Appendix
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When Can Benefit Cost Analyses Ignore Secondary Markets?

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Appendix: The sign of welfare effects in secondary markets

This appendix provides the details behind the brief sketch in Section 4 and the summary in Table 1. In particular, we describe a straightforward, graphical approach for signing the welfare effects in secondary markets when there are income effects. We assume the case in which there is only one relevant secondary market in which the price changes. We then define the conditions under which secondary market welfare effects will be negative, positive, or ambiguous.

BCA textbooks (Gramlich 1997; Boardman et al. 2018) mention that the net welfare effects in secondary markets are negative. But the result is not emphasized or proved, in part because the textbooks claim that analysts make offsetting general equilibrium adjustments in primary markets. Moreover, because the textbook treatments assume no income effects, questions remain about whether negative welfare effects in secondary markets is a special case or a more general result.

Following the standard textbook approach, we assume an initial price change in primary market followed by a price change in the secondary market. Compared to a complete general equilibrium approach, this means that the analysis will be "path dependent" (Auerbach and Hines 2002; Johansson 2021). Our estimate of welfare changes from a price increase in the primary market from p_{x0} to p_{x1} will not be the simple opposite of the welfare change due to a price decrease from p_{x1} to p_{x0}. But that simplification mimics how real-world BCA practitioners typically analyze policy interventions.

The approach we take allows us to define the conditions under which net welfare effects will be negative, positive, or ambiguous. As we show, the answer depends on the particular welfare measure used—that is, EV or CV—and on whether the goods in the two markets are substitutes or complements.

The graphical analysis in Section 2 relied on consumer surplus, which is appropriate so long as demand for x and y have no income effects. Here we assess welfare using the more general measures of EV and CV. To see how these apply in our setting, we begin with the definitions of indirect utility for a representative consumer as a function of prices p_x and p_y and income w, before and after prices change:

$$\begin{aligned} u_0 &= v(p_{x0}, p_{y0}, w) \\ u_1 &= v(p_{x1}, p_{y1}, w). \end{aligned} \tag{A.1}$$

Utility before prices change in either market is u_0, and utility after both p_x and p_y change is u_1. Utility u_1 provides the reference level of welfare for EV, and u_0 the reference welfare for CV.

Equivalent variation

The welfare effects due to a price change in a secondary market consist of changes in producer surplus and consumer welfare. When using EV to measure consumer welfare, we can write the combined effect as

$$\begin{aligned} \Delta SW_y^{EV} &= \Delta PS_y + EV_y \\ &= \int_{p_{y0}}^{p_{y1}} s_y(p_y) dp_y - \int_{p_{y0}}^{p_{y1}} h_y(p_{x1}, p_y, u_1) dp_y \end{aligned} \tag{A2}$$

where $h_y(p_{x1}, p_y, u_1)$ is the compensated demand for y holding utility at u_1. EV_y measures the income a consumer would give up to avoid a price increase in the secondary market caused by the price change in the primary market; or alternatively, it could measure the income the consumer would give up to obtain a price decrease in the secondary market. In both cases, the welfare effects in the secondary market are evaluated conditional on the final price change in the primary market. Our task is to see if we can sign (A2).

Figure A1(a) depicts the case of an increase in p_y, which might happen for two reasons: either p_x (not shown) has increased, and y is a gross substitute for x; or p_x has decreased and y is a gross complement. We use the standard terminology "gross" to refer to the sign of the uncompensated or Marshallian cross-price effects, $\partial D_y / \partial p_x$, and "net" to refer to the compensated or Hicksian cross-price effects, $\partial h_y / \partial p_x$. This distinction between gross and net will become important when we turn to CV shortly.

