# Online Supplemental Information

### Mathematical formulation of Stochastic Actor-oriented Multivariate Dynamic Model

Following the notations used by Snijders and colleagues ([Snijders, Lomi, & Torlò 2013](#_ENREF_2)), we denote by $Y$ the two-mode network, the node sets being a set $N$ of adolescents and a set $A$ of activities *a* with tie variable $Y\_{ia}$ for $i\in N$, $a\in A$, where $Y\_{ia}=1$ if an adolescent *i* participates in activity *a,* and $Y\_{ia}=0$ otherwise. The one-mode network $X$ has node set $N$ and directed tie variable $X\_{ij}$ for $i, j\in N, i\ne j$. In our study, the first node set $N$ represents a set of adolescents, who participate in one set $A$ of sports activities that constitute the second mode. In our case, both the one-mode friend network and two two-mode network was observed at two time points $t\_{1}$ and $t\_{2}$, representing $X\left(t\_{1}\right)$, $X\left(t\_{1}\right), Y\_{s}\left(t\_{1}\right)$, and $Y\_{s}\left(t\_{2}\right)$ where *s* represents sports.

In the actor-oriented stochastic model ( [Snijders et al., 2013](#_ENREF_2)), we assume the agency of adolescents in the first node set $N$, who may stochastically change their outgoing ties in the two-mode network, $Y\_{ia}$ for $a\in A$, or in the one-mode network, $X\_{ij}$ for $j\in N$ at random time moments. The result of all these changes in between two observations will produce the total change from one observation to the next. To model these changes, two model components were specified for each of the networks *X* and *Y*. The first component was to model the timing and frequency of changes. We specified a constant rate of change between observations $t\_{1}$ and $t\_{2}$ for adolescents in both the one-mode and two-mode network. The second component was to model the probability distributions for the changes in terms of the “objective functions,” which were defined separately for the one-mode and the two-mode networks, and which were sums of an “evaluation function” and an “endowment/maintenance function.” For each one-mode and two-mode network, our objective functions were expressed by a combination of structural and attribute characteristics of adolescent personal networks toward which they were assumed to be attracted.

The objective function was defined as follows:

$$f\_{i}^{net}\left(x, y;x^{0}, y^{0}\right)=\sum\_{k}^{}β\_{k}^{net}s\_{ik}^{net}\left(x, y\right)+∆(x, y;x^{0}, y^{0})\sum\_{l}^{}γ\_{l}^{net}s\_{il}^{net}\left(x, y\right)$$

where $s\_{ik}^{net}\left(x, y\right)$ represents “evaluation effects” and $β\_{k}^{net}$are corresponding parameters, where $s\_{il}^{net}\left(x, y\right)$ represents “endowment/maintenance effects” and $γ\_{l}^{net}$are corresponding parameters, and where *net* can be *X* or *Y*. Further, *x* and *y* are the potential new values of the network, while *x0* and *y0* are the current values; ∆ is 1 if the change from *x* or *y* to *x0* or *y0*, respectively, means that a tie is dropped, and 0 otherwise. For a detailed explanation about actor-oriented dynamic models, refer to [Snijders, van de Bunt, & Steglich (2010](#_ENREF_3)) and the manual, [Ripley, Snijders, Boda, Vörös, & Preciado (2017](#_ENREF_1)). For a detailed explanation of co-evolution between a one-mode and a two-mode network, refer to [Snijders, Lomi, & Torlò (2013](#_ENREF_2)). The RSiena software, used to estimate the model, is documented also in its manual Ripley et al. (2017). Table 1(S) gives the short names employed in the manual to identify the effects $s\_{ik}^{net}\left(x, y\right)$ used in the analysis. In Table 1(S), the keyword “name” defines the dependent variable, and the keyword “interaction1” defines the explanatory variable/s. These are the keywords also used in RSiena.

Some terminology for social networks: *outdegree* is the number of outgoing ties of an individual, *indegree* the number of incoming ties; in the two-mode network, ties are considered as being directed from the individuals to the activities or cognitions, respectively; *ego* refers to the focal individual (respondent); *alter* refers to the potential friend (nominee).

