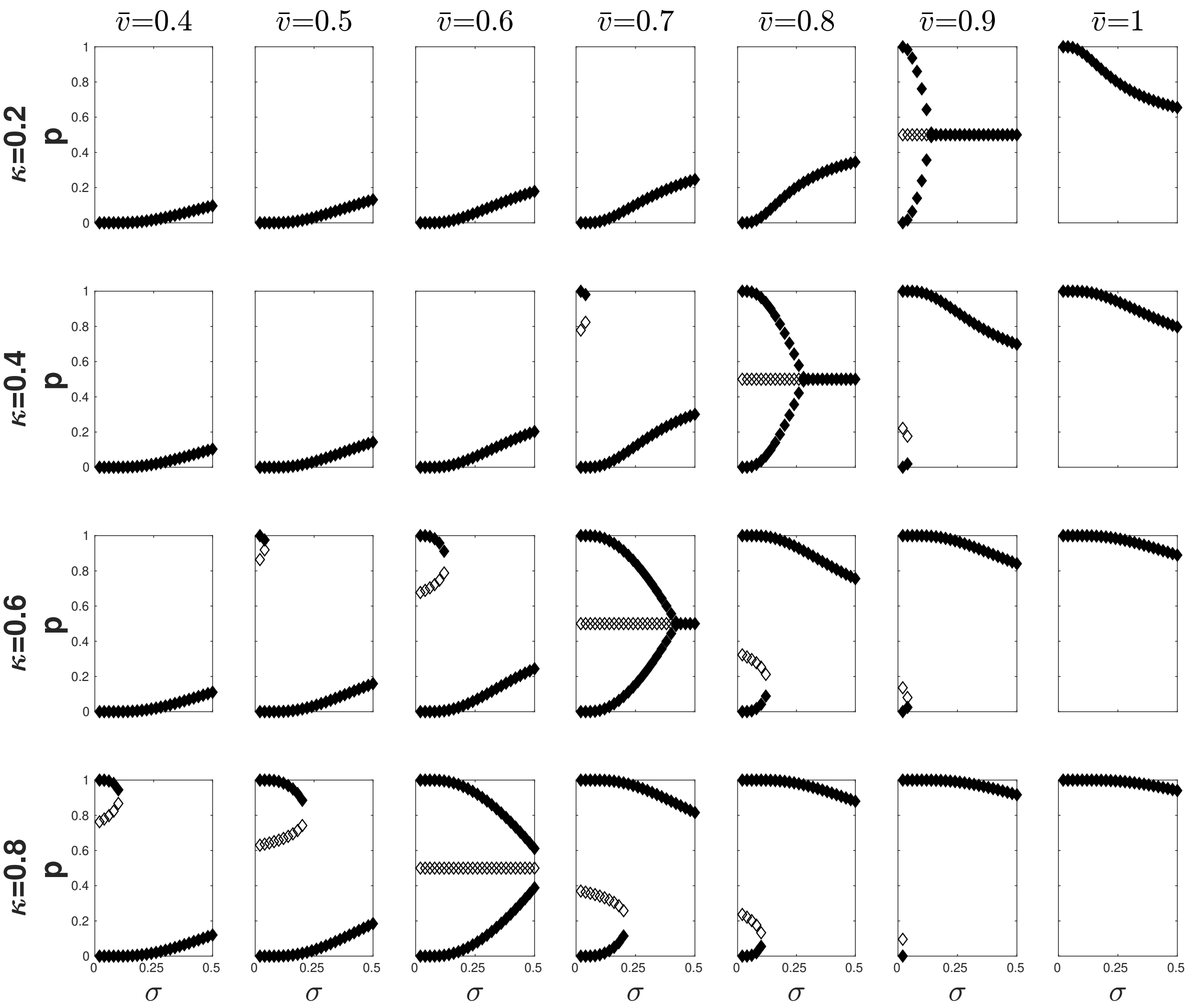

Figure S5: Equilibrium values of p in the case of Laplace distribution of v. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality.