Either way, p_y has increased and at the new equilibrium, the supply of y equals the uncompensated demand. What is more, by definition uncompensated and compensated demands are equal at these prices, $D_y(p_{x1}, p_{y1}, w) = h_y(p_{x1}, p_{y1}, u_1)$, and a further implication is that for every $p_y < p_{y1}$, compensated demand exceeds supply. Hence for a price increase in the secondary market, the EV_y loss exceeds the ΔPS_y gain. That is, the second term on the right side of (A2) exceeds the first. In Figure A1(a) the net welfare loss in the secondary market is the shaded area *cde*.[22]

The remaining two possibilities, where p_y decreases, are shown in Figure A1(b): an increase in p_x where y is a gross complement for x, and a decrease in p_x where y is a gross substitute. In either case, p_y falls and the ΔPS_y loss exceeds the EV_y gain. The first term on the right side of (A2) is negative and exceeds the second. The net welfare effect is again unambiguously negative and equal to area *cde* in Figure A1(b).

This discussion has so far focused entirely on whether *x* and *y* are gross substitutes or complements, ignoring whether they are net substitutes or complements. The reason involves the definition of EV, and its reliance on u_1 as the reference utility. For EV, the starting point for assessing welfare changes is the new price-quantity combination $D_y(p_{x1}, p_{y1}, w)$, which occurs at the new, uncompensated level of utility u_1 in both sides of Figure A1. So it is irrelevant whether, if consumers were compensated for a change in p_x, they would consume more or less *y*. All that matters is whether y is a gross substitute or complement. That changes when we turn to CV in the next subsection.

For now, however, we have shown a general result: when EV measures consumer welfare, net welfare effects in the secondary market are always negative. This result does not depend on the direction of the price change in the primary market, on whether the goods are gross or net complements or substitutes, or on whether the goods are normal or inferior.

Compensating variation

When CV measures consumer welfare, the net effect of a change in p_y is

$$\begin{aligned} \Delta SW_y^{CV} &= \Delta PS_y + CV_y \\ &= \int_{p_{y0}}^{p_{y1}} s_y(p_y)dp_y - \int_{p_{y0}}^{p_{y1}} h_y(p_{x1}, p_y, u_0)dp_y \end{aligned} \tag{A3}$$

[22] We have drawn the compensated demand in Figure A1 as steeper than uncompensated demand, consistent with y being a normal good. But *y* being normal is not necessary for our overall result in this subsection that any change p_y results in a welfare loss in the secondary market. If *y* were inferior, the compensated demand curves in Figure A1(a) and (b), h_y, would be less steep than the uncompensated demand curves D_y. As a consequence, the net welfare losses represented by the shaded areas in each graph, *cde*, would be even larger.

where CV_y is the compensating variation in the secondary market. The only difference between (A2) and (A3) is the reference level of utility, u_0, the level before either price changes. The measure of CV_y captures the amount of income that must be given to consumers to compensate for the secondary market price increase, or taken to compensate for a price decrease, leaving consumer welfare at u_0.

Using CV to measure welfare, the results do depend on whether y is a substitute or complement for x. To focus on the most plausible case, we assume that y is a normal good throughout.[23] We begin with the simpler case where y is a gross substitute for x. Figure A2(a) illustrates what happens in the secondary market as a result of an increase in p_x. As before, good y being a gross substitute for *x* means that p_y increases and the equilibrium shifts from point *c* to *d*, and PS increases by area *abcd*.

The CV_y welfare cost of the increase in p_y is based on the compensated demand curve, h_y, and we know this is to the right of the uncompensated demand curve as shown in Figure A2(a). Maintaining utility level u_0 after an increase in p_x requires an increase in income, and this must subsequently cause an increase in demand for *y*, a normal good, meaning that $h_y(p_{x1}, p_y, u_0) > D_y(p_{x1}, p_y, w)$ for all $p_y \geq p_{y0}$. Note also that the assumption of y being a gross substitute combined with its normality implies that both goods are also net substitutes.[24] The compensated demand h_y shifts further to the right than uncompensated demand D_y. It then follows that the loss is CV_y (area *abef*), which is greater than the gain ΔPS_y (area *abcd*) by the amount of the shaded area *cdef*. The result is still a net welfare loss in the secondary market.

