**Appendix: Formalizing the False Inferences Theory of Section 2**

2.1 Framework of Analysis

Suppose that a consumer is choosing between two food products, A and B. Product A carries a government-mandated “Contains Z25” disclosure. Product B does not. The consumer wants to purchase healthy food products. But she is uncertain about the health effects of Z25. For expositional purposes, we assume that there is a particular health risk, *H* (measured in dollars), that is potentially associated with Z25. In this basic framework, the outcome is binary – either Z25 is harmful or not.[[1]](#footnote-1)

The consumer knows that there are two possible reasons why the government would mandate a Z25 disclosure:

1. The government would mandate disclosure because it believes that Z25 generates the risk *H*. Formally, the government receives one of two possible signals – either that Z25 is harmful or that Z25 is harmless. The accuracy of these signals is $p\_{G}>\frac{1}{2}$. Namely, if Z25 is harmful, then there is a probability $p\_{G}>\frac{1}{2}$ that the government gets a “harmful” signal and a probability $1-p\_{G}$ that the government gets a “harmless” signal. And if Z25 is harmless, then there is a probability $p\_{G}>\frac{1}{2}$ that the government gets a “harmless” signal and a probability $1-p\_{G}$ that the government gets a “harmful” signal. The government would mandate disclosure if and only if it receives a “harmful” signal.
2. The government would mandate disclosure regardless of its beliefs about the harmfulness of Z25 (indeed, the government would mandate disclosure even if it received no signal about whether Z25 is harmful or not and, indeed, even if it received a “harmless” signal). For example, the government believes that consumers should have as much information as possible about the ingredients in food products, regardless of any associated health risks. Or the government agency succumbs to interest-group pressure and mandates the disclosure.

The consumer attributes probability *q* to reason (1) and probability 1-*q* to reason (2).[[2]](#footnote-2)

Before learning that the government mandates a Z25 disclosure, the consumer believed that Z25 generates the risk *H* with probability $p\_{0}$. This is the consumer’s prior. After learning that the government mandates a Z25 disclosure, the consumer updates her beliefs, according to a standard Bayesian updating process. We next derive the consumer’s updated, posterior probability that Z25 generates risk *H*. (The posterior probability is the consumer’s final, post-updating probability estimate.) We denote this posterior probability $p\_{1}$.

2.2 Motive for Disclosure is Harmfulness

It is instructive to begin with the special case where *q* = 1, namely, where the government would mandate disclosure if and only if it believes that Z25 is harmful. In this case, the Bayesian inference problem is depicted in the following tree diagram:

$$1-p\_{0}$$

$$p\_{0}$$

$$1-p\_{G}$$

$$p\_{G}$$

$$p\_{G}$$

$$1-p\_{G}$$

Disclosure

No

Disclosure

No

Disclosure

Disclosure

Figure A1: Bayesian Inference when Motive for Disclosure is Harmfulness

Pre-disclosure, the consumer believes that Z25 is harmful with probability $p\_{0}$ (left side of the tree) and harmless with probability $1-p\_{0}$ (right side of the tree). If Z25 is harmful, then with probability $p\_{G}>\frac{1}{2}$ the government will get a signal that Z25 is harmful and mandate disclosure; and with probability $1-p\_{G}$ the government will get a signal that Z25 is harmless and decline to mandate disclosure. If Z25 is harmless, then with probability $p\_{G}>\frac{1}{2}$ the government will get a signal that Z25 is harmless and decline to mandate disclosure; and with probability $1-p\_{G}$ the government will get a signal that Z25 is harmful and mandate disclosure.

Knowing that the government decided to mandate a Z25 disclosure, the consumer would believe that Z25 is harmful with a (posterior) probability:

$$p\_{1}=\frac{p\_{0}∙p\_{G}}{p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)}$$

In calculating her posterior, the consumer considers the likelihood that the government correctly mandates disclosure ($p\_{0}∙p\_{G}$) and compares it to the overall likelihood that the government mandates disclosure – correctly or incorrectly ($p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)$). The posterior is basically the share of correct disclosure mandates.

The mathematical formula for the posterior, $p\_{1}$, captures several forces that intuitively affect the inferences that consumers draw from government-mandated disclosure: First, the consumer’s prior has a strong effect on the posterior probability. A higher prior translates into a higher posterior:

$$\frac{dp\_{1}}{dp\_{0}}=\frac{p\_{G}∙\left(1-p\_{G}\right)}{\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]^{2}}>0$$