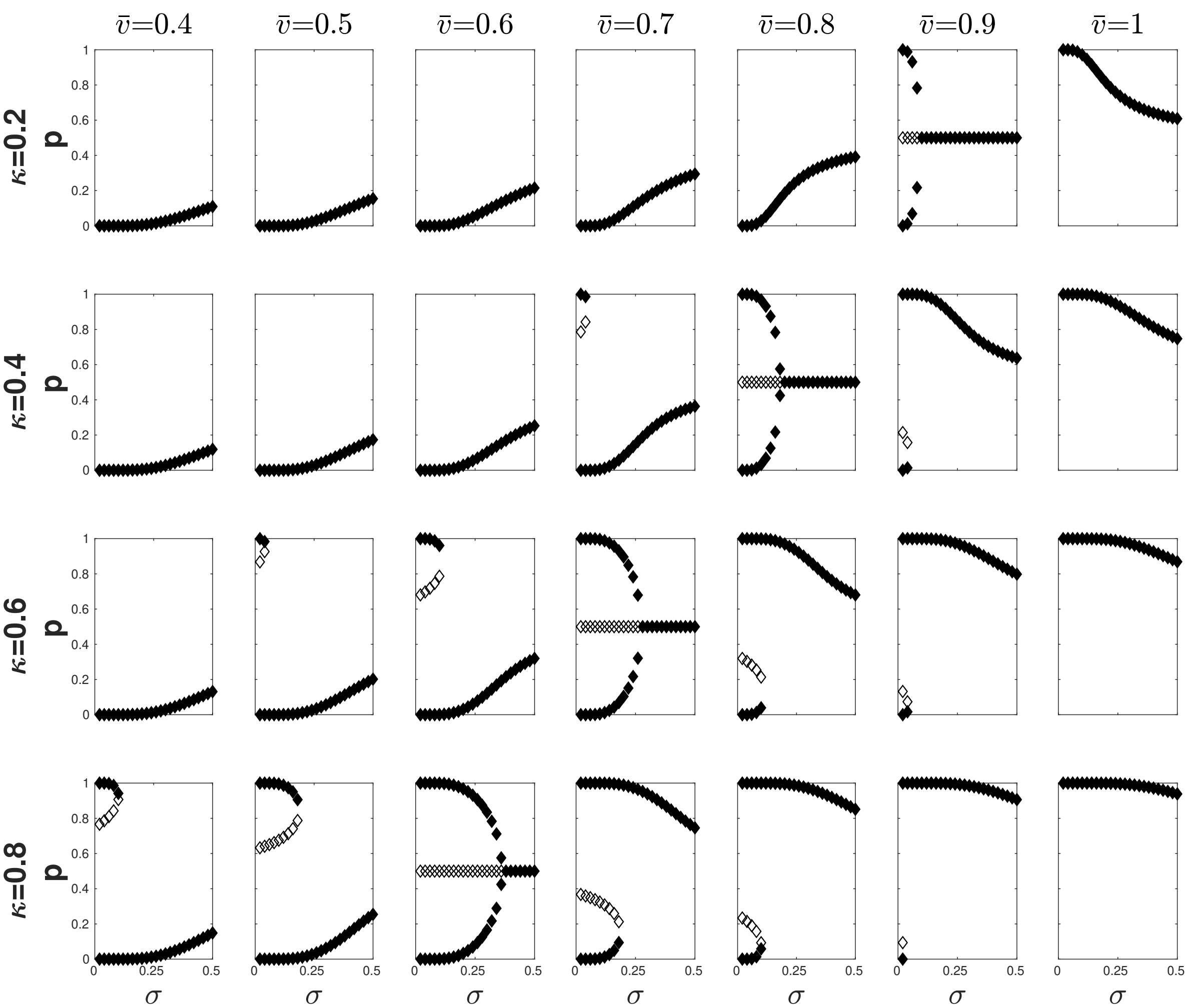

Figure S6: Equilibrium values of p in the case of logistic distribution of v. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality.

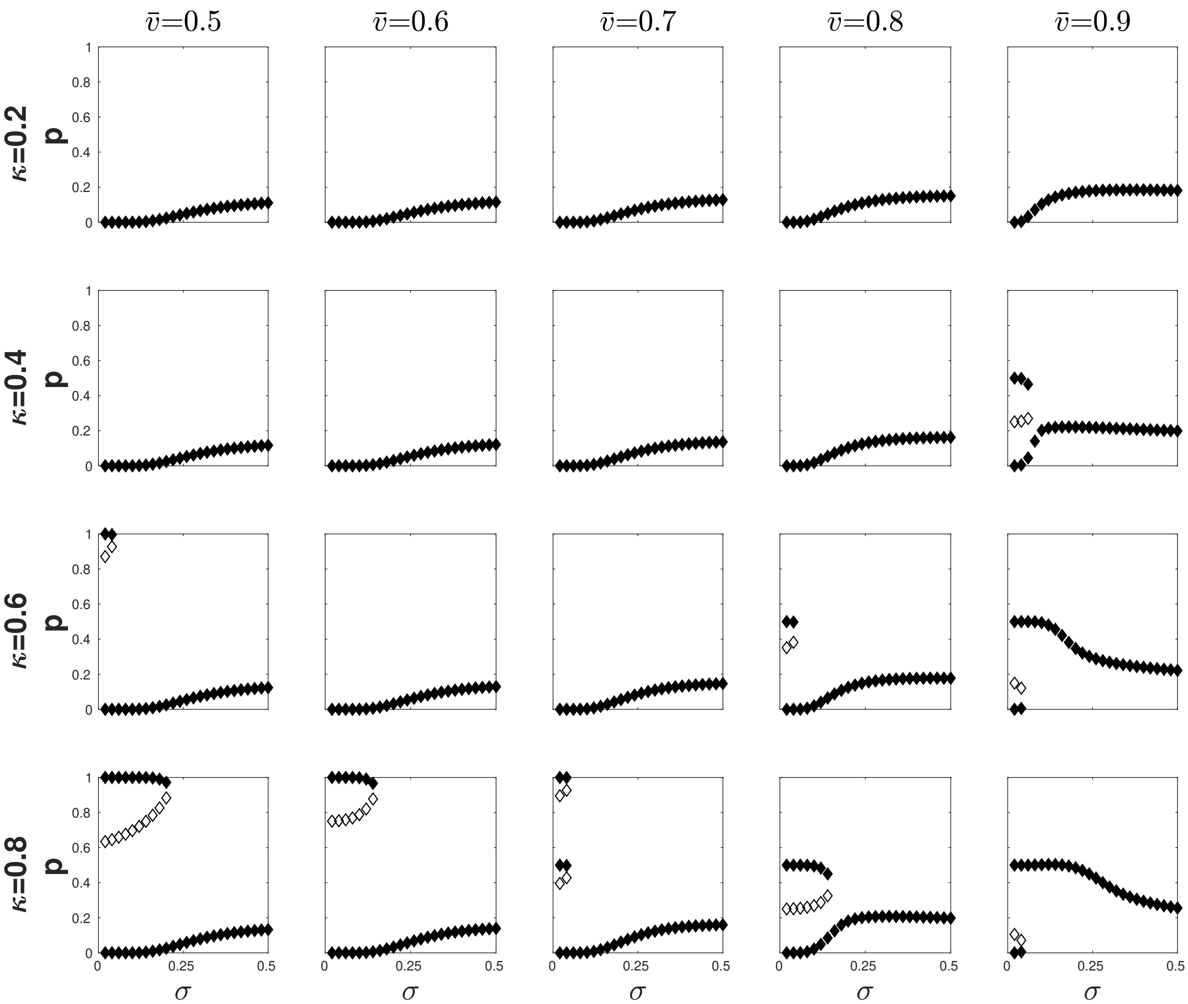

Figure S7: Equilibrium values of p in the case of a bimodal distribution of v. The c.d.f. used is an average of two c.d.f. of the log-normal distribution (S2c) with the same variance σ^2 but different means: one at $\bar{v}$ and another at $1 - \bar{v}$. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality.