Figure A2(b) illustrates how there is also a net welfare loss in the secondary market when the primary market price p_x declines, so long as we continue to assume y is a gross substitute for x. In this case, p_y decreases and the equilibrium shifts from point *d* to *c*. PS decreases by area *abcd*. To see the measure of CV_y, we again use the compensated demand curve at reference utility u_0, which in this case shifts further to the left than uncompensated demand because income must be taken away and y is a normal good. The CV_y gain in consumer welfare is

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[23] The results differ if *y* is inferior in ways that are less compelling, because unlike with EV, almost anything is possible in terms of the sign of net welfare effects in the secondary market.

[24] Recall the Slutsky equation: $\frac{\partial y}{\partial p_x} = \frac{\partial h}{\partial p_x} - \frac{\partial y}{\partial w} y$. If *x* and *y* are gross substitutes the first term is positive. If *y* is normal the third term is positive. Together that means the middle term must also be positive, so *x* and *y* are net substitutes.

therefore area *abef*, which is less than ΔPS_y, implying a net welfare loss equal to the shaded area *cdef*.

Now consider the case where y is a gross complement for x. Here, in contrast to the previous cases, there is no general result, and the sign of the secondary market welfare change will depend on whether *x* and *y* are net substitutes or complements. Assume first that *x* and *y* are net complements, and that p_x increases. Figure A3(a) illustrates the consequences in the secondary market. Uncompensated demand D_y shifts to the left and causes a decrease in p_y as the equilibrium shifts from point *d* to point *c*. Compensated demand $h_y(p_{x1}, p_y, u_0)$ does not shift as far to the left as D_y, because consumers need to be compensated for the increase in p_x to keep utility at u_0, and *y* is a normal good. Therefore for all prices of good *y* between p_{y0} and p_{y1}, it must hold that $h_y(p_{x1}, p_y, u_0) > D_y(p_{x1}, p_y, w)$. The decrease in PS_y in Figure A3(a) is area *abcd*, as always when p_y declines. The increase in consumer welfare CV_y is area *abef*. Hence the net welfare effect in this case is ambiguous and depends on the relative sizes of the two shaded triangles, with the upper representing a welfare loss and the lower a welfare gain.

Figure A3(b) illustrates what happens for a policy that decreases p_x in the primary market. In this case, the compensated demand is to the left of the uncompensated demand over the range of p_y from p_{y0} up to p_{y1}. PS increases by *abcd*. CV_y is a loss represented by *abef*. And here again, the net welfare change is ambiguous. The lower shaded triangle depicts a loss to consumers and the upper triangle a gain to producers.

Finally, consider what happens when *x* and *y* are gross complements but net substitutes. This is possible if the income effect in the Slutsky equation is sufficiently large.[25] Figure A2 can be relabeled to illustrate this case. Panel (a) still represents the response to a price increase in the primary market, but instead of p_y increasing from p_{y0} to p_{y1}, reinterpret it as decreasing from p_{y1} down to p_{y0} because x and y are complements. Producer surplus thus declines in panel (a) by *abcd*. To see the CV_y measure, note that the compensated demand curve shifts to the right even though the uncompensated demand shifts to the left. The result is a consumer welfare gain of *abef* and a net welfare gain of the shaded area. Similarly, Figure A2(b) can be reinterpreted to

[25] Again recall Slutsky: $\frac{\partial y}{\partial p_x} = \frac{\partial h}{\partial p_x} - \frac{\partial y}{\partial w} y$. For gross complements the first term is negative. The middle term can thus be positive, and the goods therefore net substitutes, so long as third term—the income effect—is sufficiently large.

