**Appendix A: Articles Included in the Review of Research Using Panel Data**

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**Appendix B: Assumptions**

This appendix reviews the assumptions for the data generating processes considered in the main text. Very small $T$ panel data is commonly used by those studying public opinion and political behavior. The assumptions are largely justified with that in mind but such data and assumptions are not limited to these fields.

*1) Linear models*: many public opinion dependent variables of interest are approximately continuous (e.g., scales of agreement or approval) and so linear models are common. Although a number of political behavior studies use dependent variables that are limited or bounded, estimators for non-linear dynamic models are still limited. Therefore, it is common under these circumstances to use a linear model as an approximation.

*2) Single equation models*: such models are the most common and are appropriate as long as the assumption of weak exogeneity can be made. In essence, this means that contemporaneous values of the dependent variable are not on the right-hand side of the DGP for the independent variables. This assumption is increasingly being made plausible by the use of (quasi-) experiments in behavioral work.

*3) Very small* $T$: panel data used by those studying public opinion and political behavior is typically survey data in which a relatively large number of individuals (anywhere from a few hundred to many thousands) are interviewed on at least three but usually less than ten occasions.

*4) Correlations within individuals but not across them*: since the data typically used by those studying public opinion and political behavior is survey data collected on a semi-random sample that makes up a very small fraction of the entire population, once we control for time specific effects the assumption of independence across individuals is highly plausible. This is because time-effects control for all interdependence that is due to common factors affecting all cases at the same point in time. These factors can vary from one point in time to the next but the response of all cases is assumed to be the same. This latter assumption can be relaxed with a common factor model (Pesaran 2015, pp. 772-774) providing a more flexible control for interdependence. However, such an approach generally does not perform well with a very small $T$. If there is interdependence that is not due to common factors across cases and a spatial weight matrix is known *a priori*, then there are ML estimators for dynamic panels with spatial dependence (Pesaran 2015, pg. 810). These estimators are biased when $T$ is small but bias corrected ML estimators have been proposed (Pesaran and Zhao 1999). These are not transformed-likelihood estimators and beyond the scope of this paper but their performance is well worth exploring.

*5) Effects of time-varying independent variables*: these effects are known as *within-effects*. They are the appropriate effects to estimate when we are testing dynamic theories about how changes in circumstances and context change opinions and behavior. Such theories tend to dominate political behavior and public opinion research. However, there are very important theories about how individual attributes that do not change (in the short to medium term) affect opinions and behaviors – e.g., education, class, gender and race.

If we are interested in the effect of a time-invariant variable, we are interested in a *between-effect*. If we are interested in the effect of average differences between individuals in a time-varying independent variable, we are again interested in a between-effect. If the independent variable of interest varies too little over time to provide a reliable within-effect estimation, then we can only estimate its between-effect (assuming it varies enough between individuals).

For static models, the classic between estimator is:

$y\_{i\overbar{t}}= β\_{0}+β\_{1}x\_{i,\overbar{t}}+\sum\_{m=2}^{M}β\_{m}z\_{m,i,\overbar{t}}+μ\_{i\overbar{t}}$ (9)

where $\overbar{t}$ indicates the over time average for each case – e.g., $y\_{i,\overbar{t}}={\sum\_{t=1}^{T}y\_{i,t}}/{T}$. If we believe the between- and within-effects are the same in the DGP, we could also use the pooled estimator (equation 1 in the main text). This estimator produces an average of the within and between-effects.[[1]](#footnote-1) We can also use Plumper and Troeger’s (2007) fixed-effect vector decomposition (Beck 2011).[[2]](#footnote-2) This procedure requires the assumption that the time-invariant variables account for all of the explanatory power of the individual-effects.

Importantly, all estimators that include a between-effect – between, pooled, or fixed-effects variance decomposition – require us to assume that the covariates, or at least the time varying covariates, in the model are independent of the unobservable individual-effects.[[3]](#footnote-3) To the extent that this independence assumption is violated, these estimators are biased. However, to the extent that the violation is small, the bias for these estimators may be small. With dynamic models, it is less clear whether there are conditions under which a small violation of the independence assumption will only result in a small bias. In fact, it is not even clear whether any of these estimators are unbiased even when the independence assumption is met for the $x\_{it}$ in the model, because the assumption cannot (by definition) be met for the lagged dependent variable (LDV). Exploring this further goes beyond the scope of this paper. However, the quasi-maximum likelihood random effects (QML-RE) estimator is consistent if the coefficient on the LDV is not too close to 1. Therefore, if we are willing to make the independence assumption for the covariates in the model, other than the LDV, we can use this estimator to estimate the between-effects for (near) time-invariant variables.

**Appendix C: Bias in the Long-Run Effect**

It is sometimes noted that the bias of an estimator for a dynamic model such as $y\_{i,t}= αy\_{i,t-1}+β\_{1}x\_{i,t}+μ\_{i,t}$ is worse for $α$ than $β\_{1}$. This is sometimes used as a justification to use an estimator despite its bias. The difficulty with this reasoning is that when estimating a dynamic model, the estimates of both $α$ and $β\_{1}$ are important for inference. Bias in the estimation of $β\_{1}$ means we will fail to understand the SRE of $x\_{it}$. Bias in the estimation of $α$ means that we will have a biased estimate of the LREs of $x\_{it}$. Consider a situation in which the estimate of $β\_{1}$ is correct but there is a negative bias in the estimation of $α$. Say the true value of $β\_{1}$ is 0.5 and of $α$ is 0.25 but negative bias results in an estimate of -0.3 for $α$. The simulations we present in the paper show that bias of this order is likely when using a standard OLS-FE estimator and $T$ is very small. Figure C.1 displays the difference between the true LRE of a one unit increase in $x\_{it}$ and that which we get from our biased estimate. While the true effect builds over the long-run to a total effect of 0.67, the estimated effect decays to a total effect of 0.38. Even if our estimate of the SRE is correct (which it generally will not be when $T$ is very small), our understanding of the LRE will not be. Further note that the estimate of the LRE is biased downwards. This increases the probability of type II errors, leading us to the conclusion that there is no effect of $x\_{i,t}$ on $y\_{i,t}$ when there is.

**Figure C.1.**



**Appendix D: Monte Carlo Method for the Orthogonal Reparameterization Approach**

This appendix describes how the Monte Carlo method can be used to obtain the OPM parameter estimates and credible intervals. Let us allow for $K$ dynamic variables in our data model:

$$y\_{it}= αy\_{i,t-1}+\sum\_{k=1}^{K}β\_{k}x\_{i,t,k}+η\_{i}+ε\_{it}$$

Let $βX\_{i,t}=\sum\_{k=1}^{K}β\_{k}x\_{i,t,k}$. The posterior density function is (Lancaster 2002)[[4]](#footnote-4):

|  |  |
| --- | --- |
| $$p\left(data\right)∝σ^{-\left(N(T-1)-2\right)}exp\left\{\frac{N}{T}\sum\_{t=1}^{T-1}\left(\frac{T-t}{t}α^{t}\right)-\frac{1}{2σ^{2}}\sum\_{i=1}^{N}\left(y\_{i,t}-αy\_{i,t-1}-βX\_{i,t}\right)'H\left(y\_{i,t}-αy\_{i,t-1}-βX\_{i,t}\right)\right\}$$ | (D.1) |

$H$ is defined as an operator that subtracts the mean. For example, if

$ω\_{i}=y\_{i,t}-αy\_{i,t-1}-β\_{1}x\_{i,t}$, then $H\left(ω\_{i}\right)≡ω\_{i}-\overbar{ω}$.

Sampling from this posterior (D.1) gives us estimates (distributions) for $α$, $β$ and $σ^{2}$. To do this we begin by integrating $β$ out of (D.1). This gives us the following joint posterior density:

$p\left(data\right)∝σ^{-\left(N\left(T-1\right)-K-2\right)}exp\left\{\frac{N}{T}\sum\_{t=1}^{T-1}\left(\frac{T-t}{t}α^{t}\right)\right\}exp\left\{-\frac{1}{2σ^{2}}\left(\left(\sum\_{i=1}^{N}\left(y\_{i,t}-αy\_{i,t-1}\right)'H\left(y\_{i,t}-αy\_{i,t-1}\right)\right)^{'}\left(\sum\_{i=1}^{N}\left(x\_{i,t}\right)'H\left(x\_{i,t}\right)\right)^{-1}\left(\sum\_{i=1}^{N}\left(y\_{i,t}-αy\_{i,t-1}\right)'H\left(y\_{i,t}-αy\_{i,t-1}\right)\right)\right)\right\}$ (D.2)

Next, we integrate out $σ^{2}$ from (D.2) giving us the marginal posterior density:

$p\left(data\right)∝\frac{exp\left\{\frac{N}{T}\sum\_{t=1}^{T-1}\left(\frac{T-t}{t} α^{t}\right)\right\}}{\left(\left(\sum\_{i=1}^{N}\left(y\_{i,t}- αy\_{i,t-1}\right)'H\left(y\_{i,t}- αy\_{i,t-1}\right)\right)^{'}\left(\sum\_{i=1}^{N}\left(x\_{i,t}\right)'H\left(x\_{i,t}\right)\right)^{-1}\left(\sum\_{i=1}^{N}\left(y\_{i,t}- αy\_{i,t-1}\right)'H\left(y\_{i,t}- αy\_{i,t-1}\right)\right)\right)^{-\left(\frac{N\left(T-1\right)-K}{2}\right)}}$ (D.3)

We can now proceed to sample triplet values $\left( α, β, 1/σ^{2}\right)$ by: first sampling $α$ from (D.3); then, given $α$, sample $1/σ^{2}$ from (D.2); then given $α$ and $1/σ^{2}$, sample$ β$ from (D.1). When we sample values of $1/σ^{2}$ from (D.2) given $α$, we are sampling from a gamma distribution. When we sample values of $β$ from (D.1) given $α$ and $1/σ^{2}$, we are sampling from a multivariate normal.

