# Online Appendix for "The Polarization Dynamics of Electoral Reforms"

## Theoretical Results

Elections consist of an incumbent from one party facing a challenger from the opposing party. The incumbent's ideological location is given by $I_p$ where $p \in L, R$ and the challenger's by $C_p$. Throughout, we use hat scripts to denote quantities when the reform is implemented and tilde scripts to denote outcomes when the reform is removed. For example, $\hat{\gamma}_p$ refers to the probability than an incumbent from party $p$ is reelected when the reform is implemented.

We make several assumptions on the voter's utility function to derive our results. We specify the voter's utility function as the sum of a deterministic spatial component that is decreasing in the distance between the voter's ideal policy and the candidate's and a stochastic shock. We assume that the deterministic component voter's utility function is continuous, single-peaked, symmetric, and strictly decreasing in the distance between the voter's ideal point and the candidate's ideology. As a result, the distance between the voter's ideal point and the candidate's ideology completely determines the spatial component of the voter's utility. For the stochastic component, we assume that the stochastic shocks for the voter's utility from the left and right party candidates are $\epsilon_L$ and $\epsilon_R$ are independent and identically distributed continuous random variables with support over the entire real line. We denote the cumulative distribution function of $\epsilon_L - \epsilon_R$ with the function $F(.)$. Because $F(.)$ is a CDF for a random variable with support over the real line it is a strictly-increasing function. Because $\epsilon_L$ and $\epsilon_R$ are iid, the probability density function of the difference in these random variables is symmetric about 0 and we have $F(x) = 1 - F(-x)$ for all $x$ on the real line.

We first make several symmetry assumptions to simplify the analysis. After proving the main results for the case of symmetrically located incumbents, we relax this assumption. We assume that the voter has ideal policy of 0 and that the incumbents are located symmetrically around the voter's ideal policy. In the baseline period, the left party incumbents are located at $I_L = -I_R$ and the right party incumbents are located at $I_R > 0$. The moderation-inducing reform shifts the location of each party's challengers by $\frac{\tau}{2} > 0$. To ensure that implementing the moderation-inducing reform does not make the left challenger more conservative than the right challenger we assume that $\tau \leq 2I_R$. We allow for the incumbent to partially adjust her ideological location toward the challenger position: $X_p = \lambda I_p + (1 - \lambda)C_p$ where $\lambda \in (0, 1]$. We assume that $\lambda > 0$ so that there is at least partial incumbent policy persistence.

Whether an individual incumbent is reelected or not is probabilistic because the election outcome depends upon the realization of the stochastic shock. Therefore, all of our results are defined in terms of expectations.

Proof of Result 1: We show that Result 1 holds by characterizing the probability that the voter reelects the incumbent under the alternative scenarios.

$$\begin{aligned}
\hat{\gamma}_R &= Pr(-g(|\hat{X}_R - 0|) + \epsilon_R > -g(|\hat{C}_L - 0|) + \epsilon_L) \\
&= Pr(-g(|\hat{X}_R|) + g(|-I_R + \frac{\tau}{2}|) > \epsilon_L - \epsilon_R) \\
&= F(-g(|\hat{X}_R|) + g(|-I_R + \frac{\tau}{2}|)) \\
\widetilde{\gamma}_R &= F(-g(|\widetilde{X}_R|) + g(|-I_R - \frac{\tau}{2}|))
\end{aligned}$$

The same logic as above gives us:

$$\begin{aligned}
\hat{\gamma}_L &= Pr(-g(|-\hat{X}_R|) + \epsilon_L > -g(|I_R - \frac{\tau}{2}|) + \epsilon_R) \\
&= Pr(\epsilon_L - \epsilon_R > g(| - \hat{X}_R|) - g(|I_R - \frac{\tau}{2}|)) \\
&= Pr(\epsilon_L - \epsilon_R > g(|\hat{X}_R|) - g(|I_R - \frac{\tau}{2}|)) \\
&= 1 - F(g(|\hat{X}_R|) - g(|I_R - \frac{\tau}{2}|)) = F(-g(|\hat{X}_R|) + g(|I_R - \frac{\tau}{2}|))
\end{aligned}$$

where the last equality follows from $F(x) = 1 - F(-x)$. Similarly, we have:

$$\widetilde{\gamma}_L = F(-g(|\widetilde{X}_R|) + g(|-I_R - \frac{\tau}{2}|))$$

As a result, we have $\hat{\gamma}_R = \hat{\gamma}_L \equiv \hat{\gamma} = F(-g(|\hat{X}_R|) + g(|-I_R + \frac{\tau}{2}|))$ and $\widetilde{\gamma}_R = \widetilde{\gamma}_L \equiv \widetilde{\gamma} = F(-g(|\widetilde{X}_R|) + g(|-I_R - \frac{\tau}{2}|))$. Now that we have shown that the respective incumbent reelection rates are equal for both parties, we can now characterize how the reelection rates differ depending upon whether the reform is implemented or reformed. First note that the incumbent reelection rate in the baseline period is

