# **B** Online Appendix - Not for Publication

# **B.1** Implications and Alternative Approaches

From our analysis in the main text we can express the updating rule when the husband's participation constraint binds as follows:

$$(21) \quad \frac{\lambda_2}{1-\lambda_2} = \left(\frac{\epsilon_m}{\epsilon_f}\right)^{(1-\eta)(1-\gamma)} \left(\left(-\frac{\frac{\xi}{\chi}\epsilon_m^{(1-\eta)(1-\gamma)}(1-\gamma)}{(A_c+\sum_g\epsilon_g w(1-\tau_N))^{1-\gamma}} + \left(\frac{\frac{A_c}{2}+\epsilon_m w(1-\tau_N)}{A_c+\sum_g\epsilon_g(1-\tau_N)w}\right)^{1-\gamma}\right)^{\frac{1}{\gamma-1}} - 1\right)^{-\gamma}$$

We think that it is important to generalize our findings, by briefly explaining what (21) would look like if we included in our model the following features: 1. Preference heterogeneity, 2. Nash bargaining each period and 3. Different costs of divorce and a different division of the household's wealth in the event of divorce.

### **B.1.1** Preference Heterogeneity

To address preference heterogeneity, we keep our parameterization of the individual utility function as non-separable but assume that male and female spouses value consumption and leisure differently through the parameters  $\eta_m \neq \eta_f$ . Note that given our specification of preferences, differences in  $\eta_m$ and  $\eta_f$  also translate into differences in the relative risk aversion coefficient of the spouses.<sup>42</sup> It can be shown that the updating rule takes the following form: (22)

$$\frac{\lambda_2}{1-\lambda_2} = \kappa_3 \left(\frac{\epsilon_m^{(1-\eta_m)(1-\gamma)}}{\epsilon_f^{(1-\eta_f)(1-\gamma)}}\right) \left[ \left(-\frac{\frac{\xi}{\chi} \epsilon_m^{(1-\eta_m)(1-\gamma)} (1-\gamma)}{(A_c + \sum_g \epsilon_g w(1-\tau_N))^{1-\gamma}} + \left(\frac{\frac{A_c}{2} + \epsilon_m w(1-\tau_N)}{A_c + \sum_g \epsilon_g (1-\tau_N)w}\right)^{1-\gamma}\right)^{\frac{1}{\gamma-1}} - 1 \right]^{-\gamma}$$

where  $\kappa_3 = (\frac{\eta_f}{\eta_m})^{1-\gamma} (\frac{\eta_m}{1-\eta_m})^{(1-\eta_m)(1-\gamma)} (\frac{1-\eta_f}{\eta_f})^{(1-\eta_f)(1-\gamma)}$ . Notice that (22) is different from (21) only in the leading term. In both equations the leading terms are positive and the crucial expression which determines sign of the effect of wealth and the tax rates is the bracketed subsequent term. Therefore under preference heterogeneity wealth continues to have the same impact on household commitment and the implications of changes in capital and labor taxes on commitment are essentially the same.

## B.1.2 Nash Bargaining

Under Nash bargaining the household contract no longer has the property that it keeps individual weights constant over a region of the state space where the participation constraints do not bind. In the Nash bargaining equilibrium the sharing rule is rebargained each period, and  $\lambda_2$  is a function of assets,  $\epsilon_m$  and  $\epsilon_f$  and solves the following equation:

(23) 
$$\lambda_2(a,\epsilon) \in \arg\max_{\lambda_2}$$

$$\left[ \left( \frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2, \epsilon))((w(1 - \tau_N)\epsilon_m))^{1-\eta}} \right)^{1-\gamma} \frac{\chi}{1-\gamma} + \xi - \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{1-\eta}} \right)^{1-\gamma} \frac{\chi}{1-\gamma} \right] \\
\left[ \left( \frac{f(\lambda_2, \epsilon)}{1 + f(\lambda_2, \epsilon)} \frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{(w(1 - \tau_N)\epsilon_f)^{1-\eta}} \right)^{1-\gamma} \frac{\chi}{1-\gamma} + \xi - \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_f}{(w(1 - \tau_N)\epsilon_f)^{1-\eta}} \right)^{1-\gamma} \frac{\chi}{1-\gamma} \right] \\$$

From (23) we can derive the impact of wealth on the equilibrium allocation that solves the Nash bargaining program. <sup>43</sup>

<sup>&</sup>lt;sup>42</sup>It is not possible to derive closed form solutions when  $\gamma_m \neq \gamma_f$ . We have however solved this case numerically and found that our results generalize.

