Appendix A. Proof of Proposition 4

Regarding the equivalent income index, we have, for two bundles (y_i, L_i) and (y_i', L_i'), equivalent income levels $\hat{y}_i$ and $\hat{y}_i'$ satisfying:

$$U_i\left(\hat{y}_i, \bar{L}\right) = U_i(y_i, L_i) \text{ and } U_i\left(\hat{y}_i', \bar{L}\right) = U_i(y_i', L_i')$$

Given the monotonicity of $U_i(\cdot)$ in y_i, if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then $\hat{y}_i > \hat{y}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{y}_i < \hat{y}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{y}_i = \hat{y}_i'$. We thus have: $\hat{y}_i' \geq \hat{y}_i \iff U_i(y_i', L_i') \geq U_i(y_i, L_i)$, that is, Respect for Preferences is satisfied.

Consider now the equivalent lifetime index. Assume $y_i, \bar{y} > \tilde{y}_i$. For two bundles (y_i, L_i) and (y_i', L_i'), equivalent lifetime $\hat{L}_i$ and $\hat{L}_i'$ satisfy:

$$U_i\left(\bar{y}, \hat{L}_i\right) = U_i(y_i, L_i) \text{ and } U_i\left(\bar{y}, \hat{L}_i'\right) = U_i(y_i', L_i')$$

If $\bar{y} > \tilde{y}_i$, it is easy to see that if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then it has to be the case, by monotonicity of $U_i(y_i, L_i)$ in L_i, that $\hat{L}_i > \hat{L}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{L}_i < \hat{L}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{L}_i = \hat{L}_i'$. Thus Respect for Preferences is satisfied when $y_i > \tilde{y}_i$ and $\bar{y} > \tilde{y}_i$.

Assume now $y_i, \bar{y} < \tilde{y}_i$. If $U_i(y_i, L_i) > U_i(y_i', L_i')$, then we need $U_i\left(\bar{y}, \hat{L}_i\right) > U_i\left(\bar{y}, \hat{L}_i'\right)$, which implies $\hat{L}_i < \hat{L}_i'$. Thus Respect for Preferences is not satisfied in that case.

Concerning the alternative equivalent lifetime index, three cases can arise.

If $y_i > y_i' > \tilde{y}_i$, Respect for Preferences is satisfied, and the proof is similar to the one for the standard equivalent lifetime index (since in that case $\breve{L}_i = \hat{L}_i$), except that the reference income is now $\bar{y}_2$.

If $y_i < y_i' < \tilde{y}_i$, we have, for two bundles (y_i, L_i) and (y_i', L_i'), alternative equivalent lifetime levels $\breve{L}_i = -\hat{L}_i$ and $\breve{L}_i' = -\hat{L}_i'$ where $\hat{L}_i$ and $\hat{L}_i'$ satisfy:

$$U_i\left(\bar{y}_2, \hat{L}_i\right) = U_i(y_i, L_i) \text{ and } U_i\left(\bar{y}_2, \hat{L}_i'\right) = U_i(y_i', L_i')$$

Given that $y_i < y_i' < \tilde{y}_i$, we have that if $U_i(y_i, L_i) > U_i(y_i', L_i')$, then it has to be the case, by monotonicity of $U_i(y_i, L_i)$ in L_i, that $\hat{L}_i < \hat{L}_i'$, leading to $\breve{L}_i > \breve{L}_i'$. Moreover, if $U_i(y_i, L_i) < U_i(y_i', L_i')$, then $\hat{L}_i > \hat{L}_i'$, leading to $\breve{L}_i < \breve{L}_i'$. Finally, if $U_i(y_i, L_i) = U_i(y_i', L_i')$, then $\hat{L}_i = \hat{L}_i'$, leading to $\breve{L}_i = \breve{L}_i'$. Thus Respect for Preferences is satisfied.

If $y_i < y_i' = \tilde{y}_i$ or $y_i > \tilde{y}_i = y_i'$, Respect for Preferences also holds, since in the former case we have $\breve{L}_i = -\hat{L}_i < 0 = \breve{L}_i'$, whereas in the latter case $\breve{L}_i = \hat{L}_i > \breve{L}_i' = 0$.

Appendix B. Proof of Proposition 6

Take the equivalent income index. When $L_i = L_j = \bar{L}$, we have:

$$\begin{aligned} U_i\left(\hat{y}_i, \bar{L}\right) &= U_i(y_i, \bar{L}) \iff \hat{y}_i = y_i \\ U_j\left(\hat{y}_j, \bar{L}\right) &= U_j(y_j, \bar{L}) \iff \hat{y}_j = y_j \end{aligned}$$

Hence it follows that: $\hat{y}_i \geq \hat{y}_j \iff y_i \geq y_j$, that is, that Resourcism is satisfied.

Take the equivalent lifetime index. When $y_i = y_j = \bar{y}$, we have

$$\begin{aligned} U_i\left(\bar{y}, L_i\right) &= U_i(\bar{y}, \hat{L}_i) \iff \hat{L}_i = L_i \\ U_j(\bar{y}, L_j) &= U_j(\bar{y}, \hat{L}_j) \iff \hat{L}_j = L_j \end{aligned}$$

Hence it follows that: $\hat{L}_i \geq \hat{L}_j \iff L_i \geq L_j$, i.e., that Lifetimism is satisfied.