5 Dynamics of p in time

Figures S8-S10 shows the dynamics of p in time for the case of lognormal distribution of p.

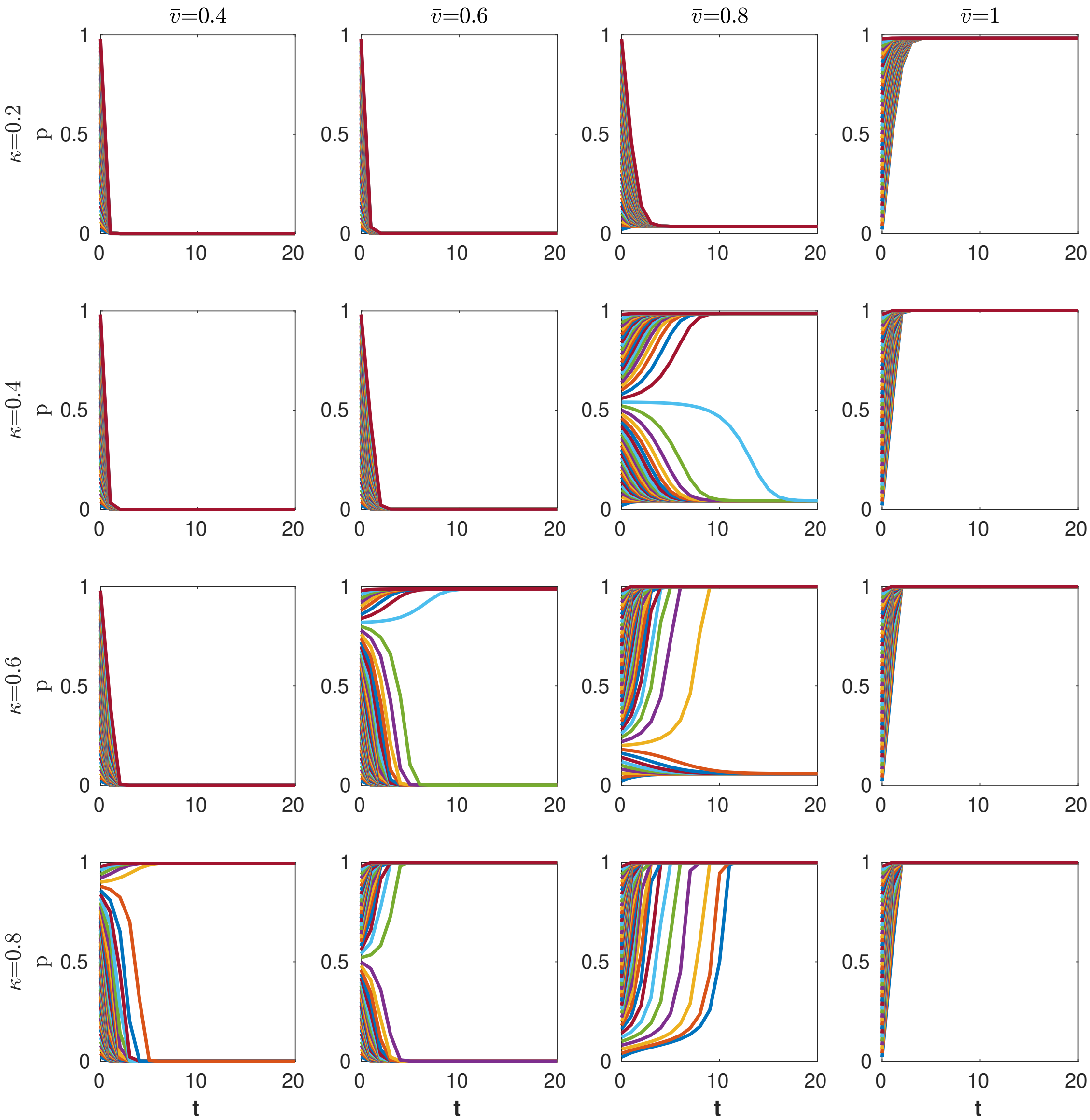

Figure S8: The dynamics of p in the case of lognormal distribution with $\sigma = 0.1$. Different lines correspond to different initial conditions equally spaced between 0 and 1. $b = 1$.