<sup>&</sup>lt;sup>43</sup>Notice that given that  $\lambda_2(a, \epsilon)$  is a function of the state vector, the equilibrium under Nash bargaining is a Markov perfect equilibrium whereby wealth and current productivity are sufficient to summarize the household's contract.

The first order derivative with respect to  $\lambda_2$  in (23) leads to the following optimality condition:

$$\frac{\Phi_m^{1-\gamma}}{\Omega_m} = \frac{\Phi_f^{1-\gamma}}{f(\lambda_2^*, \epsilon)\Omega_f}$$

where:

$$\Phi_{m} = \left(\frac{A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}}{(1 + f(\lambda_{2}^{*}, \epsilon))((w(1 - \tau_{N})\epsilon_{m}))^{1-\eta}}\right), \qquad \Phi_{f} = \left(\frac{f(\lambda_{2}^{*}, \epsilon)(A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g})}{(1 + f(\lambda_{2}^{*}, \epsilon))((w(1 - \tau_{N})\epsilon_{f}))^{1-\eta}}\right)$$
$$\Omega_{m} = \left(\frac{A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}}{(1 + f(\lambda_{2}^{*}, \epsilon))((w(1 - \tau_{N})\epsilon_{m}))^{1-\eta}}\right)^{1-\gamma} \frac{\chi}{1 - \gamma} + \xi - \left(\frac{A_{c}}{2} + w(1 - \tau_{N})\epsilon_{m}}{(w(1 - \tau_{N})\epsilon_{m})^{1-\eta}}\right)^{1-\gamma} \frac{\chi}{1 - \gamma}$$
$$\Omega_{f} = \left(\frac{f(\lambda_{2}^{*}, \epsilon)}{1 + f(\lambda_{2}^{*}, \epsilon)} \frac{A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}}{(w(1 - \tau_{N})\epsilon_{f})^{1-\eta}}\right)^{1-\gamma} \frac{\chi}{1 - \gamma} + \xi - \left(\frac{A_{c}}{2} + w(1 - \tau_{N})\epsilon_{f}}{(w(1 - \tau_{N})\epsilon_{f})^{1-\eta}}\right)^{1-\gamma} \frac{\chi}{1 - \gamma}$$

In order to obtain the derivative  $\frac{d\lambda_2^*}{dA_c}$  we utilize the Implicit Function Theorem. The derivative satisfies:

$$(24) \qquad -\frac{\chi}{\Omega_m} \frac{\Phi_m^{1-\gamma}}{(A_c + \sum_g w(1-\tau_N)\epsilon_g)} \left(1 - \frac{\left(\frac{A_c}{2} + \epsilon_m w(1-\tau_N)\right)\right)^{1-\gamma}}{\Phi_m^{1-\gamma}(\epsilon_m w(1-\tau_N)^{(1-\eta)(1-\gamma)})} \right) \\ + \frac{\chi}{\Omega_f} \frac{\Phi_f^{1-\gamma}}{(A_c + \sum_g w(1-\tau_N)\epsilon_g)} \left(1 - \frac{\left(\frac{A_c}{2} + \epsilon_f w(1-\tau_N)\right)^{1-\gamma}}{\Phi_f^{1-\gamma}(\epsilon_f w(1-\tau_N))^{(1-\eta)(1-\gamma)}}\right) \\ = -\gamma \frac{f_{\lambda_2^*}}{f(\lambda_2^*, \epsilon)} - \frac{\Phi_f^{1-\gamma}}{f(\lambda_2^*, \epsilon)\Omega_f} \frac{f_{\lambda_2^*}}{f(\lambda_2^*, \epsilon)(1+f(\lambda_2^*, \epsilon))} - \frac{\Phi_m^{1-\gamma}}{f(\lambda_2^*, \epsilon)\Omega_m} \frac{f_{\lambda_2^*}}{(1+f(\lambda_2^*, \epsilon))}$$