In the extreme cases, where the consumer is certain about the health effects of Z25, the government’s decision to mandate disclosure has no effect on the consumer’s beliefs. The posterior is equal to the prior: $p\_{1}=p\_{0}$. There are two extreme cases. The first occurs when, before learning whether or not the government mandates disclosure, the consumer was already certain that Z25 is harmful. Formally, this means that the consumer’s prior was $p\_{0}=1$. If the consumer was already 100% certain that Z25 is harmful, then any signal emanating from the government’s decision to mandate disclosure will have no effect. Indeed, plugging $p\_{0}=1$ into the posterior equation above, we get:

$$p\_{1}=\frac{1∙p\_{G}}{1∙p\_{G}+\left(1-1\right)∙\left(1-p\_{G}\right)}=1$$

The second extreme case occurs when, before learning whether or not the government mandates disclosure, the consumer was already certain that Z25 is not harmful. Formally, this means that the consumer’s prior was $p\_{0}=0$. Again, the signal emanating from the government’s decision to mandate disclosure will have no effect. Plugging $p\_{0}=0$ into the posterior equation, we get:

$$p\_{1}=\frac{0∙p\_{G}}{0∙p\_{G}+\left(1-0\right)∙\left(1-p\_{G}\right)}=0$$

The second force that affects the consumer’s posterior is the accuracy of the government’s signal, as measured by $p\_{G}$. The more accurate the signal, the more upward updating would be expected – from $p\_{0}$ to $p\_{1}$. (We expect only upward updating, since updating is triggered by the government’s decision to mandate disclosure – a decision that is motivated by a signal that Z25 is harmful.) Updating is captured by the difference

$$p\_{1}-p\_{0}=\frac{p\_{0}∙\left(1-p\_{0}\right)∙\left(2p\_{G}-1\right)}{p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)}$$

(Note that, since $p\_{G}>\frac{1}{2}$, we get $p\_{1}-p\_{0}>0$, which implies upward updating.) We see that, as explained above, in the cases of pre-disclosure certainty, when $p\_{0}=1$ or $p\_{0}=0$, there is no updating, i.e., the government’s signal has no effect on the consumer’s posterior: $p\_{1}-p\_{0}=0$, or $p\_{1}=p\_{0}$. We also see that as the pre-disclosure uncertainty increases, namely, as $p\_{0}$ moves away from $p\_{0}=1$ or $p\_{0}=0$, the effect of the government’s signal increases.[[3]](#footnote-3) Finally, we see that the level of updating increases in the accuracy of the government’s signal:

$$\frac{d\left(p\_{1}-p\_{0}\right)}{dp\_{G}}=\frac{p\_{0}∙\left(1-p\_{0}\right)}{\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]^{2}}>0$$

In fact, the posterior is influenced not by the actual $p\_{G}$, but by the perceived $p\_{G}$. In particular, if consumers overestimate the accuracy of the government’s signal, the level of updating will be higher.

2.3 Uncertainty About the Motive for Disclosure

We now reintroduce uncertainty about the government’s motives, namely, with probability *q* the government mandates disclosure because it believes that Z25 is harmful and with probability 1-*q* the government mandates disclosure for other reasons that have nothing to do with the potential harmfulness of Z25. In this case, the Bayesian inference problem is depicted in the following tree diagram:

$$1-p\_{0}$$

$$p\_{0}$$

1-*q*

1-*q*

*q*

*q*

Disclosure

No

Disclosure

$$p\_{G}$$

$$1-p\_{G}$$

Disclosure

No

Disclosure

$$p\_{G}$$

$$1-p\_{G}$$

Disclosure

$$\frac{1}{2}$$

No

Disclosure

Disclosure

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

No

Disclosure

Figure A2: Bayesian Inference when Motive for Disclosure is Unclear

On the left side of the tree (where Z25 is harmful, according to the consumer’s prior), with probability $p\_{G}>\frac{1}{2}$ the government will get a signal that Z25 is harmful and with probability $1-p\_{G}$ the government will get a signal that Z25 is harmless. But these signals determine the government’s decision whether to mandate disclosure only with probability *q*. With probability 1-*q*, the decision to mandate disclosure is completely uninformative, as captured by the 50%-50% probability distribution on the 1-*q* branch. On the right side of the tree (where Z25 is harmless), with probability $p\_{G}>\frac{1}{2}$ the government will get a signal that Z25 is harmless and with probability $1-p\_{G}$ the government will get a signal that Z25 is harmful. But these signals determine the government’s decision whether to mandate disclosure only with probability *q*.

Knowing that the government decided to mandate a Z25 disclosure, the consumer would believe that Z25 is harmful with a (posterior) probability:

$$p\_{1}=\frac{p\_{0}∙\left[qp\_{G}+\left(1-q\right)\frac{1}{2}\right]}{p\_{0}∙\left[qp\_{G}+\left(1-q\right)\frac{1}{2}\right]+\left(1-p\_{0}\right)∙\left[q\left(1-p\_{G}\right)+\left(1-q\right)\frac{1}{2}\right]}$$

Or:

$$p\_{1}=\frac{p\_{0}∙\left[qp\_{G}+\left(1-q\right)\frac{1}{2}\right]}{q∙\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]+\left(1-q\right)∙\frac{1}{2}}$$

In calculating her posterior, the consumer considers the likelihood that the government correctly mandates disclosure – based on an accurate signal that Z25 is harmful or randomly ($p\_{0}∙\left[qp\_{G}+\left(1-q\right)\frac{1}{2}\right]$), and compares it to the overall likelihood that the government mandates disclosure – correctly or incorrectly ($q∙\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]+\left(1-q\right)∙\frac{1}{2}$).