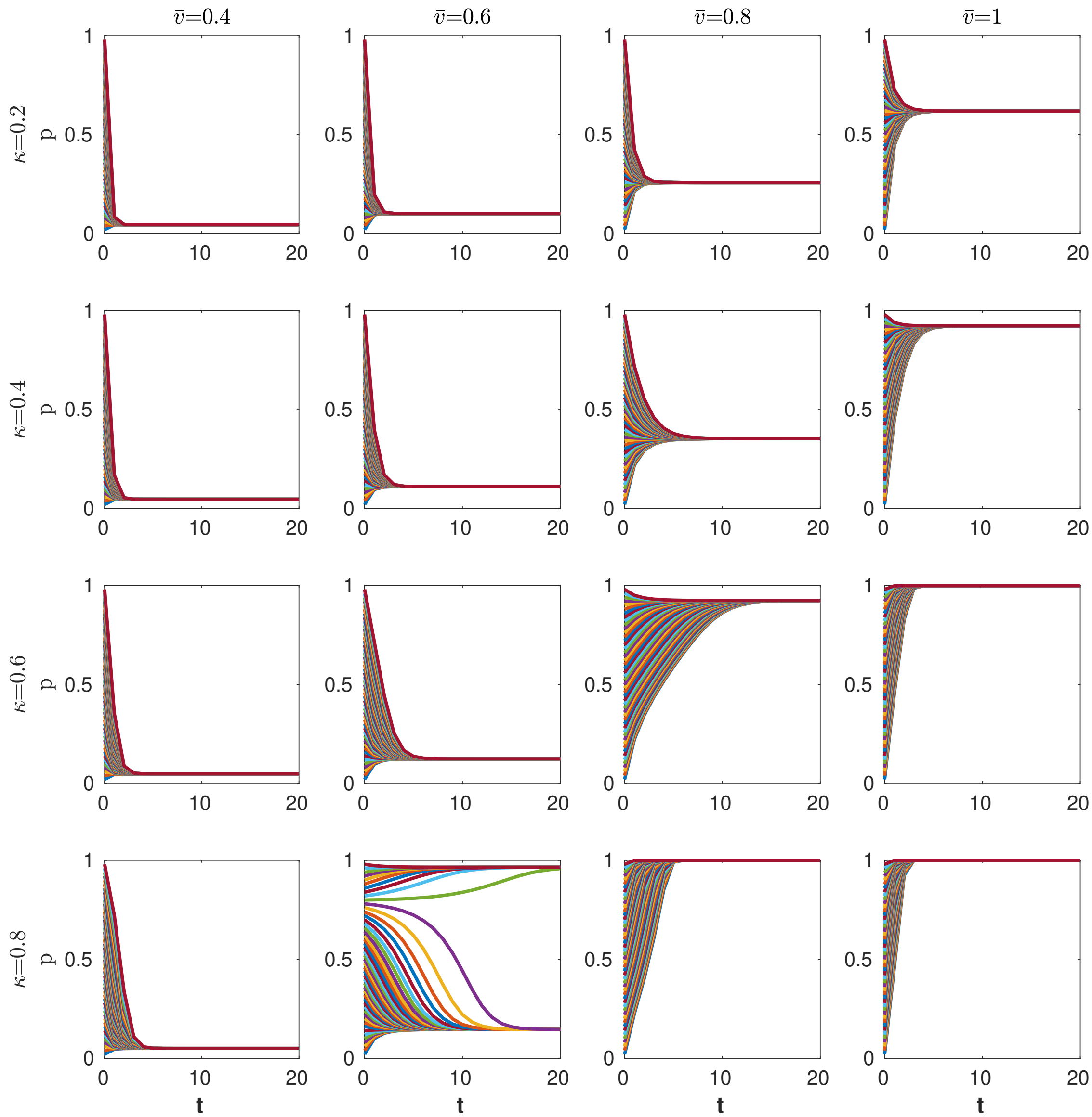

Figure S9: The dynamics of p in the case of lognormal distribution with $\sigma = 0.3$. Different lines correspond to different initial conditions equally spaced between 0 and 1. $b = 1$.

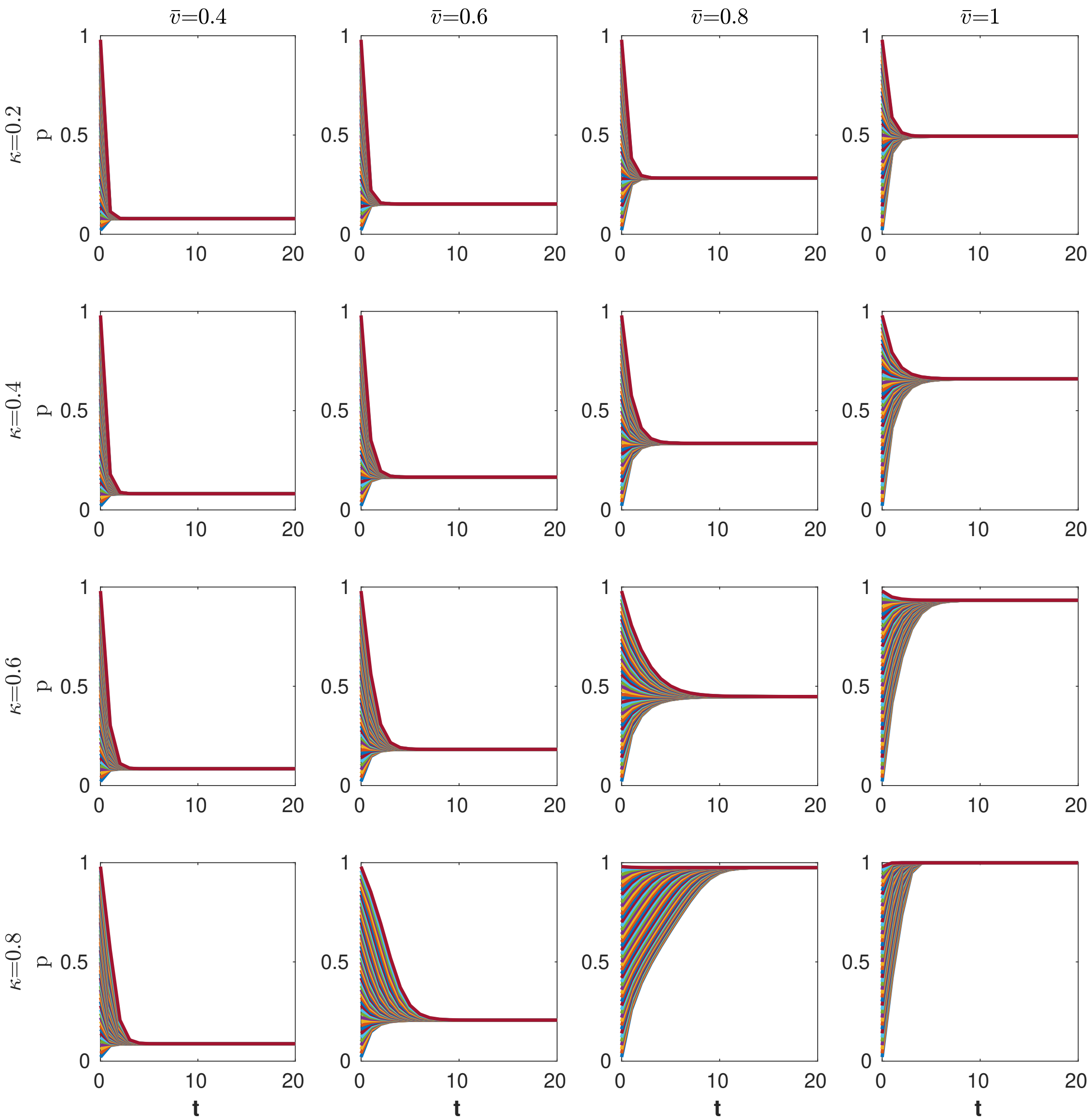

Figure S10: The dynamics of p in the case of lognormal distribution with $\sigma = 0.5$. Different lines correspond to different initial conditions equally spaced between 0 and 1. $b = 1$.