**Appendix E: General Properties and Supplementary Simulation Results**

This appendix provides the results from the simulations described in Table 1, with the addition of the pooled estimator, OLS-FE and GLS-RE. It also includes the same simulations but allowing for an autoregressive independent variable in the DGP and allowing for time specific effects within the DGP.

General Properties Simulation Results

Tables E.1 and E.2 provides the simulation results for Tables 1 and 2 in the main text, based on the DGP described by equation (13):

$y\_{i,t}= αy\_{i,t-1}+βx\_{i,t}+η\_{i}+ε\_{1,i,t}$

with

$$x\_{i,t}=0.75η\_{i}+ε\_{2,i,t}$$

$ε\_{1,i,t} \~NID(0,1)$; $ε\_{2,i,t} \~NID(0,2)$; $η\_{i}\~U(-ω, ω)$

In addition to the estimators discussed in the text, Table E.1 includes the OLS pooled estimator.

Table E.3 provides the distributional violations simulation results for Table 3 in the main text.

GMM-Sys1 sometimes ran into problems when optimizing the weighting matrix. When this occurred, the results for that particular data set ware dropped from the analysis.

**Table E.1. General Properties Simulation Results**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **T** | **Alpha** |  | **Estimator** | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** | **Sargan** |
| **Omega** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Cov.** | **Cov.** | **Cov.** | **Pow.** | **Pow.** | **Pow.** | **P-value** |
| **10** | **0.9** | **1.15** | **Pooled** | 0.064 | 0.064 | 0.0092 | 0.011 | 9.2 | 9.2 | 0.27 | 0 | 0.6 | 0 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **OLS-FE** | -0.14 | 0.14 | -0.028 | 0.029 | -3.1 | 3.1 | 0.044 | 0 | 0.001 | 0 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **GLS-RE** | 0.064 | 0.065 | 0.0084 | 0.01 | 9.3 | 9.3 | 0.28 | 0 | 0.64 | 0 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **GMM-Diff** | 0.019 | 0.04 | 0.0043 | 0.011 | 0.81 | 63 | 0.88 | 0.9 | 0.93 | 0.77 | 1 | 1 | 0.93 |  |
| **10** | **0.9** | **1.15** | **GMM-Sys1** | -0.0043 | 0.013 | 0.0011 | 0.0071 | -0.28 | 0.78 | 0.5 | 0.9 | 0.95 | 0.94 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **GMM-Sargan1** | -0.004 | 0.013 | 0.00093 | 0.0071 | -0.26 | 0.77 | 0.5 | 0.87 | 0.94 | 0.91 | 1 | 1 | 1 | 0.023 |
| **10** | **0.9** | **1.15** | **GMM-Sys2** | -0.0075 | 0.014 | -0.017 | 0.064 | -0.64 | 1.3 | 0.66 | 0.85 | 0.94 | 0.92 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **GMM-Sargan2** | -0.0073 | 0.014 | -0.017 | 0.064 | -0.63 | 1.2 | 0.65 | 0.74 | 0.88 | 0.8 | 1 | 1 | 1 | 0.048 |
| **10** | **0.9** | **1.15** | **QML** | -0.00023 | 0.0096 | -9.1E-05 | 0.0059 | -0.0054 | 0.53 | 0.35 | 0.95 | 0.95 | 0.95 | 1 | 1 | 1 |  |
| **10** | **0.9** | **1.15** | **OPM** | 0.00041 | 0.0099 | 0.00013 | 0.006 | 0.025 | 0.55 | 0.36 | 0.93 | 0.94 | 0.93 | 1 | 1 | 1 |  |
| **5** | **0.9** | **1.15** | **Pooled** | 0.064 | 0.064 | 0.0094 | 0.012 | 9.1 | 9.2 | 0.41 | 0 | 0.77 | 0 | 1 | 1 | 1 |  |
| **5** | **0.9** | **1.15** | **OLS-FE** | -0.32 | 0.32 | -0.075 | 0.076 | -4 | 4 | 0.034 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| **5** | **0.9** | **1.15** | **GLS-RE** | 0.063 | 0.063 | 0.011 | 0.013 | 8.9 | 9 | 0.4 | 0 | 0.72 | 0 | 1 | 1 | 1 |  |
| **5** | **0.9** | **1.15** | **GMM-Diff** | 0.03 | 0.13 | 0.0077 | 0.033 | 2.2 | 360 | 1.5 | 0.95 | 0.95 | 0.72 | 1 | 1 | 0.13 |  |
| **5** | **0.9** | **1.15** | **GMM-Sys1** | -0.0012 | 0.027 | 0.00073 | 0.011 | -0.2 | 1.7 | 0.93 | 0.91 | 0.96 | 0.95 | 1 | 1 | 0.99 |   |
| **5** | **0.9** | **1.15** | **GMM-Sargan1** | -0.00082 | 0.027 | 0.00078 | 0.011 | -0.2 | 1.7 | 0.92 | 0.9 | 0.95 | 0.94 | 1 | 1 | 0.99 | 0.059 |
| **5** | **0.9** | **1.15** | **GMM-Sys2** | -0.0069 | 0.026 | -0.022 | 0.14 | -0.67 | 2.7 | 1.4 | 0.89 | 0.97 | 0.96 | 1 | 0.94 | 0.86 |   |
| **5** | **0.9** | **1.15** | **GMM-Sargan2** | -0.0064 | 0.025 | -0.022 | 0.14 | -0.64 | 2.5 | 1.4 | 0.86 | 0.95 | 0.94 | 1 | 0.95 | 0.93 | 0.043 |
| **5** | **0.9** | **1.15** | **QML** | -0.00095 | 0.025 | -0.00031 | 0.011 | -0.084 | 4.5 | 0.88 | 0.95 | 0.94 | 0.92 | 1 | 1 | 0.95 |   |
| **5** | **0.9** | **1.15** | **OPM** | 0.0022 | 0.027 | 0.00056 | 0.011 | 0.12 | 2.5 | 0.94 | 0.9 | 0.93 | 0.89 | 1 | 1 | 1 |   |
| **4** | **0.9** | **1.15** | **Pooled** | 0.064 | 0.064 | 0.009 | 0.013 | 9.2 | 9.2 | 0.5 | 0 | 0.84 | 0 | 1 | 1 | 1 |   |
| **4** | **0.9** | **1.15** | **OLS-FE** | -0.43 | 0.43 | -0.1 | 0.1 | -4.2 | 4.2 | 0.028 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| **4** | **0.9** | **1.15** | **GLS-RE** | 0.063 | 0.063 | 0.011 | 0.014 | 8.8 | 8.8 | 0.47 | 0 | 0.79 | 0 | 1 | 1 | 1 |   |
| **4** | **0.9** | **1.15** | **GMM-Diff** | 0.023 | 0.22 | 0.006 | 0.056 | 3.2 | 43 | 1.7 | 0.95 | 0.95 | 0.64 | 0.97 | 1 | 0.077 |  |
| **4** | **0.9** | **1.15** | **GMM-Sys1** | 0.0015 | 0.038 | 0.0013 | 0.014 | -0.16 | 17 | 1.1 | 0.93 | 0.96 | 0.96 | 1 | 1 | 0.84 |   |
| **4** | **0.9** | **1.15** | **GMM-Sargan1** | 0.0022 | 0.038 | 0.0014 | 0.014 | -0.14 | 2.8 | 1.1 | 0.92 | 0.96 | 0.95 | 1 | 1 | 0.89 | 0.046 |
| **4** | **0.9** | **1.15** | **GMM-Sys2** | -0.0042 | 0.035 | -0.016 | 0.21 | -0.73 | 17 | 1.8 | 0.91 | 0.98 | 0.96 | 1 | 0.72 | 0.48 |   |
| **4** | **0.9** | **1.15** | **GMM-Sargan2** | -0.0043 | 0.035 | -0.016 | 0.21 | -0.72 | 33 | 1.8 | 0.89 | 0.98 | 0.96 | 1 | 0.77 | 0.59 | 0.045 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **4** | **0.9** | **1.15** | **QML** | -0.0028 | 0.039 | -0.0006 | 0.015 | -6.9 | 179 | 1.2 | 0.94 | 0.94 | 0.91 | 1 | 1 | 0.70 |  |
| **4** | **0.9** | **1.15** | **OPM** | 0.0026 | 0.038 | 0.00031 | 0.014 | 0.092 | 4.3 | 1.4 | 0.87 | 0.92 | 0.86 | 1 | 1 | 1 |  |