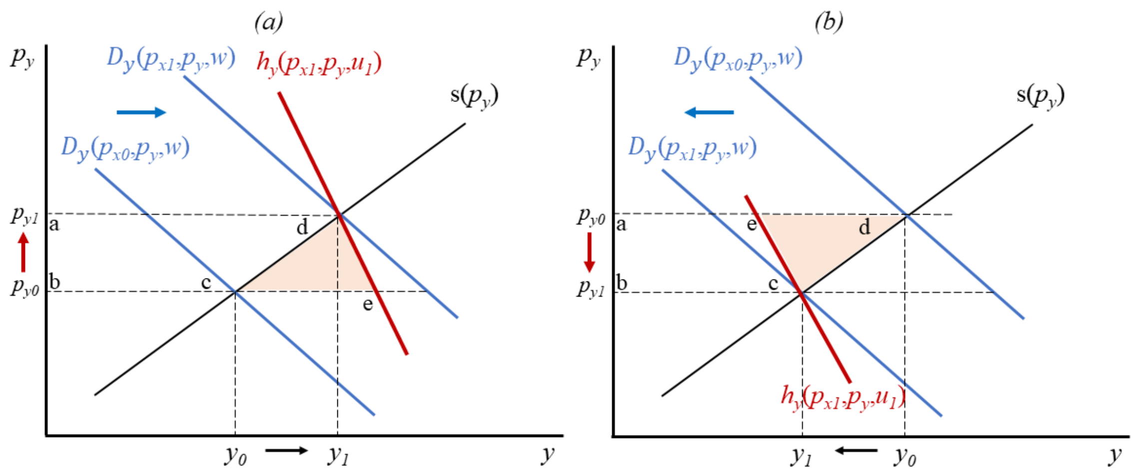
illustrate the reaction to a price decrease in the primary market, when *x* and *y* are gross complements but net substitutes. The price p_y rises from p_{y1} to p_{y0}, PS rises by *abcd*, uncompensated demand shifts right while compensated demand shifts left, and CV_y is a loss represented by area *abef*. The result is net welfare gain of *cdef*.

Summary

Table 1 provides a summary of the general results. When the price in the secondary market p_y does not change, there are no welfare effects in that market. That could happen for two reasons. Either *x* and *y* are neither substitutes nor complements, so demand does not shift, having no effect on price. Or, as we showed in Section 2, price does not change when secondary market supply curves are flat. In the latter case, even if demand for y shifts, p_y does not change and there are no welfare effects.

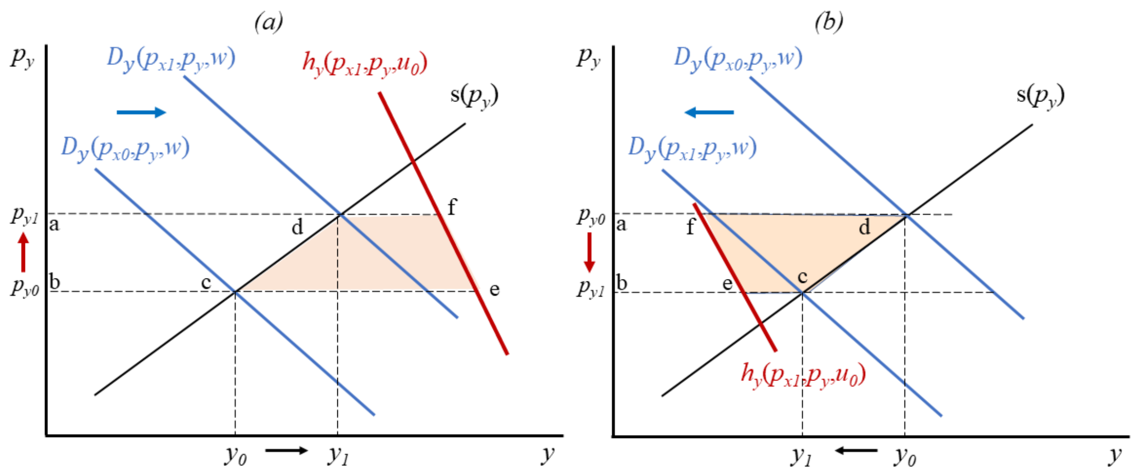
When p_y does change, and when the relevant measure of consumer welfare is EV, welfare effects in the secondary market are always negative. This result matches that for no income effects or when welfare is measured using CS. And assuming further that the good in the secondary market is normal, the same result continues to hold if welfare is measured using CV so long as y is a gross substitute for x. If, however, y is a gross complement, then different results are possible. In particular, the net welfare effects are indeterminate if the goods are net complements, whereas the effects are positive if they are net substitutes but gross complements.