6 QRE equilibrium with a uniform distribution of v

Errors in utility evaluation. To capture possible errors, one can use the Quantal Response Equilibrium (QRE) approach which generalizes classical Nash equilibria (Goeree *et al.*, 2016). In this approach, with logit errors, the probability that an individual complies with the norm is defined as

$$P(\Delta u) = \frac{1}{1+\exp(-\lambda \Delta u)}, \tag{S3a}$$

where $\Delta u = u_1 - u_0$ is the difference in utilities and λ is a non-negative precision parameter. [If $\lambda = 0$, individuals choose $x = 1$ or $x = 0$ with equal probabilities; if $\lambda = \infty$, they always choose the action with the maximum utility as was assumed in the model above.] In our model, $\Delta u = v - v^*$.

In this case, the recurrence equation for p takes form

$$p' = \int P(v - v^*) dF(v). \tag{S3b}$$

As precision parameter λ increases to infinity, the above equation reduces to equation (2) of the main text. The steady state of the above equation gives us a QRE equilibrium. The corresponding integral can be evaluated numerically or, in some cases, analytically.

If v has a uniform distribution on $[\overline{v} - \sigma, \overline{v} + \sigma]$, then $f(v) = \frac{1}{2\sigma}$, and

$$p' = \frac{1}{2\sigma\lambda} \left[H(\overline{v} + \sigma - b + \kappa p) - H(\overline{v} - \sigma - b + \kappa p) \right],$$

where $H(z) = \ln[1 + \exp(\lambda z)]$. Figure S11 illustrates the effects of precision parameter λ on the QRE equilibria in this model. One can see that decreasing precision causes the disappearance of the solution branch with the unstable equilibrium and shifts the remaining solution towards 0.5 as individuals tend to make their decision more randomly.

7 Truncated distributions

Let $F(z)$ be a c.d.f. of a continuous random variable v and $\alpha < \beta$ be two constants. Then the c.d.f of v given $\alpha < v \leq \beta$ is

$$F_{\alpha,\beta}(z) = \frac{F(z) - F(\alpha)}{F(\beta) - F(\alpha)}.$$

If the truncation removes the bottom s proportion of the population, $F(\alpha) = s, F(\beta) = 1$, and

$$F_s(z) = \max\left(\frac{F(z) - s}{1 - s}, 0\right). \tag{S4a}$$

If the truncation removes the top s proportion of the population, $F(\alpha) = 0, F(\beta) = 1 - s$.

$$F^s(z) = \min\left(\frac{F(z)}{1 - s}, 1\right). \tag{S4b}$$

Figures S13 and S12 show equilibrium values of p in the model of persuasive interventions targeting individuals with the lowest and highest values of v, respectively.