**Table E.1. General Properties Simulation Results (cont.)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** | **Sargan** |
| **T** | **Alpha** | **Omega** | **Estimator** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Cov.** | **Cov.** | **Cov.** | **Pow.** | **Pow.** | **Pow.** | **P-value** |
| **3** | **0.9** | **1.15** | **Pooled** | 0.064 | 0.064 | 0.0096 | 0.015 | 9.3 | 9.3 | 0.61 | 0 | 0.87 | 0 | 1 | 1 | 1 |  |
| **3** | **0.9** | **1.15** | **OLS-FE** | -0.62 | 0.62 | -0.16 | 0.16 | -4.5 | 4.5 | 0.023 | 0 | 0 | 0 | 1 | 1 | 1 |   |
| **3** | **0.9** | **1.15** | **GLS-RE** | 0.063 | 0.063 | 0.012 | 0.017 | 8.7 | 8.8 | 0.54 | 0 | 0.8 | 0 | 1 | 1 | 1 |   |
| **3** | **0.9** | **1.15** | **GMM-Diff** | 0.07 | 5.5 | 0.017 | 1.4 | 4.3 | 42 | 1.7 | 0.96 | 0.96 | 0.48 | 0.58 | 0.98 | 0.065 |  |
| **3** | **0.9** | **1.15** | **GMM-Sys1** | 0.022 | 0.1 | 0.005 | 0.028 | 0.18 | 53 | 1.8 | 0.99 | 1 | 0.93 | 0.95 | 1 | 0.18 |  |
| **3** | **0.9** | **1.15** | **GMM-Sargan1** | 0.028 | 0.12 | 0.015 | 0.068 | 0.31 | 40 | 1.8 | 0.94 | 0.95 | 0.89 | 0.97 | 0.97 | 0.35 | 0.050 |
| **3** | **0.9** | **1.15** | **GMM-Sys2** | -0.0022 | 0.066 | -0.053 | 0.63 | -0.66 | 1200 | 3.1 | 0.98 | 1 | 0.99 | 0.9 | 0.2 | 0.062 |  |
| **3** | **0.9** | **1.15** | **GMM-Sargan2** | 0.00044 | 0.069 | -0.044 | 0.65 | -0.39 | 500 | 3.1 | 0.97 | 1 | 0.98 | 0.99 | 0.29 | 0.11 | 0.082 |
| **3** | **0.9** | **1.15** | **QML** | -0.0074 | 0.078 | -0.0026 | 0.025 | 0.91 | 130 | 1.5 | 0.96 | 0.95 | 0.86 | 1 | 1 | 0.29 |  |
| **3** | **0.9** | **1.15** | **OPM** | -0.007 | 0.054 | -0.0016 | 0.021 | -0.018 | 4.8 | 1.9 | 0.92 | 0.95 | 0.92 | 1 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **Pooled** | 0.2 | 0.2 | 0.036 | 0.038 | 0.77 | 0.77 | 0.039 | 0 | 0.14 | 0 | 1 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **OLS-FE** | -0.46 | 0.46 | -0.12 | 0.12 | -0.6 | 0.6 | 0.015 | 0 | 0 | 0 | 0.31 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **GLS-RE** | 0.17 | 0.17 | 0.042 | 0.043 | 0.66 | 0.66 | 0.04 | 0 | 0.068 | 0 | 1 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **GMM-Diff** | -0.0056 | 0.084 | -0.0021 | 0.026 | -0.0079 | 0.27 | 0.14 | 0.98 | 1 | 0.98 | 1 | 1 | 0.99 |  |
| **3** | **0.5** | **1.15** | **GMM-Sys1** | 0.0036 | 0.047 | 0.00098 | 0.018 | 0.0093 | 0.11 | 0.077 | 1 | 1 | 0.99 | 1 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **GMM-Sargan1** | 0.0045 | 0.047 | 0.003 | 0.038 | 0.013 | 0.15 | 0.1 | 0.94 | 0.94 | 0.95 | 1 | 1 | 1 | 0.044 |
| **3** | **0.5** | **1.15** | **GMM-Sys2** | 0.0012 | 0.055 | -0.021 | 0.51 | -0.094 | 1.1 | 0.47 | 0.99 | 1 | 1 | 0.86 | 0.2 | 0.19 |  |
| **3** | **0.5** | **1.15** | **GMM-Sargan2** | 0.0021 | 0.055 | -0.013 | 0.51 | -0.059 | 1.1 | 0.49 | 0.98 | 1 | 1 | 0.99 | 0.3 | 0.3 | 0.024 |
| **3** | **0.5** | **1.15** | **QML** | -0.0024 | 0.045 | -0.0013 | 0.019 | 0.00068 | 0.13 | 0.081 | 0.97 | 0.95 | 0.96 | 1 | 1 | 1 |  |
| **3** | **0.5** | **1.15** | **OPM** | 0.006 | 0.05 | 0.0022 | 0.02 | 0.01 | 0.15 | 0.089 | 0.9 | 0.94 | 0.9 | 1 | 1 | 1 |  |
| **3** | **0.9** | **5** | **Pooled** | 0.072 | 0.072 | 0.011 | 0.015 | 13 | 13 | 0.26 | 0 | 0.85 | 0 | 1 | 1 | 1 |  |
| **3** | **0.9** | **5** | **OLS-FE** | -0.62 | 0.62 | -0.16 | 0.16 | -4.5 | 4.5 | 0.023 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| **3** | **0.9** | **5** | **GLS-RE** | 0.071 | 0.071 | 0.014 | 0.018 | 13 | 13 | 0.26 | 0 | 0.77 | 0 | 1 | 1 | 1 |  |
| **3** | **0.9** | **5** | **GMM-Diff** | 0.37 | 11 | 0.091 | 2.8 | 5 | 19 | 0.46 | 0.97 | 0.97 | 0.27 | 0.028 | 0.21 | 0.12 |  |
| **3** | **0.9** | **5** | **GMM-Sys1** | -0.013 | 0.39 | -0.003 | 0.1 | 0.27 | 230 | 4.9 | 0.81 | 1 | 0.9 | 0.77 | 0.86 | 0.14 |  |
| **3** | **0.9** | **5** | **GMM-Sargan1** | -0.00059 | 0.62 | 0.00077 | 0.35 | -0.55 | 92 | 5.2 | 0.7 | 0.92 | 0.81 | 0.82 | 0.82 | 0.25 | 0.060 |
| **3** | **0.9** | **5** | **GMM-Sys2** | -0.046 | 0.13 | -0.12 | 0.55 | -6 | 55 | 6.1 | 0.84 | 0.99 | 0.95 | 0.88 | 0.2 | 0.16 |  |
| **3** | **0.9** | **5** | **GMM-Sargan2** | -0.043 | 0.12 | -0.13 | 0.57 | -5.7 | 960 | 6.2 | 0.73 | 0.99 | 0.9 | 0.96 | 0.35 | 0.26 | 0.115 |
| **3** | **0.9** | **5** | **QML** | -0.0074 | 0.078 | -0.0026 | 0.025 | 0.91 | 130 | 1.5 | 0.96 | 0.95 | 0.86 | 1 | 1 | 0.29 |  |
| **3** | **0.9** | **5** | **OPM** | -0.0069 | 0.054 | -0.0016 | 0.02 | -0.11 | 4.8 | 1.9 | 0.92 | 0.94 | 0.92 | 1 | 1 | 1 |   |

**Table E.1. General Properties Simulation Results (cont.)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** | **Sargan** |
| **T** | **Alpha** | **Omega** | **Estimator** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Cov.** | **Cov.** | **Cov.** | **Pow.** | **Pow.** | **Pow.** | **P-value** |
| **3** | **0.5** | **5** | **Pooled** | 0.33 | 0.33 | 0.061 | 0.062 | 2.4 | 2.4 | 0.042 | 0 | 0.003 | 0 | 1 | 1 | 1 |   |
| **3** | **0.5** | **5** | **OLS-FE** | -0.46 | 0.46 | -0.12 | 0.12 | -0.6 | 0.6 | 0.015 | 0 | 0 | 0 | 0.31 | 1 | 1 |  |
| **3** | **0.5** | **5** | **GLS-RE** | 0.33 | 0.33 | 0.076 | 0.077 | 2.3 | 2.3 | 0.041 | 0 | 0 | 0 | 1 | 1 | 1 |  |
| **3** | **0.5** | **5** | **GMM-Diff** | -0.09 | 0.95 | -0.023 | 0.23 | 0.15 | 25 | 0.34 | 0.98 | 0.98 | 0.9 | 0.3 | 1 | 0.16 |  |
| **3** | **0.5** | **5** | **GMM-Sys1** | 0.013 | 0.12 | 0.0032 | 0.033 | -0.0053 | 0.28 | 0.19 | 0.95 | 1 | 0.95 | 0.74 | 1 | 0.77 |  |
| **3** | **0.5** | **5** | **GMM-Sargan1** | 0.022 | 0.13 | 0.017 | 0.1 | -0.01 | 0.66 | 0.26 | 0.89 | 0.9 | 0.88 | 0.82 | 0.93 | 0.64 | 0.044 |
| **3** | **0.5** | **5** | **GMM-Sys2** | -0.012 | 0.11 | -0.37 | 0.82 | -0.79 | 2.9 | 0.71 | 0.97 | 0.95 | 0.94 | 0.71 | 0.24 | 0.24 |  |
| **3** | **0.5** | **5** | **GMM-Sargan2** | -0.011 | 0.1 | -0.34 | 0.84 | -0.67 | 3.6 | 0.72 | 0.94 | 0.91 | 0.91 | 0.92 | 0.41 | 0.39 | 0.080 |
| **3** | **0.5** | **5** | **QML** | -0.0024 | 0.045 | -0.0013 | 0.019 | 0.00068 | 0.13 | 0.081 | 0.97 | 0.95 | 0.96 | 1 | 1 | 1 |  |
| **3** | **0.5** | **5** | **OPM** | 0.006 | 0.05 | 0.0022 | 0.02 | 0.01 | 0.15 | 0.089 | 0.9 | 0.94 | 0.9 | 1 | 1 | 1 |   |