### References

Ripley, R. M., Snijders, T. A. B., Boda, Z., Vörös, A., & Preciado, P. (2017). Manual for RSiena *Technical Report*. Oxford: University of Oxford, Department of Statistics; Nuffield College.

Snijders, T. A. B., Lomi, A., & Torlò , V. J. (2013). A model for the multiplex dynamics of two-mode and one-mode networks, with an application to employment preference, friendship, and advice. *Social Networks, 35*, 265-276.

Snijders, T. A. B., van de Bunt, G. G., & Steglich, C. E. G. (2010). Introduction to stochastic actor-based models for network dynamics. *Social Networks, 32*, 44-60.

### Table 1(S): Effect Names of Within-network and Multivariate Dynamics and Corresponding RSiena Effect Names

|  |  |
| --- | --- |
| Multivariate Dynamic Model | RSiena v. 1.1-302  |
| *Within-network effects of one-mode network* (*name=”friends")* | name = “friends” |
| *Outdegree (density)* | default |
| *reciprocity* | default |
| *reciprocal degree-related activity* | reciAct |
| *transitive triplets* (friends of friends tend to become or remain friends) | transTrip |
| *transitive reciprocated triplets* (reciprocated friendship tends to close the two path) | transRecTrip |
| *transitive ties* (the same as the *transitive triplets* effect but considering at least one indirect tie sufficing for the triadic closure) | transTies |
| *three-cycles* (triadic closure in a cyclical direction) | cycle3 |
| *indegree popularity* | inPop |
| *outdegree popularity* | outPop |
| *outdegree activity* | outAct, |
| Square root of *outdegree activity* | outActSqrt |
| *outdegree at least 1* | outTrunc, parameter = 1 |
| *gender-related popularity* | altX, interaction1 = “sex” |
| *gende -related activity* | egoX, interaction1 = “sex” |
| *gender-related similarity* | sameX, interaction1 = “sex” |
| *ethnicity-related popularity* | altX, interaction1 = “ethnicity” |
| *ethnicity-related activity* | egoX, interaction1 = “ethnicity” |
| *ethnicity-related similarity* | sameX, interaction1 = “ethnicity” |
| *grade-related popularity* | altX, interaction1 = “grade” |
| *grade-related activity* | egoX, interaction1 = “grade” |
| *grade-related similarity* | simX, interaction1 = “grade” |
| *class-related similarity* | sameX, interaction1 = “class” |
| interaction *reciprocity*\**same gender* | sameXRecip, interaction1 = “sex” |
| interaction *reciprocity*\**same class* | sameXRecip, interaction1 = “class” |
| interaction *same gender*\**same class* | sameX, sameX, interaction1 = c(‘sex’, ‘class’)note: use include Interaction |
| *Within-network effects of two-mode network (name="bsports")* | name = “sports” |
| *outdegree* | default |
| *4-cycles closure* | cycle4 |
| *4-cycles closure from same gender* | sameXCycle4, p = 1, interaction1 = “sex” |
| *outdegree activity* | outact |
| *indegree popularity* | inPop |
| *gender ego-in-alter distance 2 similarity*  | simEgoInDist2, interaction1= “sex” |
| *outdegree at least 1* | outTrunc, parameter = 1 |
| *gender-related activity* | egoX, interaction1 = “sex” |
| *ethnicity-related activity* | egoX, interaction1 = “ethnicity” |
| *grade-related activity* | egoX, interaction1 = “grade” |
| *Between-network effects of one-mode and two-mode networks* |  |
| Friendship → sports activity (increased friends’ nomination leads to increased sports participation) (evaluation effect) | name = “sports”, outActIntn, interaction1 = “friends” *‘from one-mode activity to two-mode activity’* |
| Friendship → sports activity (increased friends’ nomination maintains increased sports participation) (maintenance effect) | name = “sports”, outActIntn, interaction1 = “friends”, type= “endow” *‘from one-mode activity to two-mode activity’* |
| Sports activity → friendship (increased sports participations leads to increased friends’ nomination) | name = “friends”, outActIntn, interaction1 = “sports”) ‘*from two-mode activity to one-mode activity’* |
| Shared sports activities leading to friendship (Sports agreement leading to friendship) | name = “friends”, from, interaction1 = “sports”)‘*from two-mode agreement to one-mode out-tie’* |
| Friendship leading to shared sports activities (Friendship leading to sports agreement)(evaluation effect) | name = “sports”, to, interaction1 = “friends”*‘from one-mode out-tie to two-mode agreement’* |
| Friendship maintains shared sports activities (Friendship maintains sports agreement)(maintenance effect) | name = “sports”, to, interaction1 = “friends”, type= “endow”) *‘from one-mode out-tie to two-mode agreement’* |