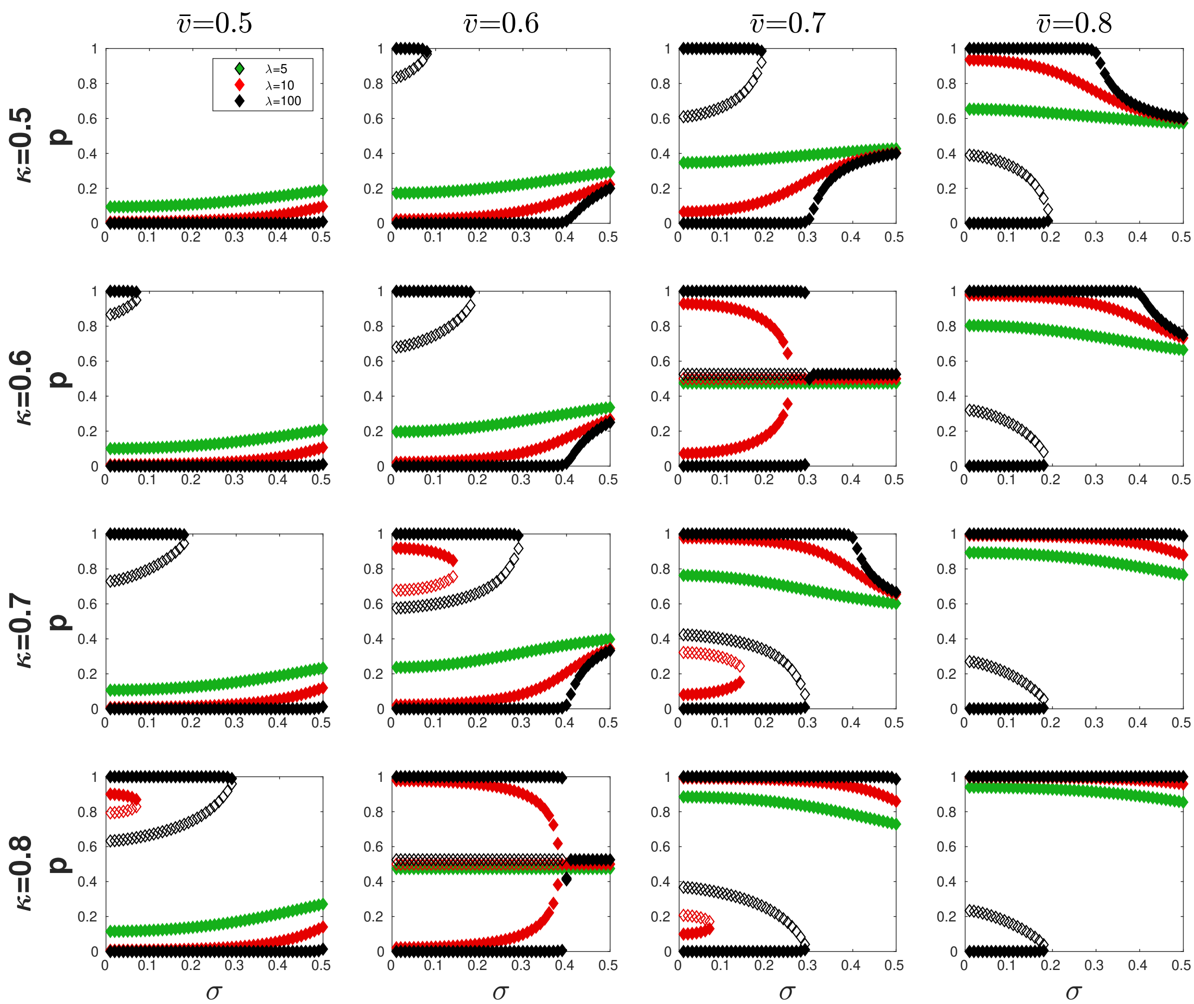

Figure S11: Equilibrium values of frequency p in the model with stochastic errors (equation S3b). The distribution of v is uniform on $[\overline{v} - \sigma, \overline{v} + \sigma]$ (see the SI). Green, red, and black diamonds correspond to three different values of the precision parameter: $\lambda = 5, 10$, and 100, respectively. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. $b = 1$. The symbols describing the equilibrium at $p = 0.5$ for different λ are slightly displaced relative to each other.

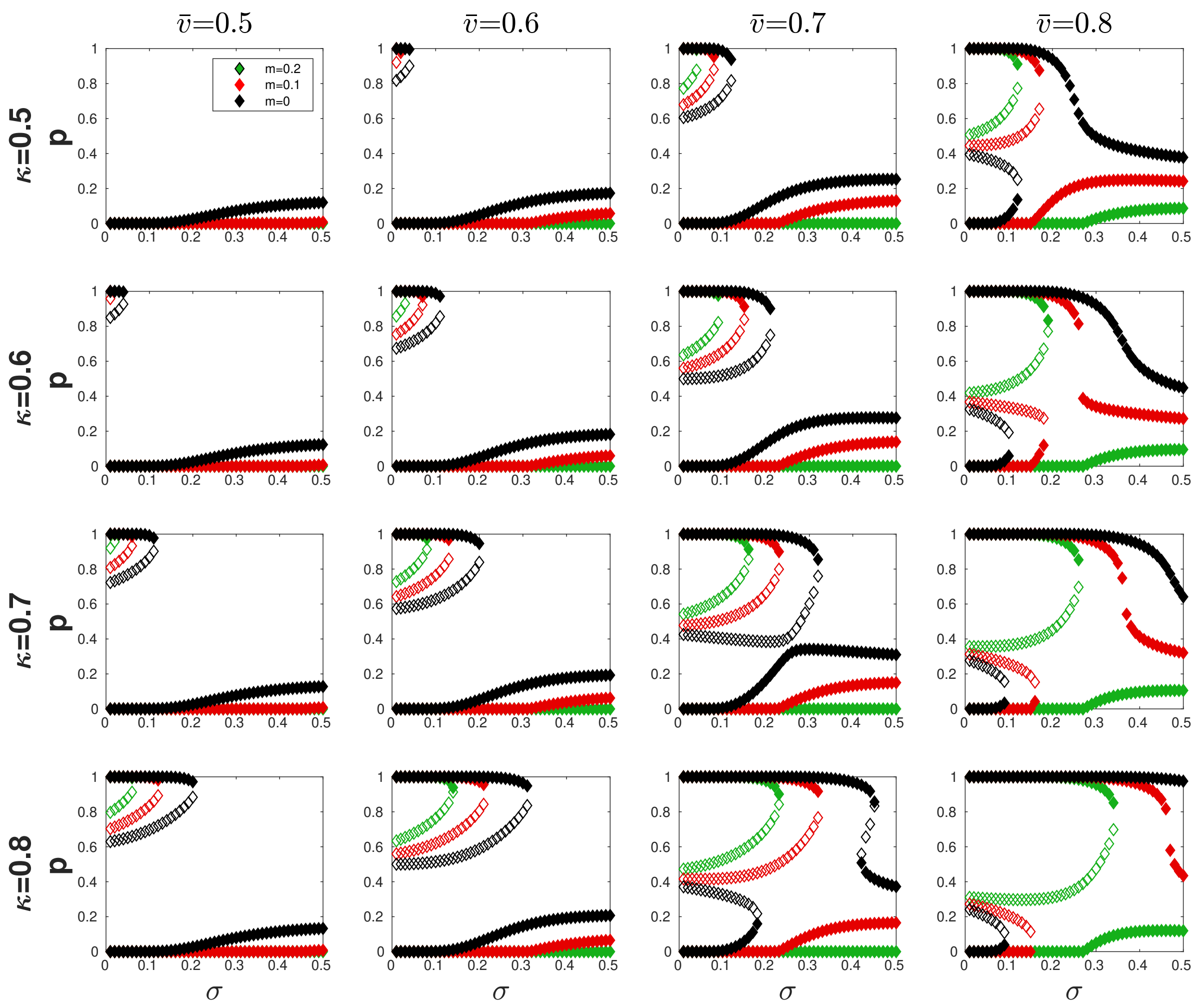

Figure S12: Equilibrium values of frequency p in the model of persuasive interventions targeting individuals with the highest values of v. Green, red, and black diamonds correspond to three different proportions of "removed" individuals with the highest values of v: $m = 0.2, 0.1$, and 0, respectively. Solid symbols are stable equilibria. Open symbols are unstable equilibria separating the two stable ones. The initial distribution of v is log-normal. $b = 1$.