Figure A1. Equivalent Variation



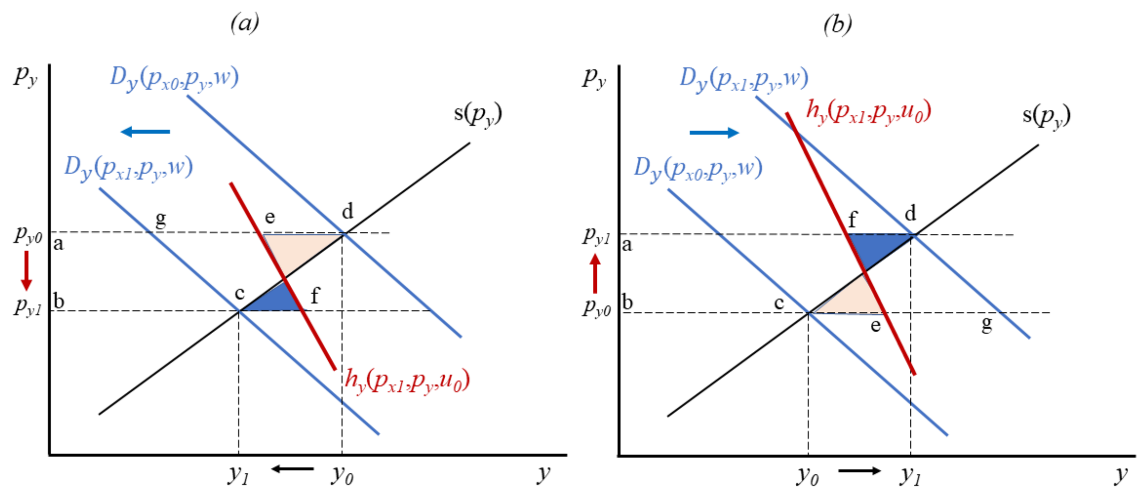
Notes: Panel (a) depicts an increase in p_y, either because a regulation increased p_x and *x* and *y* are substitutes, or because p_x decreased and the goods are complements. Panel (b) depicts a decline in p_y. PS increases by *abcd* in (a) and decreases by *abcd* in (b). Consumer welfare measured by EV_y falls by *abde* in (a) and grows by *abce* in (b). Both panels show a net loss in welfare equal to the shaded areas *cde*.

Figure A2. Compensating variation



Notes: As in Figure A1, (a) depicts an increase in p_y and (b) depicts a decrease. And as in Figure A1, PS increases by *abcd* in (a) and decreases by *abcd* in (b). Consumer welfare measured by CV_y falls by *abef* in (a) and grows by *abef* in (b). Both panels show a net loss in welfare equal to the shaded areas *cdef*.

Figure A3. Compensating variation, x and y are complements



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Notes: In this figure the goods are both gross and net complements. So an increase in p_x causes p_y to fall in (a), and a decrease in p_x causes p_y to rise in (b). Compensated demand h_y doesn't shift as far left in (a) as the uncompensated demand D_y, because of the extra income necessary to offset the increase in p_x to maintain utility u_0. And so PS falls by *abcd*, consumer welfare rises by CV_y=*abef*, and the net welfare change is ambiguous. In (b) PS rises by *abcd*, compensated demand shifts to the right by less than uncompensated demand, consumer welfare falls by CV_y=*abef*, and the net welfare change is ambiguous.