### Table 2(S): School-wise Results of Multivariate Dynamic Models: Within-network and Between-network Effect Sizes of Coefficients and their Standard Errors (SE) and Significance Level

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|   | School 1 | School 2 | School 3 | School 4 | School 5 |
| Dependent Variable | Friendship | Sports Participation | Friendship | Sports Participation | Friendship | Sports Participation | Friendship | Sports Participation | Friendship | Sports Participation |
| *Within-network effects of multivariate dynamics:* |
| Rate | 19.479(0.750) | 2.652(0.239) | 18.850(0.753) | 2.295(0.248) | 13.327(0.672) | 3.369(0.581) | 16.475(0.655) | 2.483(0.291) | 22.258(0.864) | 2.009 (0.223) |
| Outdegree (density) | -3.400\*\*\*(0.339) | -0.941(2.115) | -2.076\*\*\*(0.381) | -2.199\*\*\*(0.377) | -3.090\*\*\*(0.518) | -3.263\*\*\*(0.491) | -3.218\*\*\*(0.404) | -5.332\*\*\*(0.873) | -3.519\*\*\*(0.413) | -2.369\*\*\* (0.613) |
| Reciprocity | 2.886\*\*\*(0.157) |  | 2.705\*\*\*(0.167) |  | 2.744\*\*\*(0.238) |  | 2.989\*\*\*(0.166) |  | 2.775\*\*\*(0.171) |   |
| Reciprocal degree-related activity | -0.092\*\*\*(0.014) |  | -0.069\*\*\*(0.016) |  | -0.085\*\*(0.028) |  | -0.104\*\*\*(0.019) |  | -0.079\*\*\*(0.015) |   |
| Transitive triplets (friends of friends tend to become or remain friends) | 0.357\*\*\*(0.032) |  | 0.297\*\*\*(0.030) |  | 0.410\*\*\*(0.058) |  | 0.463\*\*\*(0.036) |  | 0.398\*\*\*(0.032) |   |
| Transitive reciprocated triplets (reciprocated friendship tends to close the two path) | -0.390\*\*\*(0.068) |  | -0.327\*\*\*(0.052) |  | -0.233†(0.121) |  | -0.504\*\*\*(0.067) |  | -0.293\*\*\*(0.071) |   |
| Transitive ties (the same as the *transitive triplets* effect but considering at least one indirect tie as sufficing for the triadic closure) | 0.570\*\*\*(0.057) |  | 0.713\*\*\*(0.052) |  | 0.453\*\*\*(0.092) |  | 0.546\*\*\*(0.064) |  | 0.425\*\*\*(0.073) |   |
| Three cycles (triadic closure in a cyclical direction) | 0.133\*(0.058) |  | 0.138\*\*(0.052) |  | 0.055(0.102) |  | 0.220\*\*\*(0.057) |  | 0.101(0.064) |   |
| Four cycles |  | 0.007(0.023) |  | 0.069\*(0.027) |  | 0.006(0.029) |  | -0.092†(0.049) |  | 0.041 (0.030) |
| Four cycles from same gender |  | 0.040(0.033) |  | -0.065†(0.034) |  | -0.005(0.042) |  | 0.123†(0.064) |  | -0.085\* (0.039) |
| Indegree popularity | 0.013\*(0.006) | -0.005(0.007) | 0.026\*\*\*(0.007) | 0.009(0.007) | 0.035\*\*\*(0.010) | 0.034\*\*\*(0.008) | 0.010(0.008) | 0.001(0.008) | 0.022\*\*\*(0.005) | 0.020\*\* (0.007) |
| Gender ego-in-alter distance 2 similarity  |  | 0.833\*(0.399) |  | 1.724\*\*\*(0.375) |  | 1.860\*\*\*(0.441) |  | 1.168\*(0.462) |  | 1.750\*\*\*(0.416) |
| Outdegree popularity | -0.069\*\*\*(0.009) |  | -0.070\*\*\*(0.010) |  | -0.092\*\*\*(0.019) |  | -0.077\*\*\*(0.010) |  | -0.065\*\*\*(-0.009) |  |
| Outdegree activity | 0.025(0.022) | -0.365(0.448) | 0.096\*\*\*(0.025) | 0.005(0.055) | 0.041(0.035) | 0.086(0.058) | 0.034(0.025) | 0.293\*\*\*(0.073) | 0.011(0.023) | -0.029(0.103) |
| Square root of Outdegree activity | 0.035(0.165) |  | -0.536\*(0.186) |  | -0.116(0.261) |  | -0.029(0.196) |  | 0.154(0.187) |  |
| Outdegree ≥ 1 | -1.408\*(0.583) | -2.487(1.686) | -2.806\*\*\*(0.576) | -1.897\*\*\*(0.431) | -1.590\*(0.706) | -1.588\*\*(0.509) | -0.829(0.702) | -0.427(0.657) | -0.776(0.763) | -2.116\*\*\*(0.592) |