**Table E.2. Finite Sample Properties**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **T** | **N** | **Alpha** |  | **Estimator** | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** |
| **Omega** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Cov.** | **Cov.** | **Cov.** | **Pow.** | **Pow.** | **Pow.** |
| **6** | **500** | **0.9** | **5** | **GMM-Diff** | 0.16 | 0.27 | 0.04 | 0.067 | 3.4 | 22 | 0.82 | 0.91 | 0.91 | 0.5 | 0.89 | 1 | 0.083 |
| **6** | **500** | **0.9** | **5** | **GMM-Sys1** | -0.057 | 0.066 | -0.0087 | 0.017 | -6.2 | 240 | 3 | 0.22 | 0.9 | 0.69 | 1 | 1 | 0.63 |
| **6** | **500** | **0.9** | **5** | **GMM-Sys2** | -0.059 | 0.062 | -0.07 | 0.14 | -9.2 | 39 | 3 | 0.096 | 0.9 | 0.46 | 1 | 0.99 | 0.83 |
| **6** | **500** | **0.9** | **5** | **QML** | -0.00041 | 0.028 | -3.4E-05 | 0.013 | 0.078 | 3.1 | 0.93 | 0.95 | 0.95 | 0.92 | 1 | 1 | 0.92 |
| **6** | **500** | **0.9** | **5** | **OPM** | 0.0015 | 0.028 | 0.00022 | 0.013 | 0.073 | 2.5 | 1 | 0.9 | 0.94 | 0.91 | 1 | 1 | 1 |
| **6** | **300** | **0.9** | **5** | **GMM-Diff** | 0.23 | 0.35 | 0.057 | 0.087 | 3.8 | 23 | 0.67 | 0.9 | 0.9 | 0.42 | 0.76 | 0.99 | 0.071 |
| **6** | **300** | **0.9** | **5** | **GMM-Sys1** | -0.062 | 0.068 | -0.0095 | 0.021 | -8 | 170 | 4.1 | 0.18 | 0.91 | 0.74 | 1 | 1 | 0.56 |
| **6** | **300** | **0.9** | **5** | **GMM-Sys2** | -0.063 | 0.065 | -0.055 | 0.13 | -10 | 38 | 3.3 | 0.064 | 0.92 | 0.44 | 1 | 0.98 | 0.8 |
| **6** | **300** | **0.9** | **5** | **QML** | -0.0005 | 0.036 | -0.00083 | 0.017 | 0.1 | 15 | 1.1 | 0.95 | 0.94 | 0.91 | 1 | 1 | 0.76 |
| **6** | **300** | **0.9** | **5** | **OPM** | 0.001 | 0.034 | 0.00081 | 0.017 | -0.055 | 3.5 | 1.2 | 0.92 | 0.94 | 0.91 | 1 | 1 | 1 |
| **6** | **100** | **0.9** | **5** | **GMM-Diff** | 0.36 | 0.49 | 0.09 | 0.13 | 4.1 | 12 | 0.5 | 0.84 | 0.83 | 0.31 | 0.52 | 0.95 | 0.051 |
| **6** | **100** | **0.9** | **5** | **GMM-Sys1** | -0.067 | 0.072 | -0.0065 | 0.032 | -10 | 160 | 4.6 | 0.14 | 0.94 | 0.74 | 1 | 1 | 0.51 |
| **6** | **100** | **0.9** | **5** | **GMM-Sys2** | -0.068 | 0.069 | -0.033 | 0.12 | -12 | 380 | 3.4 | 0.046 | 0.94 | 0.48 | 1 | 0.98 | 0.76 |
| **6** | **100** | **0.9** | **5** | **QML** | -0.008 | 0.074 | -0.0023 | 0.03 | 0.72 | 170 | 1.6 | 0.92 | 0.95 | 0.84 | 1 | 1 | 0.4 |
| **6** | **100** | **0.9** | **5** | **OPM** | -0.004 | 0.049 | -0.0012 | 0.027 | -0.024 | 4.2 | 1.8 | 0.94 | 0.95 | 0.94 | 1 | 1 | 1 |
| **6** | **40** | **0.9** | **5** | **GMM-Diff** | 0.4 | 0.54 | 0.1 | 0.14 | 4.2 | 46 | 0.48 | 0.83 | 0.82 | 0.3 | 0.44 | 0.92 | 0.059 |
| **6** | **40** | **0.9** | **5** | **GMM-Sys1** | -0.069 | 0.072 | -0.0045 | 0.052 | -11 | 87 | 4.9 | 0.12 | 0.93 | 0.76 | 1 | 1 | 0.53 |
| **6** | **40** | **0.9** | **5** | **GMM-Sys2** | -0.069 | 0.07 | -0.02 | 0.12 | -12 | 280 | 3.6 | 0.016 | 0.93 | 0.45 | 1 | 0.96 | 0.78 |
| **6** | **40** | **0.9** | **5** | **QML** | -0.019 | 0.12 | -0.0063 | 0.05 | 1.6 | 440 | 1.7 | 0.86 | 0.93 | 0.74 | 1 | 1 | 0.29 |
| **6** | **40** | **0.9** | **5** | **OPM** | -0.02 | 0.062 | -0.0048 | 0.042 | -0.57 | 3.3 | 1.7 | 0.97 | 0.95 | 0.97 | 1 | 1 | 1 |
| **3** | **500** | **0.9** | **5** | **GMM-Diff** | 1.4 | 31 | 0.33 | 7.6 | 5 | 17 | 0.39 | 0.98 | 0.99 | 0.27 | 0.015 | 0.17 | 0.11 |
| **3** | **500** | **0.9** | **5** | **GMM-Sys1** | -0.012 | 0.54 | -0.003 | 0.13 | -0.46 | 140 | 6.6 | 0.8 | 1 | 0.93 | 0.8 | 0.86 | 0.094 |
| **3** | **500** | **0.9** | **5** | **GMM-Sys2** | -0.052 | 0.14 | -0.067 | 0.58 | -6.9 | 140 | 7.3 | 0.79 | 0.99 | 0.95 | 0.88 | 0.17 | 0.11 |
| **3** | **500** | **0.9** | **5** | **QML** | -0.012 | 0.13 | -0.0043 | 0.039 | 1.7 | 52 | 1.7 | 0.93 | 0.95 | 0.78 | 1 | 1 | 0.21 |
| **3** | **500** | **0.9** | **5** | **OPM** | -0.021 | 0.069 | -0.0049 | 0.027 | -0.43 | 3.8 | 2 | 0.94 | 0.94 | 0.94 | 1 | 1 | 1 |
| **3** | **300** | **0.9** | **5** | **GMM-Diff** | -3.4 | 100 | -0.85 | 23 | 5 | 11 | 0.38 | 0.98 | 0.98 | 0.29 | 0.012 | 0.11 | 0.12 |
| **3** | **300** | **0.9** | **5** | **GMM-Sys1** | -0.037 | 0.43 | -0.0079 | 0.11 | -1.5 | 320 | 7.2 | 0.8 | 1 | 0.95 | 0.81 | 0.88 | 0.065 |
| **3** | **300** | **0.9** | **5** | **GMM-Sys2** | -0.061 | 0.13 | -0.034 | 0.55 | -6.8 | 190 | 7.5 | 0.8 | 1 | 0.96 | 0.86 | 0.17 | 0.1 |
| **3** | **300** | **0.9** | **5** | **QML** | -0.025 | 0.17 | -0.0078 | 0.053 | 2.2 | 1000 | 2.3 | 0.9 | 0.93 | 0.73 | 1 | 1 | 0.18 |
| **3** | **300** | **0.9** | **5** | **OPM** | -0.035 | 0.084 | -0.0089 | 0.034 | -0.76 | 3.3 | 1.9 | 0.93 | 0.95 | 0.93 | 1 | 1 | 1 |