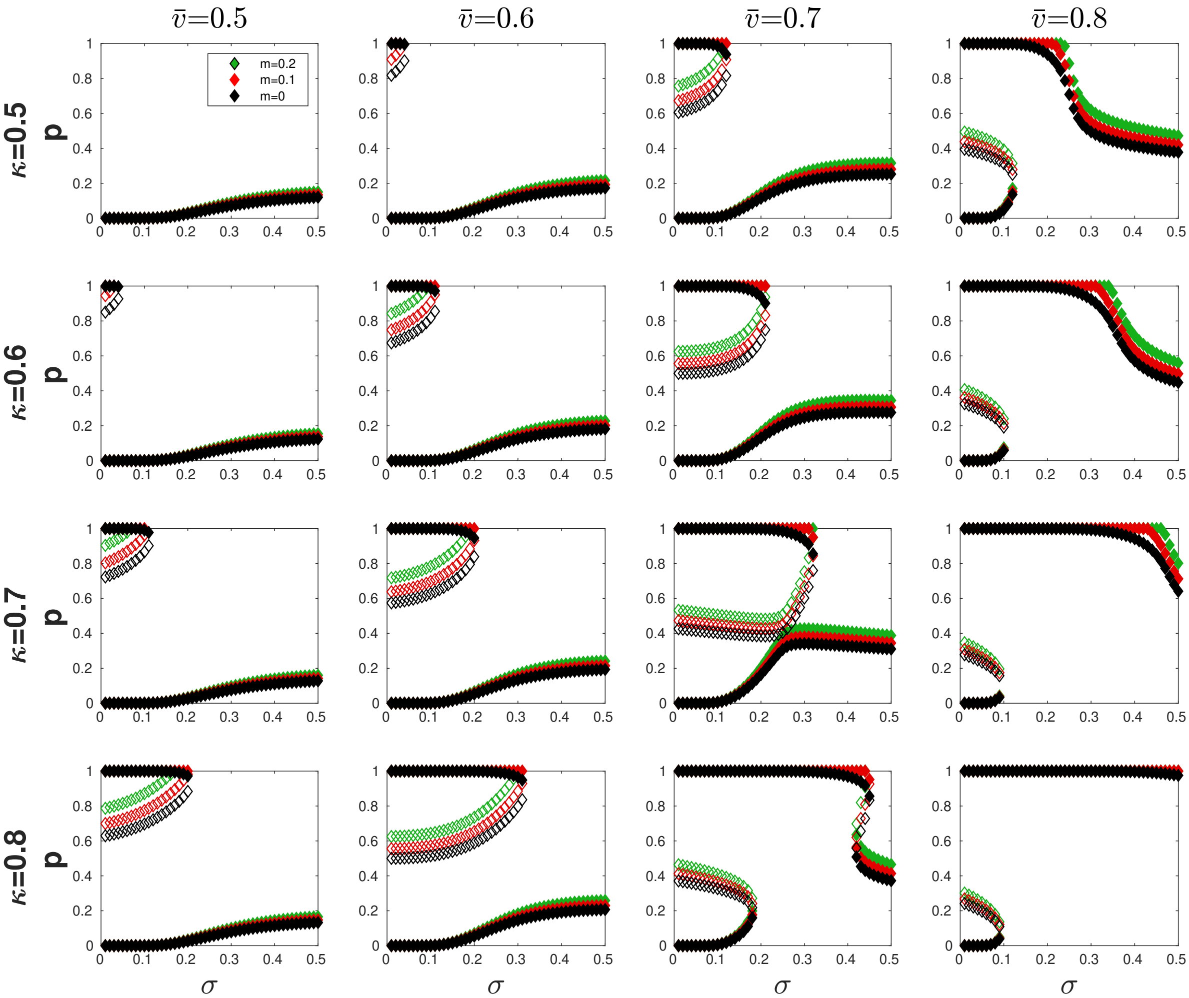

Figure S13: Equilibrium values of frequency p in the model of persuasive interventions targeting individuals with the lowest values of v. Green, red, and black diamonds correspond to three different proportions of "removed" individuals with the lowest values of v: $m = 0.2, 0.1$, and 0, respectively. Filled symbols are stable equilibria. Open symbols are unstable equilibria separating the two stable ones. The initial distribution of v is log-normal. $b = 1$.