where  $f_{\lambda_2^*}$  represents the partial derivative. Note that for the sake of brevity the detailed derivations of (24) are omitted. Moreover, note that since the sign of the denominator terms in (24) is positive, the sign of  $\frac{d\lambda_2^*}{dA_c}$  is basically the sign of the numerator terms. These terms may be written as follows:

(25) 
$$-\frac{\chi}{\Omega_m} \frac{\Phi_m^{1-\gamma}}{(A_c + \sum_g w(1-\tau_N)\epsilon_g)} (1 - \frac{U_{s,m}}{U_{m,m}}) + \frac{\chi}{\Omega_f} \frac{\Phi_f^{1-\gamma}}{(A_c + \sum_g w(1-\tau_N)\epsilon_g)} (1 - \frac{U_{s,f}}{U_{m,f}})$$

where  $U_{j,g}, j \in \{s, m\}$  is the utility of the spouse of gender g as a single (j = s) or under the marriage (subscript j = m). Making use of the optimality condition of program 23 we can write:

(26) 
$$\frac{\chi}{\Omega_m} \frac{\Phi_m^{1-\gamma}}{(A_c + \sum_g w(1-\tau_N)\epsilon_g)} \left[-\left(1 - \frac{U_{s,m}}{U_{m,m}}\right) + \left(f(\lambda_2^*,\epsilon) - f(\lambda_2^*,\epsilon)\frac{U_{s,f}}{U_{m,f}}\right)\right]$$

Note that in the case where  $\epsilon_m = \epsilon_f$ ,  $\lambda_2^* = \frac{1}{2}$  and  $f(\lambda_2^*, \epsilon) = 1$ . In this case (26) equals zero because the utility that either spouse gets from the marriage (net of  $\xi$ ) is precisely the utility they get as singles). If however,  $\epsilon_m > \epsilon_f$  then  $\lambda_2^* > \frac{1}{2}$  and  $f(\lambda_2^*, \epsilon) < 1$ . In this case the male spouse gets a higher utility from being married because his bargaining power (relative to the outside option) increases. This makes the leading term in brackets negative. Analogously with  $f(\lambda_2^*, \epsilon) < 1$  the second (positive) term in the brackets is reduced, though positive. Overall, the derivative  $\frac{d\lambda_2^*}{dA_c}$  is negative. To put it differently when  $\lambda_2^* > \frac{1}{2}$ , higher wealth reduces the value and when  $\lambda_2^* < \frac{1}{2}$  the opposite holds; there is a rise in  $\lambda_2^*$  with higher wealth.

In contrast, under our benchmark contract the entire history of shocks matters for the allocation (even beyond the current stock of wealth and productivity). Ours is a model where the weight is constant over a part of the state space, and in this region perfect risk sharing applies. Nash bargaining is therefore less efficient because it leads to more frequent renegotiations over the intrahousehold allocation.

### B.1.3 Divorce Costs

Finally, assume that instead of an equal division of assets in the case of divorce we have a rule of the form  $A_c \ge D_m(A_c) + D_f(A_c)$ . As in Regalia and Ríos-Rull [2001] this specification allows for divorce costs to be proportional to the wealth stock but also for male and female shares to be different. Under this rule we can establish that the sign of the effect of wealth on rebargaining, when the male spouse's participation constraint binds, is determined by:

