Appendix A.

(A.1)  which implies 

 *where*, 

(A.2)  for 0≤*t*<*T*-τ, which implies

 

*where*  and

 .

(A.3)  for *T−τ* ≤*t*≤ *T*, which implies



(A.4) for 0≤*t*≤*T*, which implies

 

(A.5)  for *0*≤*t*≤*T*, which implies

 

(A.6)   implies

 

(A.7)    implies

 

(A.8)  implies 

(A.9)  implies 

(A.10) *lim* *λ*1*e*-δ*t* ≥0, *lim* *λ*2e-δ*t* = 0, *lim* *λ*3*e*-δ*t* = 0, *lim* *λ*4*e*-δ*t* = 0,

 *t*→*T* *t*→*T**t*→*T**t*→*T*

Equation (A.1), representing the adjoint equation, illustrates that the pumping costs create value associated with user costs. Similarly, the adjoint equations (A.2) and (A.3) demonstrate that the nitrate stock in the aquifer create value associated with user costs. The expected foregone economic benefits from failing to adopt an induced irrigation technology, due to failure to develop an induced irrigation technology, in equation (A.4) represent the user costs associated with the probablity of developing an induced irrigation technology. Similarly, the expected foregone economic benefits from failing to adopt an induced irrigation technology in equation (A.5) create the user costs associated with the probablity of adopting an induced irrigation technology. Equations (A.6) through (A.9) are state equations associated with state variables, *h*(*t*), *N*(*t*), *M*(*t*), and *V*(*t*), respectively. Finally, equation (A.10) represents conventional transversality conditions.

**Appendix B**

The approximated marginal contributions of the state variables at time *t*0 to the expected optimal action-value function are presented as follows:

(B.1) 

#####

*where*, =< 0 and

 =< 0.

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