8 Heterogeneity in costs of passive disapproval κ

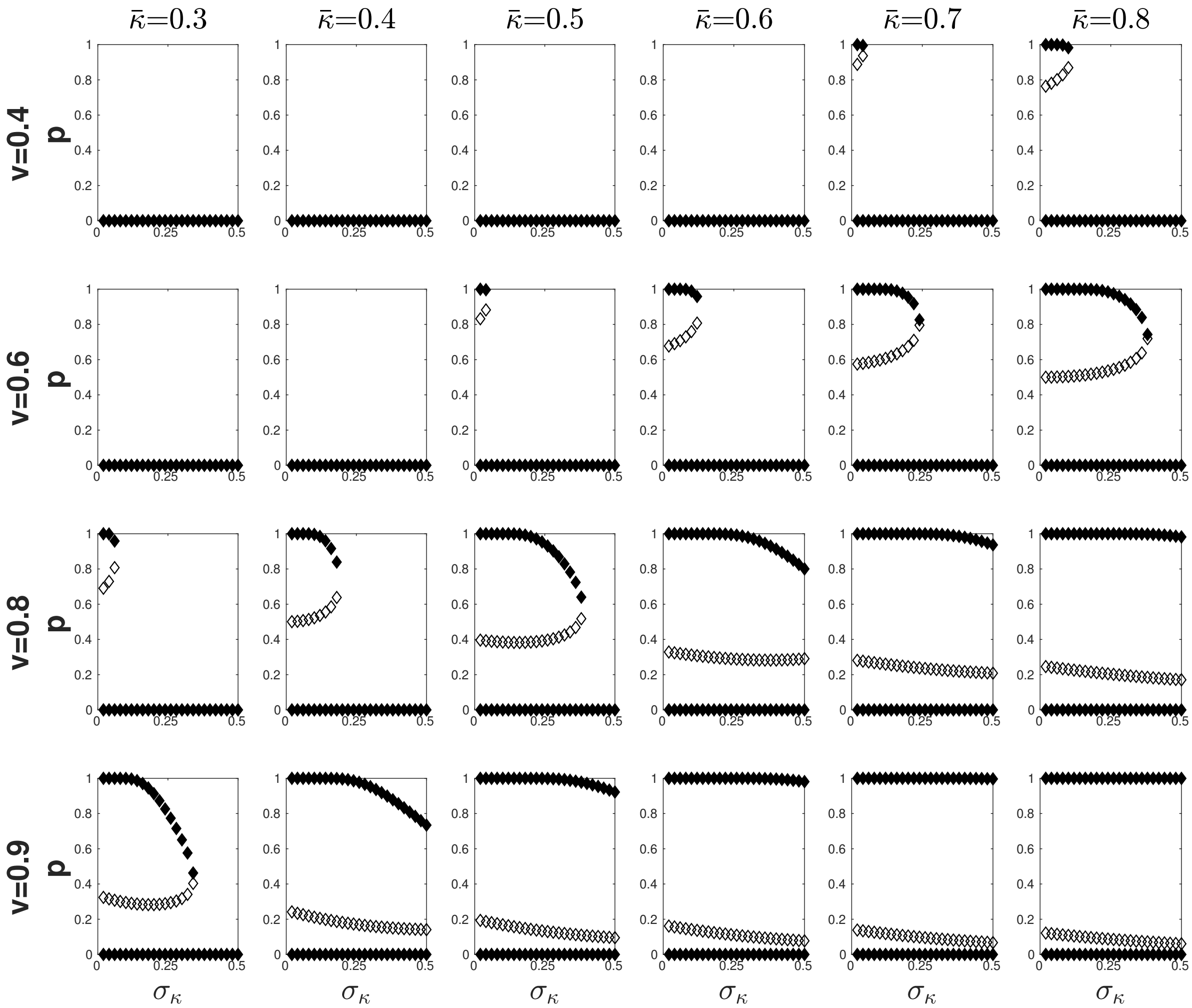

Figure S14: Equilibrium values of frequency p for the case of log-normal distribution of κ with different values of the mean $\overline{\kappa}$ and standard deviation σ_κ. Filled diamonds are stable equilibria. Open diamonds are unstable equilibria separating the two stable ones. Parameter b is set to 1 without loss of generality.

9 Equilibrium values of punishment frequency q

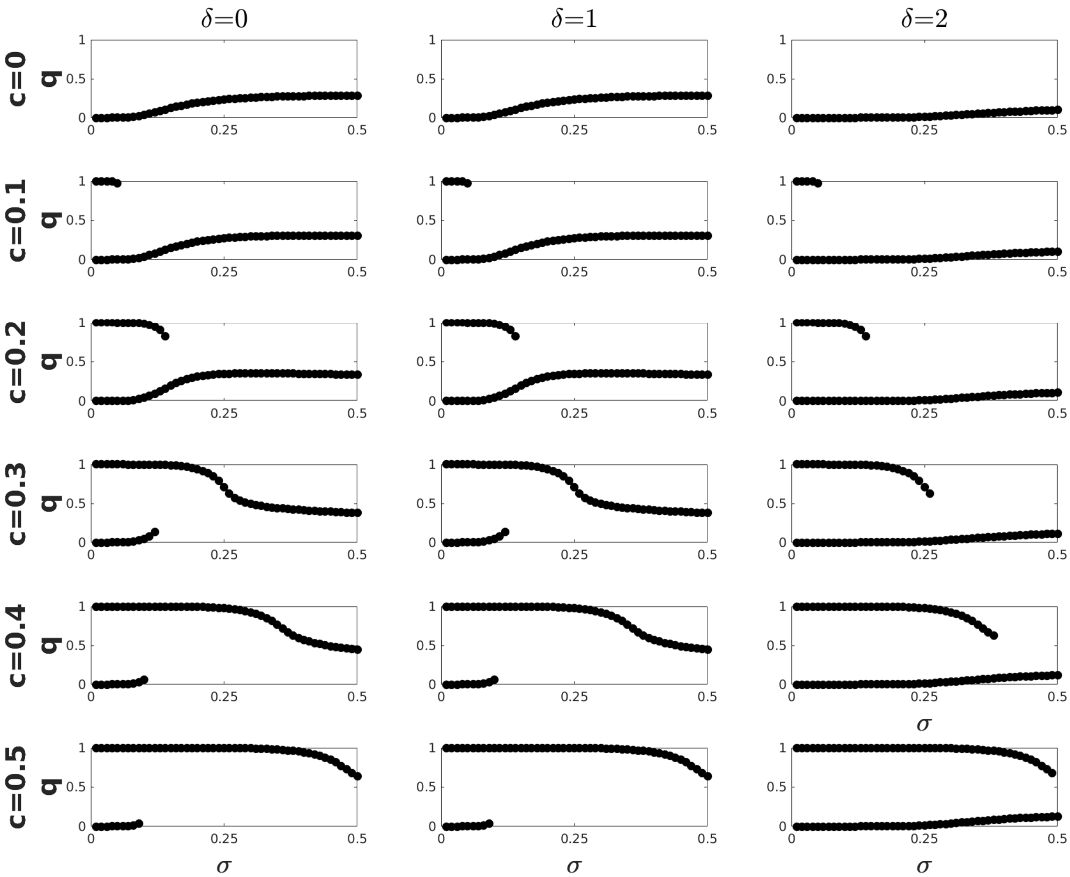

Figure S15: Equilibrium values of q in the model with both passive and active disapproval of norm violators with σ as the bifurcation parameter for different values of the maximum cost of being punished c and the cost of punishing others δ. Only stable equilibria in q are shown. $\overline{v} = 0.8, \kappa = 0.2$. Lognormal distribution of v. See Figure 3 in the main text for the corresponding diagram for p.