(27) 
$$\frac{(\gamma - 1)^2 \xi \epsilon_m^{(1-\eta)(1-\gamma)}}{\chi (A_c + \sum \epsilon_i)^{1-\gamma}} + (1-\gamma) \kappa_4 [D'_m A_c - D_m (A_c) + w(1-\tau_N) (D'_m \sum \epsilon_g - \epsilon_m))]$$

where  $\kappa_4 = \frac{(D_m(A_c) + \epsilon_m w(1-\tau_N))^{-\gamma}}{(\sum \epsilon_g w(1-\tau_N) + A_c)^{2-\gamma}}$ . The first term captures more commitment when wealth increases. The second term has the same impact if positive. To simplify, let  $\theta_m$  be the male spouses share on wealth in the event of divorce. Then the second term in (27) becomes  $(1-\gamma)\kappa_4[w(1-\tau_N)(\theta_m \sum \epsilon_g - \epsilon_m))]$ . This is positive surely over the relevant region if  $\theta_m \leq \frac{1}{2}$ , because (27) is pertinent only when  $\epsilon_m > \epsilon_f$ . Moreover, even if this condition is violated, i.e. when  $(\theta_m > \frac{\epsilon_m}{\sum_g \epsilon_g})$ ,  $\theta_m$  still needs to be large enough to compensate for the first term in (27).

As discussed previously, the relevant empirical evidence suggests that US courts in principle on average split equally the marital property (assets). In this respect, the weight  $\theta_m$  could be large, but only as a realization of a stochastic  $\theta_m$  that the couple draws after the divorce is final. Notice that if we had made  $\theta_m$  stochastic, then for risk averse individuals divorcing would possibly be even less attractive the higher wealth is, because the variability in post divorce consumption would be higher. We conclude this paragraph by noting that under reasonable alternative assumptions our model's implications continue to hold.<sup>44</sup>

# **B.2** Home Production

We consider the implications of home production, which requires household members' time. In the two-period model we illustrate first what this implies for the specialization of labor within the household, the intrahousehold allocation and commitment. Then we analyze how these depend on the tax code.

As a simplifying assumption, following Knowles [2007]), there is a subsistence requirement in home production  $\overline{x}$  and agents do not derive any further utility from home production. The amount required for a single household,  $\overline{x_S}$ , may be different to what is required in a couple household,  $\overline{x_M}$ . The home good is produced using male and female time,  $h_m$  and  $h_f$  respectively, which are perfect substitutes, according to  $x = h_m + h_f$ . Leisure time is therefore  $l_g = 1 - n_g - h_g$  for  $g \in \{m, f\}$ .

To characterize the households program we solve backwards. Given the wealth endowment and

<sup>&</sup>lt;sup>44</sup>For the sake of completeness we briefly comment on how our model's implications would be affected if we allowed divorce settlements to depend on the income of the male and the female spouses such as in the case of alimony. First, note that alimony payments are more likely to concern a permanent component of individual productivity rather than a transitory shock such as the  $\epsilon$  endowment considered in this section. In our quantitative model we allow for a fixed effect component to labor income and a life cycle component of male and female productivity implicit to which is the gender pay gap. We argue that by and large these predictable components of individual productivity are factored in the initial allocation in the first period, and do not lead to renegotiations of the marital contract. Household rebargaining (or its most part) stems from the time varying transitory shocks which are similar to the shocks considered in this section.

Second, note that when wealth is a state variable, as it is in our model, past realizations of temporary shocks do affect the outside options of individuals through wealth accumulation. Families that have experienced a stream of good shocks in productivity, are likely to have accumulated more wealth. Given our formalization of how wealth is divided in the event of separation it is therefore obvious that in the multiperiod model of the next section, outside options, through wealth, are influenced by the history of individual labor income and are thus (partially) contingent on the realization of shocks. If one of the spouses experiences a stream of high productivity shocks, he (she) contributes more to wealth accumulation and in the event of a separation, the division of common marital property favors the other spouse. Notice also that for couples with enough wealth, modeling such transfers through the division of wealth or modeling them as per period lump sum payments is essentially the same.

the levels of productivity,  $M_2(a_1, \lambda_2, \epsilon)$  is a solution to:

(28) 
$$M_2(a_1, \lambda_2, \epsilon) = \max_{c_2^g, h_2^g, n_2^g} \lambda_2 u(c_2^m, l_2^m) + (1 - \lambda_2) u(c_2^f, l_2^f) + \xi$$

subject to:

$$\sum_{g} c_{2}^{g} = \sum_{g} n_{2}^{g} w(1 - \tau_{N}) \epsilon_{g} + a_{1}(1 + r(1 - \tau_{K}))$$
$$\overline{x_{M}} = h_{m} + h_{f}$$
$$l_{g} = 1 - n_{g} - h_{g}$$
$$0 \le l_{g} \le 1, 0 \le n_{g} \le 1, 0 \le h_{g} \le 1 \text{ for } g \in \{m, f\}$$

Provided that  $\overline{x_M}$  is sufficiently small, the optimal allocation is such that only the spouse with the lower wage rate, i.e. the secondary earner, produces the home good. The optimal allocation satisfies  $\lambda_2 u_c(c_2^m, l_2^m) = (1 - \lambda_2) u_c(c_2^f, l_2^f)$  and  $\frac{u_l(c_2^g, l_2^g)}{u_c(c_2^g, l_2^g)} = w(1 - \tau_N)\epsilon_g$ . The *relative* allocation of consumption and leisure is the same as in the baseline model, since the secondary earner who shoulders the home production is compensated by having to work fewer hours in the market, leaving relative leisure unaffected. However, there is a level effect as with home production the equilibrium budget constraint is

(29) 
$$\sum_{g} c_2^g = \sum_{g} (1 - l_2^g) w (1 - \tau_N) \epsilon_g + a_1 (1 + r(1 - \tau_K)) - \overline{x_M} w (1 - \tau_N) \epsilon_f$$

where  $\int$  denotes the secondary earner, i.e.  $\int = m$  if  $\epsilon_m < \epsilon_f$  and  $\int = f$  otherwise.

Comparing equations (29) and (1) in the main text shows that the need for home production is isomorphic to lowering the time endowment of the secondary earner by  $\overline{x_M}$ , which in turn is isomorphic to lowering the households wealth by  $\overline{x_M}w(1-\tau_N)\epsilon_f$ .

Apart from this negative wealth effect nothing changes compared to the baseline model, since couples can decide period-by-period who shoulders the home production. The participation constraints that need to be satisfied are:

(30) 
$$u(c_2^g, l_2^g) + \xi \ge S_g(D_g(a_1), \epsilon_g) \quad g \in \{m, f\}$$

where, as before,  $S_g(D_g(a_1), \epsilon_g)$  is the utility of the household member of gender g if the marriage breaks up, in which case they receive a fraction  $D_g(a_1)$  of family wealth, but notice that the single household too has to devote some time to home production,  $h_g = \overline{x_S}$ . Therefore

(31) 
$$S^{2}(a_{1},\epsilon) = \max_{c_{2},n_{2}} u(c_{2},l_{2})$$

subject to:

(32) 
$$c_2 = (1 - \overline{x_S} - l_2)w(1 - \tau_N)\epsilon + a_1(1 + r(1 - \tau_K))$$

The effects of capital taxes are therefore the same as in the baseline model. However, the implications of labor taxes differ slightly. First, notice that a change in labor taxation will have no effect on the pattern of specialization within the household as it only depends on relative wages. Second, note that a reduction in labor taxes increases the time loss of the secondary earner due to home production and acts as if the household's wealth was reduced. Through this channel it reduces intrahousehold insurance, abstracting from the effect through the home production requirement on  $S_g$  for the moment. (However, lowering  $\tau_N$  also acts as a negative wealth effect on single households and if  $\overline{x_S}$  is big the bargaining set may increase, which would tend to increase risk-sharing possibilities.)

# **B.3** Optimal Savings

In this section, we investigate what determines a couple household's savings. Since our focus is on the intertemporal behavior of families we want to assess whether limited commitment affects the households demand for assets. Moreover, given our results that policies that lower capital taxation lead to gains in terms of commitment we want to investigate here whether more commitment is an additional channel that encourages asset accumulation in the model. To preview our results we find that household bargaining has an impact on the savings schedule but that it is impossible to sign its effect, meaning that over some parts of the state space more commitment may encourage savings but may discourage them in other parts.