Appendix Table A1: EPA Regulatory Impact Analyses

Rule	Year	Welfare: Consumer surplus (CS) or CV/EV	Partial or general equilibrium demand	Considers substitutes or complements
Pulp and Paper NESHAP and NSPS	1997	--	--	No
Architectural Coatings VOCs	1998	CS	Partial	No
Nonroad Nonhandheld Engines	1999	--	--	No
Regional Haze Rule	1999	--	--	No
Tier 2 Motor Vehicle Emissions Standards	1999	--	--	No
Heavy-Duty Engine and Fuel Sulfur	2000	--	--	No
Highway Heavy Duty Engines	2000	--	--	No
Small Nonroad Engines	2000	--	--	No
Industrial Boilers and Process Heaters NESHAP	2004	CS	Multimarket	No
Light Duty Vehcile NESHAP	2004	CS	Partial	No
Nonroad Diesel Engines	2004	CS	Multimarket	No
Plywood and Composite Wood Products	2004	CS	Partial	No
Stationary Internal Combustion Engine (RICE) NESHAP	2004	CS	Multimarket	No
Clean Air Interstate Rule	2005	--	--	No
Clean Air Mercury Rule	2005	--	--	No
Visibility Rule, Regional Haze Regulations	2005	--	--	No
PM2.5 NAAQS	2006	--	--	No
Stationary Compression Ignition Engines	2006	CS	Partial	No
Hazardous Mobile Pollutants	2007	CS	Partial	No
Stationary Spark-Ignition NSPS and NESHAP	2007	--	Partial	No
Locomotive and Marine Engines	2008	CS	Partial	No
Marine Engines	2008	CS	Partial	No
Ozone NAAQS	2008	--	--	No
Petroleum Refineries NSPS	2008	CS	Partial	No
Small Marine Engines	2008	CS	Partial	No
Mandatory reporting of GHGs	2009	--	--	No
Marine Diesel Engines	2009	CS	Partial	No

Existng Stationary Spark Ignition NESHAP	2010	CS	Partial	No
NO2 NAAQS	2010	--	--	No
Portland Cement NESHAP and NSPS	2010	CS	Partial	No
Prevention of Significant Deterioration, and Title V GHG Tailoring	2010	--	--	No
SO2 NAAQS	2010	--	--	No
Stationary Compression Engines	2010	CS	Partial	No
Industrial Boilers and Process Heaters NESHAP	2011	CS	Multimarket	No
Interstate Transport of PM2.5 and Ozone	2011	CS	Multimarket	No
Murcury and Air Toxics Standards	2011	--	--	No
Solid Waste Incinerators	2011	CS	Multimarket	No
Oil and Gas NESHAP	2012	--	--	No
Petroleum Refineries NSPS	2012	--	--	No
PM2.5 NAAQS	2012	--	--	No
Stationary Compression Engines	2013	CS	Partial	No
Stationary Spark Ignition Engines NESHAP	2013	--	--	No
Tier 3 Motor Vehicle Emission Standards	2014	--	--	No
2017–2025 Light-Duty Vehicle GHG and Fuel Economy Standards	2015	--	--	No
Clean Power Plan	2015	--	--	No
Final Brick and Structural Clay Products NESHAP	2015	CS	Partial	No
GHGs from Electric Utilities	2015	--	--	No
Ozone NAAQS	2015	--	--	No
Residential Wood Heaters NSPS	2015	CS	Partial	No
Cross-State Air Pollution Rule Update	2016	--	--	No
GHG and Efficiency Standards for Trucks	2016	--	--	No
Municipal Solid Waste Landfills	2016	--	--	No
Oiil and Gas NSPS	2016	--	--	No
Repeal of Clean Power Plan	2019	--	--	No

Industrial Boilers NESHAP	2020	--	--	No
Cross-State Air Pollution Rule Update	2021	--	--	No

Appendix References

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