| Gender alter (F) | -0.023(0.037) |  | 0.014(0.038) |  | -0.005(0.060) |  | 0.093\*(0.044) |  | -0.035(0.041) |  |
| Gender ego (F) | 0.013(0.037) | 0.433\*(0.186) | 0.010(0.042) | -0.119(0.174) | -0.013(0.061) | -0.526\*(0.223) | 0.051(0.046) | -0.490†(0.280) | 0.093\*(0.042) | 0.023(0.184) |
| Same gender | 0.380\*\*\*(0.045) |  | 0.349\*\*\*(0.043) |  | 0.525\*\*\*(0.069) |  | 0.384\*\*\*(0.050) |  | 0.363\*\*\*(0.047) |  |
| Hispanic alter | -0.087\*(0.040) |  | -0.035(0.042) |  | -0.089(0.079) |  | -0.024(0.049) |  | 0.013(0.065) |  |
| Hispanic ego | 0.129\*\*\*(0.039) | -0.235(0.178) | 0.095\*(0.045) | -0.166(0.169) | -0.258\*\*(0.079) | -0.319(0.267) | 0.056(0.049) | 0.183(0.292) | -0.045(0.063) | -0.599\*\*(0.222) |
| Same Hispanic | 0.138\*\*\*(0.037) |  | 0.033(0.041) |  | 0.158\*(0.080) |  | 0.303\*\*\*(0.043) |  | 0.144\*(0.064) |  |
| Grade alter | -0.022(0.023) |  | -0.023(0.022) |  | 0.060(0.038) |  | 0.015(0.027) |  | -0.021(0.022) |  |
| Grade ego | -0.013(0.020) | 0.208†(0.112) | 0.002(0.022) | -0.000(0.103) | 0.078\*(0.036) | 0.005(0.147) | -0.058\*(0.027) | 0.427\*(0.194) | -0.026(0.023) | 0.299\*\*(0.115) |
| Grade similarity | 0.313\*\*\*(0.079) |  | 0.425\*\*\*(0.084) |  | 0.279\*(0.134) |  | 0.502\*\*\*(0.089) |  | 0.317\*\*\*(0.088) |  |
| Same class | 0.525\*\*\*(0.094) |  | 0.414\*\*\*(0.096) |  | 0.797\*\*\*(0.120) |  | 0.560\*\*\*(0.099) |  | 0.564\*\*\*(0.091) |  |
| Reciprocity × Same gender | -0.285\*(0.131) |  | -0.137(0.14) |  | -0.261(0.194) |  | -0.266†(0.138) |  | -0.442\*\*(0.136) |  |
| Reciprocity × Same class | -0.273†(0.150) |  | -0.265†(0.16) |  | -0.435\*(0.188) |  | -0.396\*(0.160) |  | -0.210(0.154) |  |
| Same gender × Same class | -0.078(0.113) |  | 0.130(0.113) |  | -0.240†(0.140) |  | -0.009(0.119) |  | -0.041(0.107) |  |
| *Between-network effects of multivariate dynamics:*  |
| *Actor level outdegrees* |
| Friendship → sports activity (evaluation effect) |  | -0.520(0.576) |  | -1.113†(0.626) |  | -0.592(0.485) |  | -4.957\*\*(1.518) |  | 0.171(0.468) |
| Friendship → sports activity(maintenance effect) |  | 0.276(1.448) |  | 1.726(1.444) |  | 1.953(1.220) |  | 6.864\*\*(2.469) |  | -1.386(1.063) |
| Sports activity → friendship | -0.197\*\*\*(0.038) |  | -0.127\*\*(0.042) |  | -0.119\*(0.057) |  | -0.004(0.044) |  | -0.106\*\*(0.038) |  |
| *Mixed triads* |
| Sports agreement leading to friendship(“Shared sports activities leading to friendship,” as illustrated in Figure 1 (A)) | 0.276\*\*\*(0.044) |  | 0.239\*\*\*(0.063) |  | 0.251\*\*\*(0.073) |  | 0.235\*\*\*(0.053) |  | 0.159\*\*(0.057) |  |
| Friendship leading to sports agreement (“Friendship leading to shared sports activities,” as illustrated in Figure 1 (B))(evaluation effect) |  | 0.426\*\*\*(0.157) |  | 0.293(0.216) |  | -0.180(0.314) |  | 1.800\*\*\*(0.521) |  | 0.442\*(0.187) |
| Friendship leading to sports agreement (“Friendship maintains shared sports activities”)(maintenance effect) |  | 0.928\*(0.413) |  | 0.966†(0.519) |  | 3.344\*\*\*(1.219) |  | 0.727(0.962) |  | 0.646(0.452) |
| All convergence *t* ratios | < 0.04 | < 0.05 | < 0.06 | < 0.04 | < 0.04 |
| Overall maximum convergence ratio | 0.14 | 0.14 | 0.16 | 0.16 | 0.14 |