**Table E.2. Finite Sample Properties (cont.)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** |
| **T** | **N** | **Alpha** | **Omega** | **Estimator** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Cov.** | **Cov.** | **Cov.** | **Pow.** | **Pow.** | **Pow.** |
| **3** | **100** | **0.9** | **5** | **GMM-Diff** | 0.69 | 9.7 | 0.15 | 2.5 | 5 | 6.4 | 0.37 | 0.99 | 0.98 | 0.28 | 0.007 | 0.085 | 0.085 |
| **3** | **100** | **0.9** | **5** | **GMM-Sys1** | -0.033 | 0.58 | -0.0026 | 0.15 | -2.2 | 550 | 8.3 | 0.76 | 1 | 0.97 | 0.83 | 0.87 | 0.064 |
| **3** | **100** | **0.9** | **5** | **GMM-Sys2** | -0.063 | 0.17 | -0.013 | 0.49 | -7.4 | 560 | 7.5 | 0.74 | 1 | 0.95 | 0.89 | 0.16 | 0.12 |
| **3** | **100** | **0.9** | **5** | **QML** | -0.089 | 0.3 | -0.021 | 0.093 | 3.6 | 58 | 2.6 | 0.82 | 0.89 | 0.55 | 1 | 1 | 0.21 |
| **3** | **100** | **0.9** | **5** | **OPM** | -0.071 | 0.12 | -0.022 | 0.058 | -1.7 | 2.7 | 1.5 | 0.96 | 0.93 | 0.96 | 1 | 1 | 1 |
| **6** | **500** | **0.9** | **1.15** | **GMM-Diff** | 0.058 | 0.14 | 0.015 | 0.037 | 2.4 | 49 | 1.3 | 0.94 | 0.94 | 0.66 | 1 | 1 | 0.17 |
| **6** | **500** | **0.9** | **1.15** | **GMM-Sys1** | -0.0056 | 0.03 | 0.0013 | 0.014 | -0.54 | 2.1 | 1.1 | 0.9 | 0.94 | 0.97 | 1 | 1 | 0.95 |
| **6** | **500** | **0.9** | **1.15** | **GMM-Sys2** | -0.013 | 0.029 | -0.02 | 0.11 | -1.1 | 3 | 1.5 | 0.84 | 0.95 | 0.95 | 1 | 0.98 | 0.88 |
| **6** | **500** | **0.9** | **1.15** | **QML-FE** | -0.00041 | 0.028 | -3.4E-05 | 0.013 | 0.078 | 3.1 | 0.93 | 0.95 | 0.95 | 0.92 | 1 | 1 | 0.92 |
| **6** | **500** | **0.9** | **1.15** | **OPM** | 0.0015 | 0.028 | 0.00022 | 0.013 | 0.073 | 2.5 | 1 | 0.9 | 0.94 | 0.91 | 1 | 1 | 1 |
| **6** | **300** | **0.9** | **1.15** | **GMM-Diff** | 0.11 | 0.21 | 0.028 | 0.054 | 3 | 80 | 0.99 | 0.92 | 0.92 | 0.58 | 0.95 | 1 | 0.12 |
| **6** | **300** | **0.9** | **1.15** | **GMM-Sys1** | -0.0073 | 0.041 | 0.0013 | 0.018 | -0.8 | 5.9 | 1.6 | 0.88 | 0.95 | 0.96 | 1 | 1 | 0.73 |
| **6** | **300** | **0.9** | **1.15** | **GMM-Sys2** | -0.018 | 0.037 | -0.023 | 0.12 | -1.9 | 10 | 1.9 | 0.8 | 0.95 | 0.96 | 1 | 0.98 | 0.66 |
| **6** | **300** | **0.9** | **1.15** | **QML-FE** | -0.0005 | 0.036 | -0.00083 | 0.017 | 0.1 | 15 | 1.1 | 0.95 | 0.94 | 0.91 | 1 | 1 | 0.76 |
| **6** | **300** | **0.9** | **1.15** | **OPM** | 0.001 | 0.034 | 0.00081 | 0.017 | -0.055 | 3.5 | 1.2 | 0.92 | 0.94 | 0.91 | 1 | 1 | 1 |
| **6** | **100** | **0.9** | **1.15** | **GMM-Diff** | 0.26 | 0.39 | 0.065 | 0.1 | 3.9 | 48 | 0.67 | 0.88 | 0.86 | 0.38 | 0.68 | 0.98 | 0.066 |
| **6** | **100** | **0.9** | **1.15** | **GMM-Sys1** | -0.021 | 0.063 | 0.0025 | 0.032 | -1.8 | 52 | 2.8 | 0.83 | 0.95 | 0.97 | 1 | 1 | 0.27 |
| **6** | **100** | **0.9** | **1.15** | **GMM-Sys2** | -0.032 | 0.052 | -0.021 | 0.12 | -3.1 | 33 | 3 | 0.74 | 0.94 | 0.98 | 1 | 0.98 | 0.31 |
| **6** | **100** | **0.9** | **1.15** | **QML-FE** | -0.0082 | 0.074 | -0.0023 | 0.03 | 0.72 | 170 | 1.6 | 0.92 | 0.95 | 0.84 | 1 | 1 | 0.4 |
| **6** | **100** | **0.9** | **1.15** | **OPM** | -0.004 | 0.049 | -0.0012 | 0.027 | -0.024 | 4.2 | 1.8 | 0.94 | 0.95 | 0.94 | 1 | 1 | 1 |
| **6** | **40** | **0.9** | **1.15** | **GMM-Diff** | 0.34 | 0.48 | 0.087 | 0.13 | 4.1 | 21 | 0.55 | 0.84 | 0.81 | 0.33 | 0.54 | 0.94 | 0.069 |
| **6** | **40** | **0.9** | **1.15** | **GMM-Sys1** | -0.031 | 0.081 | 0.0034 | 0.052 | -2 | 590 | 3.9 | 0.79 | 0.92 | 0.95 | 1 | 1 | 0.16 |
| **6** | **40** | **0.9** | **1.15** | **GMM-Sys2** | -0.038 | 0.057 | -0.014 | 0.12 | -3.5 | 71 | 3.5 | 0.73 | 0.94 | 0.98 | 1 | 0.96 | 0.26 |
| **6** | **40** | **0.9** | **1.15** | **QML-FE** | -0.019 | 0.12 | -0.0062 | 0.049 | 1.6 | 440 | 1.7 | 0.86 | 0.93 | 0.74 | 1 | 1 | 0.29 |
| **6** | **40** | **0.9** | **1.15** | **OPM** | -0.02 | 0.062 | -0.0048 | 0.042 | -0.57 | 3.3 | 1.7 | 0.97 | 0.95 | 0.97 | 1 | 1 | 1 |
| **3** | **500** | **0.9** | **1.15** | **GMM-Diff** | 0.48 | 18 | 0.13 | 4.8 | 4.6 | 47 | 1.1 | 0.95 | 0.96 | 0.39 | 0.31 | 0.83 | 0.1 |
| **3** | **500** | **0.9** | **1.15** | **GMM-Sys1** | 0.052 | 0.2 | 0.013 | 0.051 | 0.59 | 150 | 2.6 | 0.99 | 0.99 | 0.87 | 0.85 | 0.97 | 0.054 |
| **3** | **500** | **0.9** | **1.15** | **GMM-Sys2** | -0.00048 | 0.16 | -0.027 | 0.55 | -0.26 | 66 | 3.8 | 0.98 | 0.99 | 0.98 | 0.84 | 0.18 | 0.03 |
| **3** | **500** | **0.9** | **1.15** | **QML-FE** | -0.011 | 0.12 | -0.0041 | 0.039 | 1.7 | 52 | 1.6 | 0.93 | 0.95 | 0.78 | 1 | 1 | 0.21 |
| **3** | **500** | **0.9** | **1.15** | **OPM** | -0.021 | 0.069 | -0.0049 | 0.027 | -0.43 | 3.8 | 2 | 0.94 | 0.94 | 0.94 | 1 | 1 | 1 |

**Table E.2. Finite Sample Properties (cont.)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Alpha | Alpha | Beta | Beta | LRE | LRE | LRE | Alpha | Beta | LRE | Alpha | Beta | LRE |
| **T** | **N** | **Alpha** | **Omega** | **Estimator** | Bias | RMSE | Bias | RMSE | Bias | RMSE | MD | Cov. | Cov. | Cov. | Pow. | Pow. | Pow. |
| **3** | **300** | **0.9** | **1.15** | **GMM-Diff** | 0.51 | 14 | 0.13 | 3.6 | 4.8 | 42 | 0.87 | 0.94 | 0.96 | 0.37 | 0.15 | 0.64 | 0.11 |
| **3** | **300** | **0.9** | **1.15** | **GMM-Sys1** | 0.042 | 0.22 | 0.011 | 0.059 | 1.3 | 42 | 2.7 | 0.98 | 0.99 | 0.87 | 0.8 | 0.95 | 0.028 |
| **3** | **300** | **0.9** | **1.15** | **GMM-Sys2** | 0.0048 | 0.14 | 0.0075 | 0.6 | 0.18 | 160 | 3.7 | 0.98 | 1 | 0.97 | 0.81 | 0.18 | 0.02 |
| **3** | **300** | **0.9** | **1.15** | **QML-FE** | -0.019 | 0.16 | -0.0065 | 0.051 | 2.2 | 1300 | 2.2 | 0.9 | 0.93 | 0.74 | 1 | 1 | 0.18 |
| **3** | **300** | **0.9** | **1.15** | **OPM** | -0.035 | 0.084 | -0.0089 | 0.034 | -0.75 | 3.3 | 1.9 | 0.93 | 0.95 | 0.93 | 1 | 1 | 1 |
| **3** | **100** | **0.9** | **1.15** | **GMM-Diff** | -0.23 | 9.9 | -0.077 | 2.8 | 5 | 14 | 0.53 | 0.96 | 0.96 | 0.3 | 0.046 | 0.3 | 0.11 |
| **3** | **100** | **0.9** | **1.15** | **GMM-Sys1** | 0.023 | 0.38 | 0.013 | 0.11 | 2.6 | 76 | 3.7 | 0.99 | 0.99 | 0.84 | 0.71 | 0.84 | 0.017 |
| **3** | **100** | **0.9** | **1.15** | **GMM-Sys2** | -0.016 | 0.19 | -0.0058 | 0.47 | 0.69 | 380 | 4.8 | 0.98 | 1 | 0.98 | 0.75 | 0.17 | 0.019 |
| **3** | **100** | **0.9** | **1.15** | **QML-FE** | -0.077 | 0.29 | -0.018 | 0.091 | 3.6 | 58 | 2.6 | 0.83 | 0.9 | 0.56 | 0.99 | 1 | 0.2 |
| **3** | **100** | **0.9** | **1.15** | **OPM** | -0.071 | 0.12 | -0.022 | 0.058 | -1.7 | 2.7 | 1.5 | 0.96 | 0.93 | 0.96 | 1 | 1 | 1 |