As in the simpler case, where the government’s disclosure motives were clear, we find that a higher prior leads to a higher posterior. We also find that, in the extreme cases, when, pre-disclosure, the consumer is certain about the health effects of Z25, the government’s decision to mandate disclosure has no effect on the consumer’s beliefs, and the posterior is equal to the prior: $p\_{1}=p\_{0}$. As $p\_{0}$ moves away from $p\_{0}=1$ or $p\_{0}=0$, the level of updating increases. Updating is captured by the difference

$$p\_{1}-p\_{0}=\frac{q∙p\_{0}∙\left(1-p\_{0}\right)∙\left(2p\_{G}-1\right)}{q∙\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]+\left(1-q\right)∙\frac{1}{2}}$$

Also, as in the simpler case, the level of updating increases in the accuracy of the government’s signal:

$$\frac{d\left(p\_{1}-p\_{0}\right)}{dp\_{G}}=\frac{q∙p\_{0}∙\left(1-p\_{0}\right)}{\left[q∙\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]+\left(1-q\right)∙\frac{1}{2}\right]^{2}}>0$$

And now that we have uncertainty about the motive for disclosure, we can also measure the effect of this uncertainty on the level of updating. As can be expected, the consumer will update more when disclosure is likely motivated by a signal that Z25 is harmful (i.e., when *q* is large) and the consumer will update less when disclosure is likely motivated by other reasons (i.e., when *q* is small). Formally, the level of updating is increasing in *q*:

$$\frac{d\left(p\_{1}-p\_{0}\right)}{dq}=\frac{\frac{1}{2}∙p\_{0}∙\left(1-p\_{0}\right)∙\left(2p\_{G}-1\right)}{\left[q∙\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]+\left(1-q\right)∙\frac{1}{2}\right]^{2}}>0$$

We can also identify two special cases. When *q* = 0, the disclosure is not informative and $p\_{1}=p\_{0}$. When *q* = 1, we are back in the special case of a clear motive to mandate disclosure only if Z25 is harmful and the posterior is:

$$p\_{1}=\frac{p\_{0}∙p\_{G}}{p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)}$$

In fact, the posterior is influenced not by the actual *q*, but by the perceived *q*. In particular, if consumers overestimate the likelihood that the government’s decision to mandate disclosure is motivated by a finding of harmfulness, the level of updating will be higher.

The preceding analysis is summarized in the following proposition.

**Proposition:** The consumer’s posterior probability, $p\_{1}$, is determined by the following factors -

1. Consumer’s Prior
	1. The posterior, $p\_{1}$, is increasing with the prior, $p\_{0}$.
	2. In the extreme cases, where the consumer is certain about the health effects of Z25, the government’s decision to mandate disclosure has no effect on the consumer’s beliefs: (i) When $p\_{0}$ = 0, the disclosure is irrelevant and $p\_{1}=p\_{0}=0$; and (ii) When $p\_{0}$ = 1, the disclosure is irrelevant and $p\_{1}=p\_{0}=1$. As the pre-disclosure uncertainty increases, namely, as $p\_{0}$ moves away from $p\_{0}=1$ or $p\_{0}=0$, the effect of the government’s decision to mandate disclosure increases.
2. Accuracy of the Government’s Information: The posterior, $p\_{1}$, is increasing with the perceived accuracy of the government’s information about the harm from Z25, i.e., $p\_{1}$ is increasing in $p\_{G}$.
3. Government Motives: The posterior, $p\_{1}$, is increasing with the perceived likelihood that the disclosure mandate was motivated by the government’s belief that Z25 is harmful, i.e., $p\_{1}$, is increasing in *q*.
1. We can extend this framework to allow for a continuous outcome variable measuring the probability that Z25 is harmful. [↑](#footnote-ref-1)
2. This assumes that the two reasons are mutually exclusive. We can relax this assumption. [↑](#footnote-ref-2)
3. The derivative of the difference $p\_{1}-p\_{0}$ w.r.t. $p\_{0}$ is:

$$\frac{d\left(p\_{1}-p\_{0}\right)}{dp\_{0}}=\left(2p\_{G}-1\right)\frac{\left(1-p\_{0}\right)^{2}∙\left(1-p\_{G}\right)-p\_{0}^{2}∙p\_{G}}{\left[p\_{0}∙p\_{G}+\left(1-p\_{0}\right)∙\left(1-p\_{G}\right)\right]^{2}}$$

The derivative is increasing as we move upward from $p\_{0}=0$ and as we move downward from $p\_{0}=1$. (For subtle reasons, the difference $p\_{1}-p\_{0}$ is not maximized at $p\_{0}=\frac{1}{2}$; for one, as $p\_{0}$ increases there is less room for upward updating.) [↑](#footnote-ref-3)