Take the alternative equivalent lifetime index. Suppose that $\bar{y}_1 < \tilde{y}_i < y_i < y_j < \tilde{y}_j < \bar{y}_2$. We have:

$$\begin{aligned} \breve{L}_i &= \hat{L}_i \text{ where } U_i\left(\bar{y}_2, \hat{L}_i\right) = U_i(y_i, L_i) \\ \breve{L}_j &= -\hat{L}_j \text{ where } U_j\left(\bar{y}_1, \hat{L}_j\right) = U_j(y_j, L_j) \end{aligned}$$

Hence it follows that: $\breve{L}_i \geq \breve{L}_j \iff L_i \geq -L_j$, i.e. Alternative Lifetimism is satisfied.

Appendix C. Proof of Proposition 9

We have:

$$\frac{\Delta \hat{y}}{\hat{y}} = \frac{\left[\left(\frac{(y'')^{1-\sigma}}{1-\sigma} - \alpha\right)\frac{L''}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}}{\left[\left(\frac{(y')^{1-\sigma}}{1-\sigma} - \alpha\right)\frac{L'}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}} - 1 \text{ and } \frac{\Delta \hat{L}}{\hat{L}} = \frac{L''\left[\frac{(y'')^{1-\sigma}}{1-\sigma} - \alpha\right]}{L'\left[\frac{(y')^{1-\sigma}}{1-\sigma} - \alpha\right]} - 1$$

Hence we have:

$$\frac{\Delta \hat{y}}{\hat{y}} \geq \frac{\Delta \hat{L}}{\hat{L}} \iff \frac{\left[\left(\frac{(y'')^{1-\sigma}}{1-\sigma} - \alpha\right)\frac{L''}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}}{\left[\left(\frac{(y')^{1-\sigma}}{1-\sigma} - \alpha\right)\frac{L'}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}} \geq \frac{L''\left[\frac{(y'')^{1-\sigma}}{1-\sigma} - \alpha\right]}{L'\left[\frac{(y')^{1-\sigma}}{1-\sigma} - \alpha\right]}$$

Let us define $U' \equiv L'\left(\frac{(y')^{1-\sigma}}{1-\sigma} - \alpha\right)$ and $U'' \equiv L''\left(\frac{(y'')^{1-\sigma}}{1-\sigma} - \alpha\right)$. We have, given $0 < \sigma < 1$:

$$\begin{aligned} \frac{\Delta \hat{y}}{\hat{y}} &\geq \frac{\Delta \hat{L}}{\hat{L}} \iff \frac{\left[\frac{U''}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}}{\left[\frac{U'}{\bar{L}} + \alpha\right]^{\frac{1}{1-\sigma}}} \geq \frac{U''}{U'} \\ \frac{\Delta \hat{y}}{\hat{y}} &\geq \frac{\Delta \hat{L}}{\hat{L}} \iff \frac{U'' + \alpha\bar{L}}{U' + \alpha\bar{L}} \geq \left(\frac{U''}{U'}\right)^{1-\sigma} \end{aligned}$$

Hence we have the condition of Proposition 9. When $\alpha = 0$, and given $0 < \sigma < 1$, this condition becomes:

$$\frac{\Delta \hat{y}}{\hat{y}} \geq \frac{\Delta \hat{L}}{\hat{L}} \iff U'' \geq U'$$

This completes the proof of Proposition 9.

Appendix D. Derivation of the VSL

To derive the VSL, remind first that expected lifetime utility can be written as:

$$U = \sum_{i=0}^{m-1} s_{i+1} \left[\frac{y_i^{1-\sigma}}{1-\sigma} - \alpha \right]$$

where m is the maximum length of life, $s_{i+1} = \prod_{j=0}^{i} (1-d_j)$ is the (unconditional) probability of survival to age $i+1$, and d_j is the probability of death at age j conditionally on survival to age j.

We have:

$$\frac{\partial U}{\partial d_0} = -\left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha \right] - \sum_{i=1}^{m-1} \frac{s_{i+1}}{(1-d_0)} \left[\frac{y_i^{1-\sigma}}{1-\sigma} - \alpha \right]$$

Assuming constant income per period, we obtain:

$$\begin{aligned}
\frac{\partial U}{\partial d_0} &= \left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha \right] \left[-1 - \frac{1}{1-d_0} \sum_{i=1}^{m-1} s_{i+1} \right] \\
&= \frac{-1}{1-d_0} \left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha \right] \left[\underbrace{1-d_0}_{s_1} + \sum_{i=1}^{m-1} s_{i+1} \right] \\
&= -\frac{1}{s_1} \left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha \right] L
\end{aligned}$$

since life expectancy $L = \sum_{i=0}^{m-1} s_{i+1}$.

We also have:

$$\frac{\partial U}{\partial y_0} = s_1 y_0^{-\sigma}$$

Hence, assuming, as a proxy, that $s_1 \approx 1$, the VSL can be written as:

$$VSL = -\frac{\frac{\partial U}{\partial d_0}}{\frac{\partial U}{\partial y_0}} = \frac{L \left[\frac{y_0^{1-\sigma}}{1-\sigma} - \alpha \right]}{y_0^{-\sigma}}$$