Consider a couple that starts the first period of the life cycle with a weight  $\lambda_1 = \frac{1}{2}$ . Note that an increase in savings in the first period entails a cost in terms of marginal utility. If utility is log- separable, this cost is given by the expression  $\frac{1}{-a_1+2w(1-\tau_N)}$ , and if  $\gamma > 1$  it is given by  $\frac{(-a_1+2w(1-\tau_N))^{-\gamma}}{(w(1-\tau_N))^{(1-\gamma)}(1-\gamma)}\chi$  where  $\chi = \eta^{\eta(1-\gamma)}(1-\eta)^{(1-\eta)(1-\gamma)}$ .<sup>45</sup> The second period benefit from higher savings is given by:

(33) 
$$\beta(1+r(1-\tau_K))E_1\left[\lambda_2\frac{du_m}{dA_c}+(1-\lambda_2)\frac{du_f}{dA_c}+\frac{d\lambda_2}{dA_c}(u_m-u_f)\right]$$

The leading two bracketed terms in 33 represent the marginal benefit, keeping the household sharing rule constant, whereas the last term measures the effect of higher savings on the sharing rule. If the couple was able to commit to the first period contract, setting  $\lambda_2 = \frac{1}{2}$  everywhere on the state space, the derivative  $\frac{d\lambda_2}{dA_c}$  would equal zero; the marginal utility terms would be the only ones that would count for the marginal benefit of household savings.

Under log separable utility, we can show that

(34) 
$$E_1\left[\lambda_2 \frac{du_m}{dA_c} + (1-\lambda_2) \frac{du_f}{dA_c}\right] = \frac{1}{\sum_g \epsilon_g w(1-\tau_N) + A_c}$$

Notice that the right hand side of 34 is independent of the weight  $\lambda_2$ . In fact, this expression is the same as the one we would get if we were to solve for the full commitment allocation that sets  $\lambda_2 = \frac{1}{2}$  regardless of the productivity levels of the male and female spouse. Under log utility, therefore, it is the last term in 33  $\left(\frac{d\lambda_2}{dA_c}(u_m - u_f)\right)$  that describes the impact of limited commitment on the households savings. We therefore have to focus on that term.

We consider separately each relevant region in the state space where  $\frac{d\lambda_2}{dA_c}(u_m - u_f)$  is different from zero, that is every region where the marital contract is rebargained. As discussed previously, for male productivity  $\epsilon_m$  less than a lower bound  $\underline{\epsilon_m}(A_c, \epsilon_f)$  the weight  $\lambda_2$  will fall to  $\lambda_2^U$  (there is an increase in the female spouse's share). Conversely, if  $\epsilon_m > \overline{\epsilon_m}(A_c, \epsilon_f)$  (upper bound) then  $\lambda_2 = \lambda_2^L$ . In any other region there is no rebargaining of the households allocation and, therefore,  $\lambda_2 = \frac{1}{2}$ . Thus we can write the conditional expectation of  $\frac{d\lambda_2}{dA_c}(u_m - u_f)$  as:

(35) 
$$E_1 \frac{d\lambda_2}{dA_c} (u_m - u_f) = \int_0^{\underline{\epsilon_m}(A_c,\epsilon_f)} \int \frac{d\lambda_2^U}{dA_c} (u_m - u_f) dF(\epsilon_f,\epsilon_m) + \int_{\overline{\epsilon_m}(A_c,\epsilon_f)}^{\infty} \int \frac{d\lambda_2^L}{dA_c} (u_m - u_f) dF(\epsilon_f,\epsilon_m)$$

where F is the joint density of idiosyncratic productivity in the household.