**Table E.3. Distributional Assumption Violations**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Alpha** | **Alpha** | **Beta** | **Beta** | **LRE** | **LRE** | **LRE** | **Alpha** | **Beta** | **LRE** | **Alpha** | **Beta** | **LRE** |
| **DGP** | **Estimator** | **Bias** | **RMSE** | **Bias** | **RMSE** | **Bias** | **RMSE** | **MD** | **Coverage** | **Coverage** | **Coverage** | **Power** | **Power** | **Power** |
| **Outliers**  |   |  |  |  |  |  |  |  |  |  |  |  |  |   |
| Y~t(10); X~ N | **GMM-Sys1** | 0.0017 | 0.09 | 0.0031 | 0.026 | 0.24 | 530 | 1.8 | 0.98 | 1 | 0.92 | 0.97 | 1 | 0.19 |
| Y~ t(10); X~ N  | **QML-FE** | 0.0041 | 0.095 | -0.00037 | 0.03 | 1.3 | 81 | 1.7 | 0.93 | 0.94 | 0.8 | 1 | 1 | 0.27 |
| Y~ t(10); X~ N  | **OPM** | -0.003 | 0.066 | -0.0032 | 0.023 | -0.18 | 5.2 | 2.1 | 0.88 | 0.92 | 0.87 | 1 | 1 | 1 |
| Y~ t(10); X~ t(4)  | **GMM-Sys1** | 0.0004 | 0.13 | 0.0046 | 0.038 | 0.37 | 960 | 2.2 | 0.97 | 1 | 0.89 | 0.93 | 0.99 | 0.15 |
| Y~ t(10); X~ t(4)  | **QML-FE** | -0.0039 | 0.14 | -0.00094 | 0.043 | 2.2 | 120 | 2.3 | 0.87 | 0.91 | 0.71 | 1 | 1 | 0.23 |
| Y~ t(10); X~ t(4)  | **OPM** | -0.0082 | 0.087 | -0.0062 | 0.032 | -0.41 | 6 | 2.4 | 0.85 | 0.9 | 0.85 | 1 | 1 | 1 |
| **Skewed Distribution**  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Y~ B(2,9); X~ N  | **GMM-Sys1** | -0.006 | 0.062 | -0.0013 | 0.016 | -0.26 | 3.5 | 1.5 | 1 | 1 | 1 | 0.91 | 0.99 | 0.077 |
| Y~ B(2,9); X~ N  | **QML-FE** | -0.00031 | 0.0069 | -2.8E-05 | 0.0025 | -0.015 | 0.37 | 0.25 | 0.94 | 0.93 | 0.95 | 1 | 1 | 1 |
| Y~ B(2,9); X~ N  | **OPM** | 0 | 0.0069 | 0.000051 | 0.0025 | 0.017 | 0.37 | 0.25 | 0.95 | 0.93 | 0.94 | 1 | 1 | 1 |
| Y~ B(2,9); X~ B(2,5)  | **GMM-Sys1** | -0.065 | 0.85 | -0.015 | 0.22 | -3.5 | 300 | 7.9 | 0.61 | 1 | 0.87 | 0.79 | 0.85 | 0.22 |
| Y~ B(2,9); X~ B(2,5)  | **QML-FE** | 0.00053 | 0.13 | -0.00037 | 0.038 | 2 | 310 | 2 | 0.9 | 0.93 | 0.76 | 1 | 1 | 0.23 |
| Y~ B(2,9); X~ B(2,5)  | **OPM** | -0.0028 | 0.076 | -0.0053 | 0.028 | -0.16 | 5.6 | 2.4 | 0.87 | 0.92 | 0.87 | 1 | 1 | 1 |
| **Bimodal Distribution**  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Y~ B(0.9,0.9); X~ N  | **GMM-Sys1** | -0.0062 | 0.071 | -0.0011 | 0.018 | -0.2 | 5.1 | 1.6 | 1 | 1 | 0.99 | 0.94 | 0.99 | 0.11 |
| Y~ B(0.9,0.9); X~ N  | **QML-FE** | 0.00029 | 0.017 | 0.0002 | 0.0066 | 0.025 | 1 | 0.64 | 0.96 | 0.96 | 0.95 | 1 | 1 | 1 |
| Y~ B(0.9,0.9); X~ N  | **OPM** | 0 | 0.017 | -1.1E-05 | 0.0066 | -0.0048 | 1.1 | 0.64 | 0.96 | 0.95 | 0.95 | 1 | 1 | 1 |
| Y~ B(0.9,0.9); X~ B(0.5,0.5)  | **GMM-Sys1** | -0.043 | 0.44 | -0.0066 | 0.12 | 0.48 | 490 | 4.6 | 0.85 | 1 | 0.89 | 0.76 | 0.84 | 0.11 |
| Y~ B(0.9,0.9); X~ B(0.5,0.5)  | **QML-FE** | 0.0038 | 0.14 | -0.00092 | 0.045 | 2.2 | 360 | 2 | 0.9 | 0.93 | 0.73 | 1 | 1 | 0.2 |
| Y~ B(0.9,0.9); X~ B(0.5,0.5)  | **OPM** | -0.007 | 0.068 | -0.0069 | 0.032 | -0.32 | 3.7 | 2.1 | 0.95 | 0.94 | 0.95 | 1 | 1 | 1 |
| **Chi2 Distribution**  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Y~ Chi2(2); X~ N  | **GMM-Sys1** | -0.0026 | 0.1 | 0.0022 | 0.028 | -0.052 | 21 | 1.9 | 0.99 | 0.99 | 0.93 | 0.95 | 1 | 0.15 |
| Y~ Chi2(2); X~ N  | **QML-FE** | 0.0074 | 0.095 | 0.0013 | 0.029 | 1.4 | 290 | 1.7 | 0.86 | 0.9 | 0.75 | 1 | 1 | 0.37 |
| Y~ Chi2(2); X~ N  | **OPM** | -0.004 | 0.079 | -0.0035 | 0.025 | -0.19 | 7.8 | 2.5 | 0.74 | 0.86 | 0.74 | 1 | 1 | 1 |
| Y~ Chi2(2); X~ Chi2(2)  | **GMM-Sys1** | 0.0017 | 0.18 | 0.0048 | 0.053 | 0.4 | 120 | 2.2 | 0.96 | 0.99 | 0.89 | 0.89 | 0.97 | 0.14 |
| Y~ Chi2(2); X~ Chi2(2)  | **QML-FE** | -0.0091 | 0.18 | -0.00036 | 0.054 | 3 | 220 | 3.4 | 0.75 | 0.86 | 0.63 | 1 | 1 | 0.28 |
| Y~ Chi2(2); X~ Chi2(2)  | **OPM** | -0.008 | 0.13 | -0.012 | 0.045 | -0.44 | 9.4 | 3.2 | 0.65 | 0.86 | 0.65 | 1 | 1 | 1 |
| **Heteroskedasticity** |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| $ε\_{it}$~$e^{0.2x\_{it}}N(0,1)$ | **GMM-Sys1** | 0.0034 | 0.11 | 0.0048 | 0.034 | 0.39 | 67 | 1.9 | 0.99 | 0.99 | 0.91 | 0.95 | 1 | 0.17 |
| $ε\_{it}$~$e^{0.2x\_{it}}N(0,1)$ | **QML-FE** | -0.0061 | 0.12 | -0.00075 | 0.036 | 1.6 | 300 | 2 | 0.88 | 0.89 | 0.75 | 1 | 1 | 0.27 |
| $ε\_{it}$~$e^{0.2x\_{it}}N(0,1)$ | **OPM** | 0.006 | 0.082 | -0.0051 | 0.03 | 0.27 | 7.4 | 3 | 0.77 | 0.87 | 0.78 | 1 | 1 | 1 |

Simulation Results with Autoregressive Independent Variable

We also test if an autoregressive DGP for $x\_{i,t}$ affects the performance of the transformed-likelihood estimators. The empirical model is the same as for the simulations in the main text but the DGP for the simulations is:

$$y\_{i,t}= αy\_{i,t-1}+βx\_{i,t}+η\_{i}+ε\_{1,i,t} $$

with

$$x\_{i,t}=0.5x\_{i,t-1}+0.75η\_{i}+ε\_{2,i,t}$$

$ε\_{1,i,t} \~NID(0,1)$; $ε\_{2,i,t} \~NID(0,2)$; $η\_{i}\~U(-1.15, 1.15)$

The simulations are first run with $T=3$; $ α=0.9$ and then with the less extreme scenario $T=3$; $ α=0.5$. This provides two useful points of comparison with the results from the DGP without an autoregressive $x\_{it}$. Looking to Table E.4, we see that the performance of the transformed-likelihood estimators is not substantively affected by the autoregressive DGP for $x\_{i,t}.$