†*p* < 0.1, \**p* < 0.05, \*\**p* < 0.01, \*\*\* *p* < 0.001

**Table 3(S): School-wise Results of the Average Centered Jaccard Coefficient for the Five Model Specifications**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model | School 1 | School 2 | School 3 | School 4 | School 5 |
| Trend  | 0.0038 | 0.0049 | 0.0038 | 0.0044 | 0.0054 |
| Control  | 0.0074 | 0.0091 | 0.0061 | 0.0052 | 0.0087 |
| Selection-only  | 0.0190 | 0.0195 | 0.0171 | 0.0217 | 0.0185 |
| Influence-only  | 0.0098 | 0.0116 | 0.0088 | 0.0093 | 0.0104 |
| Full  | 0.0222 | 0.0226 | 0.0203 | 0.0271 | 0.0210 |
| Observed | 0.0231 | 0.0264 | 0.0191 | 0.0241 | 0.0198 |

### Table 4(S): Results per School of Relative Contributions of Parts of the Model to Explain the Association between Friendship and Sports

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Model | School 1 | School 2 | School 3 | School 4 | School 5 | Mean |
| Trend  | 0.1733 | 0.2169 | 0.1858 | 0.1633 | 0.2576 | 0.1994 |
| Control  | 0.1599 | 0.1859 | 0.1133 | 0.0295 | 0.1557 | 0.1289 |
| Influence  | 0.5251 | 0.4604 | 0.5429 | 0.6081 | 0.4671 | 0.5207 |
| Selection  | 0.1070 | 0.1120 | 0.1321 | 0.1512 | 0.0823 | 0.1169 |
| Synergy | 0.0347 | 0.0247 | 0.0258 | 0.0479 | 0.0374 | 0.0341 |