**Appendix: Summary of generalized optimum for specific functional forms**

**Common assumptions:**

* Risk = *P*\**D*: probability times consequence (damages)
* Security (risk avoided): *S* = Max(Risk) – Risk
* Costs are a function of risk (security): C(*S*) with C’(*S*)>0 and C’’(*S*)>0

**Issues:**

A) Does S\* increase as technology improves?

B) How does a constraint affect that result?

C) What is the value function and its derivatives?

**Objective Function:**

* Unconstrained: Max *S* – C(*S*)
* When constrained: subject to C(*S*) = , assuming the organization spends all its funds.

**Selected cost functional forms:**

* Squared: C(*S*) = b*S*2 0<b<1
* Linear quadratic: C(*S*) = a*S* + b*S*2 0 < a,b < 1; a+b<1
* Exponential: C(*S*) = a*S* , a > 0; only specific results available from authors.

**Table 1A: Summary of optimized results**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Cost function** | **S\*** | **C\*** | **Net Benefit\*** | **Derivatives[[1]](#footnote-1)** |
| Squared | 1/2b | 1/4b | 1/4b | S\*’b < 0; C\*’b < 0; NB\*’b < 0 |
| Linear Quadratic |  (1-a)/2b | (1-a2)/4b | (1-a)2/4b | S\*’b < 0; C\*’b < 0; NB\*’b < 0; same for derivative w.r.t. a  |
| Squared equality constrained  | ( /b).5 |  |  .5(1-b.5)/b.5  λ = (1/(2bS))-1 | S\*’b < 0; NB\*’b < 0 |
| Linear quadratic equality constrained | [-a+ SQRT(a2+4b)] /2b |  | S\*(1-a-bS\*)λ = (1/(a+2bS))-1 | Indeterminate, depends on size of a, b,  |

1. Note that for cost-improving technological change, the coefficients *b* or *a* decrease so that the sign of the derivative is opposite that of the derivative indicated in the table below. [↑](#footnote-ref-1)