**Table E.4. Simulation Results with Autoregressive Independent Variable**

(T=3; N=1000; Alpha=0.9; Beta=0.5; LRE=5)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Alpha****Bias** | **Alpha****RMSE** | **Beta****Bias** | **Beta****RMSE** | **LRE****Bias** | **LRE****RMSE** | **LRE****MD** |
| QML-FE | -0.012 | 0.082 | -0.0021 | 0.032 | 0.78 | 584.41 | 1.87 |
| OPM | -0.0039 | 0.056 | -0.0023 | 0.028 | 0.27 | 4.43 | 2.12 |
|  | **Alpha****Coverage** | **Beta****Coverage** | **LRE****Coverage** | **Alpha****Power** | **Beta****Power** | **LRE****Power** |  |
| QML-FE | 0.94 | 0.95 | 0.86 | 1 | 1 | 0.26 |  |
| OPM | 0.92 | 0.94 | 0.92 | 1 | 1 | 1 |  |

(T=3; N=1000; Alpha=0.5; Beta=0.5; LRE=1)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Alpha****Bias** | **Alpha****RMSE** | **Beta****Bias** | **Beta****RMSE** | **LRE****Bias** | **LRE****RMSE** | **LRE****MD** |
| QML-FE | -0.0063 | 0.056 | -0.00038 | 0.028 | -0.0095 | 0.15 | 0.091 |
| OPM | 0.012 | 0.061 | 0.0013 | 0.028 | 0.019 | 0.19 | 0.10 |
|  | **Alpha****Coverage** | **Beta****Coverage** | **LRE****Coverage** | **Alpha****Power** | **Beta****Power** | **LRE****Power** |  |
| QML-FE | 0.95 | 0.95 | 0.95 | 1 | 1 | 1 |  |
| OPM | 0.88 | 0.94 | 0.88 | 1 | 1 | 1 |  |

Simulation Results with Time-Specific Effects

To test if the inclusion of time-specific effects affect the performance of the transformed-likelihood estimators, we include them in the DGP for the simulations:

$$y\_{i,t}= αy\_{i,t-1}+βx\_{i,t}+η\_{i}+τ\_{t}+ε\_{1,i,t}$$

with

$$x\_{i,t}=0.75η\_{i}+ε\_{2,i,t}$$

$ε\_{1,i,t} \~NID(0,1)$; $ε\_{2,i,t} \~NID(0,4)$; $η\_{i}\~U(-1.15, 1.15)$; $τ\_{i}\~U(-1, 1)$

We also include time-specific effects in the empirical model. The simulations are first run with $T=3$; $ α=0.9$ and then with the less extreme scenario $T=3$; $ α=0.5$. This provides two useful points of comparison with the results from the DGP without time-specific effects. The results presented in Table E.5 shows that both the QML-FE and OPM estimators perform just as well controlling for such time-specific effects. They do not exhibit any bias, the coverage statistics are around 0.95 and the power is around 1. The exception is for the LRE estimated by QML-FE when $α=0.9$. Under these circumstances, the estimator exhibits some bias and poor power for the LRE. However, the problems are no worse than they were without the time-specific effects, in the DGP.

**Table E.5. Simulation Results with Time-Specific Effects**

(T=3; N=1000; Alpha=0.9; Beta=0.5; LRE=5)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Alpha****Bias** | **Alpha****SD** | **Beta****Bias** | **Beta****RMSE** | **LRE****Bias** | **LRE****RMSE** | **LRE****MD** |
| QML-FE | -0.0096 | 0.082 | -0.0023 | 0.027 | 0.85 | 95.62 | 1.76 |
| OPM | -0.0035 | 0.056 | -0.0016 | 0.02 | 0.25 | 0.82 | 0.58 |
|  | **Alpha****Coverage** | **Beta****Coverage** | **LRE****Coverage** | **Alpha****Power** | **Beta****Power** | **LRE****Power** |  |
| QML-FE | 0.96 | 0.96 | 0.85 | 1 | 1 | 0.29 |  |
| OPM | 0.92 | 0.93 | 0.92 | 1 | 1 | 1 |  |

(T=3; N=1000; Alpha=0.5; Beta=0.5; LRE=1)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Alpha****Bias** | **Alpha****SD** | **Beta****Bias** | **Beta****RMSE** | **LRE****Bias** | **LRE****RMSE** | **LRE****MD** |
| QML-FE | -0.0028 | 0.047 | -0.00065 | 0.02 | 0.002 | 0.13 | 0.080 |
| OPM | 0.0075 | 0.051 | 0.0018 | 0.02 | 0.011 | 0.82 | 0.58 |
|  | **Alpha****Coverage** | **Beta****Coverage** | **LRE****Coverage** | **Alpha****Power** | **Beta****Power** | **LRE****Power** |  |
| QML-FE | 0.95 | 0.95 | 0.96 | 1 | 1 | 1 |  |
| OPM | 0.89 | 0.93 | 0.9 | 1 | 1 | 1 |  |

Simulation Results with Misspecified Lagged Dependent Variable

Finally, we repeat many of the simulations when $α=0.$ We include a LDV in the empirical model even though $α=0$ in the DGP for the simulations:

$$y\_{i,t}= βx\_{i,t}+η\_{i}+ε\_{1,i,t}$$

with

$$x\_{i,t}=0.75η\_{i}+ε\_{2,i,t}$$

$ε\_{1,i,t} \~NID(0,1)$; $ε\_{2,i,t} \~NID(0,2)$; $η\_{i}\~U(-1.15, 1.15)$

The results presented in Table E.6 show that the QML-FE and OPM estimators provide unbiased estimates of all the parameters and the coverage probabilities are approximately 0.95.

**Table E.6. Simulation Results – No LDV**

|  |
| --- |
| (T=3; Alpha=0; Beta=0.5) |
|  | **Alpha****Bias** | **Alpha****RMSE** | **Beta****Bias** | **Beta****RMSE** |
| QML-FE (N=1000) | -0.0022 | 0.03 | 0.0002 | 0.017 |
| OPM (N=1000) | 0.0036 | 0.031 | 0.00011 | 0.017 |
| QML-FE (N=500) | 0.00086 | 0.043 | -0.0010 | 0.024 |
| OPM (N=500) | 0.0023 | 0.044 | 0.0018 | 0.024 |
| QML-FE (N=40) | -0.020 | 0.19 | -0.0090 | 0.097 |
| OPM (N=40) | 0.068 | 0.21 | 0.021 | 0.1 |
|  | **Alpha****Coverage** | **Beta****Coverage** | **Alpha****Power** | **Beta****Power** |
| QML-FE (N=1000) | 0.95 | 0.94 | NA | 1 |
| OPM (N=1000) | 0.93 | 0.94 | NA | 1 |
| QML-FE (N=500) | 0.95 | 0.96 | NA | 1 |
| OPM (N=500) | 0.92 | 0.95 | NA | 1 |
| QML-FE (N=40) | 0.93 | 0.94 | NA | 1 |
| OPM (N=40) | 0.91 | 0.94 | NA | 1 |

**Appendix F: Empirical Examples**

Below, we apply the GMM, QML-FE, and OPM estimators to three empirical examples.

*Empirical Example 1*

In the first example, the dependent variable is credit given to the government by domestic banks, measured as a percent of GDP (*public credit*). The two independent variables used in Table 2 of Menaldo (2015) are an indicator of quality government on a 0 to 1 scale (*state capacity*) and credit provided to the private sector by domestic banks as a percent of GDP (*private credit*).

Menaldo (2015) uses a GMM, with the specification of GMM-Sys2 to estimate a LDV fixed-effects model with time trend:

$$PublicCredit\_{i,t}=αDirectCredit\_{i,t-1}+β\_{1}StateCapacity\_{i,t}+β\_{2}PrivateCredit\_{i,t}+τ+η\_{i}+ε\_{i,t}$$

The original article used yearly data from 1984 to 2011 and found a negative effect for *state capacity* and a positive effect for *private credit*. The article also found a high degree of autoregression in the data. With a $T$ of 28, the GMM estimates are likely to be valid (although Menaldo does not use a standard error correction, so these are likely biased downwards). What if we wanted to check if the effects hold for the most recent period, for example 2006 to 2011? As we have seen under these circumstances, GMM-Sys2 has moderate RMSEs (about 20%) for $β$ and is biased (30 to 60%) with very large RMSEs for the LRE. Further, as is often the case, the between variance is much greater than the within variance (124 vs 9). We apply the GMM-Diff, GMM-Sys1, GMM-Sys2, QML-FE and OPM to this data and the results are reported in Table F1.

GMM-Sys1 and GMM-Sys2 both indicate high autoregression (an $α$ of greater than .90) and neither independent variable to be statistically significant. However, neither of these estimators pass the Sargan test. In fact, we cannot find any set of moment conditions for the system estimator that pass the test. GMM-Diff does pass but finds no autoregression and neither independent variable to be statistically significant.