From equation 4 it is easy to establish that the derivative  $\frac{d\lambda_2^U}{dA_c}$  is positive and the derivative  $\frac{d\lambda_2^L}{dA_c}$  is negative. In order to sign  $\frac{d\lambda_2}{dA_c}(u_m - u_f)$ , in each relevant region of 35, we need to determine the difference in the welfare levels of husbands and wives. As it turns out, this difference is not of one sign. This is so because the limited commitment model has nothing to say about the absolute

<sup>&</sup>lt;sup>45</sup>Notice that the period one idiosyncratic productivity endowments are normalized to unity for both spouses.

level of utility; it simply states that if ever participation is violated, a correction has to be made that makes one of the spouses as well off as if they were single. Since the model does not admit an analytical solution for the conditional expectation, we used numerical methods to compute the relevant integrals. Depending on the level of assets, we found that  $E_1[\frac{d\lambda_2}{dA_c}(u_m - u_f)]$  could be both positive or negative, which implies that the effect of limited commitment on household savings is ambiguous.<sup>46</sup>

The more general case for  $\gamma > 1$  yields similar results. For this model we can derive the following expression for the leading term in 33:

(36) 
$$E_{1}\left[\lambda_{2}\frac{du_{m}}{dA_{c}} + (1-\lambda_{2})\frac{du_{f}}{dA_{c}}\right] = E_{1}\left[\left(Ac + \sum_{g}w(1-\tau_{N})\epsilon_{g}\right)^{-\gamma}\chi\right]$$
$$\left(\left(\frac{1}{1+f(\lambda_{2},\epsilon)}\right)^{1-\gamma}\frac{\lambda_{2}}{(w(1-\tau_{N})\epsilon_{m})^{(1-\eta)(1-\gamma)}} + \left(\frac{f(\lambda_{2},\epsilon)}{1+f(\lambda_{2},\epsilon)}\right)^{1-\gamma}\frac{1-\lambda_{2}}{(w(1-\tau_{N})\epsilon_{f})^{(1-\eta)(1-\gamma)}}\right)\right]$$

The above expression suggests that the intrahousehold allocation affects the optimal savings of the family relative to the full commitment model, even beyond the term  $E_1 \frac{d\lambda_2}{dA_c}(u_m - u_f)$ . To see how, first note that the bottom line of 36 can be further simplified into:

(37) 
$$((\frac{1}{1+f(\lambda_2,\epsilon)})^{-\gamma} \frac{\lambda_2}{(w(1-\tau_N)\epsilon_m)^{(1-\eta)(1-\gamma)}} = (\lambda_2^{\frac{1}{\gamma}} \epsilon_m^{\omega} + (1-\lambda_2)^{\frac{1}{\gamma}} \epsilon_f^{\omega})^{\gamma}$$

where  $\omega = -(1 - \eta)(1 - \gamma)/\gamma < 0$ . Second, assume that male and female productivities in period 2 are perfectly negatively correlated, so that  $\epsilon_m + \epsilon_f = \bar{\epsilon}$  which is constant. It is obvious that 37 is the only term that matters for household savings, as under these assumptions  $(A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{-\gamma}\chi$ would be constant, no matter the realizations of  $\epsilon_m$  and  $\epsilon_f$ .<sup>47</sup> The term in 37 exerts an influence to household savings because it changes the marginal benefit to the household. Since it is a concave function in  $\epsilon_m$ , higher uncertainty decreases the marginal utility, even under full commitment when the shares are constant. Moreover, if the shares  $\lambda_2$  change with the endowment, as they do under limited commitment, they contribute further to the variability of 37. This lowers the marginal gain from an extra unit of savings even further. In this example limited commitment means less rather than more savings. The analysis of the term  $E_1 \frac{d\lambda_2}{dA_c}(u_m - u_f)$  is similar with the log-log case and, for the sake of brevity, is omitted.

<sup>&</sup>lt;sup>46</sup>This result emerges also in Ligon et al. [2000].

<sup>&</sup>lt;sup>47</sup>Note that the covariance structure of wages within the household is extremely important for the sharing rule and, of course, the optimal allocation. In the two period model of this section, more negatively correlated shocks imply that the limited commitment problem in the household is more severe. If, on the other hand, shocks were perfectly correlated yielding  $\epsilon_m = \epsilon_f$ , then it is trivial to show that household rebargaining would not occur in equilibrium. The optimal weight  $\lambda_2$  would equal a half, and the demand for savings of a two member household would be identical to the demand of a single earner household. When shocks are not perfectly correlated, the need to accumulate assets to buffer shocks to the labor income is less, and as a consequence couple households accumulate less wealth.