QML-FE and OPM estimate $α$ to be about 0.997 and 0.970 respectively. The OPM estimate is much more precise (small confidence interval). While the TLE estimates for *state capacity* are not statistically significant and have large confidence intervals (like the GMM-Diff intervals), the estimates for *private credit* are much more precise and statistically significant. The estimated LRE is 9.128 (-433.506, 451.761) for QML-FE and 1.151 (0.224, 6.902) for OPM. The estimate of the LRE is much more precise for OPM. This is as we would expect from the simulations.

**Table F1. Empirical Example 1 (Menaldo 2015)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Alpha** | **State Capacity** | **Private Credit** | **Sargan P-value** |
| **GMM-Sys1** | 0.910 | 0.0644 | .0121 | < 0.001 |
|  | (.579, 1.242) | (-12.0827, 12.172) | (-0.0467, 0.0710) |  |
| **GMM-Sys2** | 0.954 | -2.887 | 0.0233 | < 0.001 |
|  | (0.845, 1.064) | (-9.329, 3.556) | (-0.00383, 0.0429) |  |
| **GMM-Diff** | -0.0245 | -2.3 | 0.0611 | 0.0658 |
|  | (-0.190, 0.141) | (-13.563, 8.962) | (-0.0466, 0.169) |  |
| **QML** | 0.997 | -2.389 | 0.0294 | NA |
|  | (0.840, 1.154) | (-11.112, 6.334) | (0.0159, 0.0430) |  |
| **OPM** | 0.971 | 0.764 | 0.0344 | NA |
|  | (0.889, 0.995) | (-14.631, 16.277) | (0.0123, 0.0567) |  |

The weak instrument difficulties that GMM can run into when $α$ is close to 1, particularly with a high between to within variance ratio, are evident. The greater efficiency of OPM in estimating the LRE is also evident. This is important because not only do inefficiencies result in greater standard errors, but in a single estimate, the larger tails of the estimate distribution increase the probability of a poor estimate.

*Empirical Example 2*

The second empirical example employs the three wave panel data used by Green and Palmquist (1990) to explore the autoregression in US party identification (PID) and the effect of short-term forces on PID. This data was also used by Wawro (2002) to demonstrate the use of GMM estimators. The dependent variable runs from 0 (Strong Democrat) to 5 (Strong Republican). We focus on the short-term forces (STF) for which Wawro (2002) found significant effects: *approve*, a general approval rating for Carter (1 to 5); and *inflation,* assessment of Carter's handling of inflation (1 to 5). Both variables are scored such that larger, positive scores indicate a less positive approval/assessment for Carter.

We estimate the LDV fixed-effects model with time specific effects for each short-term force:

$$PID\_{i,t}=αPID\_{i,t-1}+β\_{j}STF\_{i,t,j}+τ+η\_{i}+ε\_{i,t}$$

A pooled estimator finds a high degree of autoregression in PID and significant effects for STFs (Table F2). Green and Palmquist (1990) find that once one corrects for measurement error, the autoregression remain high but the STF effects are no longer significant. Subsequently, Wawro (2002) demonstrates that estimates from the GMM estimator that he calls orthogonal deviations plus levels (ODL) shows a statistically significant but lower degree of autoregression and significant effects for *approve* and *inflation*. GMM-Sys1 (as we operationalized it for the simulations) finds no evidence of autoregression but does find an effect for *approve*. Both GMM estimators pass the Sargant test and yet indicate different dynamics in PID. This is a challenge encountered when using GMM estimators -- different moment conditions can produce results that are different in substantively meaningful ways.

Given the pooled estimate suggests substantial autoregression and the potential problems with the GMM estimator when $α$ is close to one, a researcher might want to employ a TLE (OPM in particular), especially since the between variance is over 3 times the within variance in this dataset. As it turns out, QML-FE and OPM both estimate $α$ and both STFs to be 0. Not only are the TLE estimates 0 but they exhibit smaller confidence intervals than the GMM estimates (1/3 to 1/2 smaller).

The TLEs suggest that partisan identity is static and not affected by the short-term forces, consistent with the hypothesis originally proposed by Green and Palmquist (1990). GMM provides mixed results depending on the moment conditions used. TLEs do not require the researcher to choose between specifications and provide smaller confidence intervals.

**Table F2. Empirical Example 2 (Green and Palmquist 1990)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **Alpha** | **Approve** | **Inflation** |
| **Pooled** | (1) | 0.865 | 0.0935 |  |
|  |  | (0.834, 0.897) | (0.0500, 0.137) |  |
|  | (2) | 0.874 |  | 0.0783 |
|  |  | (0.843, 0.904) |  | (0.0313, 0.125) |
| **GMM-ODL** | (1) | 0.203 | 0.148 |  |
|  |  | (0.0054, 0.401) | (0.00688, 0.289) |  |
|  | (2) | 0.23 |  | 0.142 |
|  |  | (0.0497, 0.410) |  | (0.0362, 0.248) |
| **GMM-Sys** | (1) | 0.149 | 0.27 |  |
|  |  | (-0.0991, 0.398) | (0.0859, 0.454) |  |
|  | (2) | 0.173 |  | 0.123 |
|  |  | (-0.0774, 0.424) |  | (-0.00831, 0.254) |
| **QML** | (1) | -0.00481 | 0.0395 |  |
|  |  | (-0.140, 0.131) | (-0.0323, 0.111) |  |
|  | (2) | -0.00359 |  | 0.0213 |
|  |  | (-0.139, 0.131) |  | (-0.0471, 0.0898) |
| **OPM** | (1) | 0.003 | 0.0393 |  |
|  |  | ( -0.103, 0.126) | (-0.0335, 0.111) |  |
|  | (2) | 0.005 |  | 0.021 |
|  |  | (-0.104, 0.126) |  | (-0.0479, 0.0903) |

*Empirical Example 3*

The third example uses three waves of data from the 2015/16 British Election Panel Study (*N*=11,202). Panel data, such as this, have often been used to examine the effect of subjective economic evaluations on vote intention (e.g., Lewis-Beck et al 2008, Evans and Andersen 2006). This example uses the same variables used in those studies but for convenience, it does not include control variables. The dependent variable is an 11 point rating of how the respondent feels about the incumbent Conservative party from (0) Strongly Dislike to (10) Strongly like. The independent variable is a 5 point rating of how the economy is changing: (1) Getting a lot worse;(2) Getting a little worse; (3) Staying about the same; (4) Getting a little better; (5) Getting a lot better.[[5]](#footnote-5)

We estimate a LDV fixed-effects model with time specific effects:

$$Approve\_{i,t}=αApprove\_{i,t-1}+β\_{1}Econ\_{i,t}+η\_{i}+ε\_{i,t}$$

The LDV model is commonly used for economic approval and voting models using panel data. The pooled estimator typically suggests a high degree of autoregression in government/party approval, as it does in this data set (Table F.3). Meanwhile, GMM-Sys1 suggests less autocorrelation but with a wider confidence interval (about 8X wider). Given the potential problems with GMM estimators when $α$ is close to one, a researcher might want to employ a TLE (OPM in particular). As it turns out, QML-FE and OPM both estimate $α$ to be only 0.16 and have small conference intervals.

As for the estimated effect of economic evaluations on feeling towards the incumbent party, GMM estimates a short-run effect of about 0.32. That is a 0.32 increase in the 11 point feeling scale due to a one unit increase on the five point economic evaluation scale. QML-FE and OPM estimate short-run effects of a similar of a similar magnitude: 0.26 and 0.21. The long-run effect estimated by GMM-Sys1 is 0.45 (0.38, 0.52). For QML-FE it is 0.31 (0.28, 0.34) and for OPM 0.25 (0.22, 0.28). In other words, the results are similar but the greater efficiency of TLEs over GMM is evident. This is important because not only do inefficiencies result in greater standard errors, but in a single estimate, the larger tails of the estimate distribution increase the probability of an estimate further from the truth.

**Table F.3. Empirical Example 3 (BES)**

|  |  |  |
| --- | --- | --- |
|  | **Alpha** | **Economy** |
| **Pooled** | 0.81 | 0.40 |
|  | (0.805 ,0.816) | (0.38 ,0.42) |
| **GMM-Sys** | 0.28 | 0.32 |
|  | (0.25, 0.32) | (0.28, 0.37) |
| **QML-FE** | 0.16 | 0.26 |
|  | (0.14 ,0.18) | (0.23 ,0.29) |
| **OPM** | 0.16 | 0.21 |
|  | (0.14 ,0.18) | (0.18 ,0.23) |

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1. A fixed effects estimator eliminates any cross sectional variance and so it cannot be used to estimate between effects, unless the between effects are identical to the within effects, in which case the fixed effects estimator is unbiased but inefficient. [↑](#footnote-ref-1)
2. The effects of time-invariant and near time-invariant variables are estimated by regressing the estimated fixed-effects (from an OLS-FE estimation) on the (near) time-invariant variables. [↑](#footnote-ref-2)
3. The procedure described by Hausman and Taylor (1981) only requires some of the covariates to be independent of the individual effects. [↑](#footnote-ref-3)
4. This varies slightly from 3.24 in Lancaster (2002). The prior density for $σ^{2}$ had not been included. [↑](#footnote-ref-4)
5. Don't know responses are coded as missing and the observations are dropped. [↑](#footnote-ref-5)