$F(-g(|I_R|)+g(|-I_R|)) = F(0)$. As $|\hat{X}_R| = |\lambda I_R+(1-\lambda)(I_R-\frac{\tau}{2})| > |-I_R+\frac{\tau}{2}|$ it follows that $-g(|\hat{X}_R|) + g(|-I_R + \frac{\tau}{2}|) < 0$ and therefore $\hat{\gamma} = F(-g(|\hat{X}_R|) + g(|-I_R + \frac{\tau}{2}|)) < F(0)$. By the same logic, we have $\widetilde{\gamma} = F(-g(|\widetilde{X}_R|) + g(|-I_R - \frac{\tau}{2}|)) > F(0)$. As a result, $\hat{\gamma} < F(0) < \widetilde{\gamma}$, which completes the proof of Result 1.

Proof of Result 2: The ideological location of the average party legislator is a weighted average of the locations of the party's incumbents who win reelection and the challengers who win election. The weights are determined by the relative probabilities that the incumbents and challengers win their electoral contests. The expected location of the legislators in office in each party are:

$$\hat{\mu}_R = \frac{\hat{\gamma}}{\hat{\gamma}+1-\hat{\gamma}}\hat{X}_R + \frac{(1-\hat{\gamma})}{\hat{\gamma}+1-\hat{\gamma}}\hat{C}_R = \hat{\gamma}\hat{X}_R + (1-\hat{\gamma})\hat{C}_R$$
$$\hat{\mu}_L = \hat{\gamma}\hat{X}_L + (1-\hat{\gamma})\hat{C}_L$$
$$\widetilde{\mu}_R = \widetilde{\gamma}\widetilde{X}_R + (1-\widetilde{\gamma})\widetilde{C}_R$$
$$\widetilde{\mu}_L = \widetilde{\gamma}\widetilde{X}_L + (1-\widetilde{\gamma})\widetilde{C}_L$$

The level of polarization in each state is then:

$$\hat{Polar} = \hat{\gamma}\hat{X}_R + (1-\hat{\gamma})\hat{C}_R - \{\hat{\gamma}\hat{X}_L + (1-\hat{\gamma})\hat{C}_L\}$$
$$= I_R - I_L - \tau(1-\hat{\gamma}\lambda)$$
$$\widetilde{Polar} = I_R - I_L + \tau(1-\widetilde{\gamma}\lambda)$$

The changes in polarization relative to the baseline period when polarization is $I_R - I_L$

are then:

$$\hat{\Delta} = -\tau(1 - \hat{\gamma}\lambda)$$
$$\widetilde{\Delta} = \tau(1 - \widetilde{\gamma}\lambda)$$

Noting that $\hat{\Delta} < 0$ and $\widetilde{\Delta} > 0$ because $\tau > 0$, $\gamma \in (0, 1)$ and $\lambda \in (0, 1]$ proves the first part of Result 2. To prove the second part, we see that the magnitudes of the changes are:

$$|\hat{\Delta}| = |-\tau(1 - \hat{\gamma}\lambda)| = \tau(1 - \hat{\gamma}\lambda)$$
$$|\widetilde{\Delta}| = |\tau(1 - \widetilde{\gamma}\lambda)| = \tau(1 - \widetilde{\gamma}\lambda)$$

By Result 1, $\hat{\gamma} < \widetilde{\gamma}$ so $|\hat{\Delta}| > |\widetilde{\Delta}|$, which completes the proof of Result 2.

Proof of Result 3: After $t \in 1, 2, ..$ elections, the composition of the legislature consists of the baseline incumbents and the new entrants. The stochastic utility shocks are independent and identically distributed across periods so the probability that an incumbent is able to remain in office after $t$ elections are given by $\hat{\gamma}^t$ and $\widetilde{\gamma}^t$. By assumption, the new entrants take the same position each period. As a result, the party means as the number of elections $t$ goes to infinity are:

$$\lim_{t\to\infty} \hat{\mu}_{Rt} = \lim_{t\to\infty} \hat{\gamma}^t \hat{X}_R + (1-\hat{\gamma}^t)\hat{C}_R = \hat{C}_R$$
$$\lim_{t\to\infty} \hat{\mu}_{Lt} = \lim_{t\to\infty} \hat{\gamma}^t \hat{X}_L + (1-\hat{\gamma}^t)\hat{C}_L = \hat{C}_L$$
$$\lim_{t\to\infty} \widetilde{\mu}_{Rt} = \lim_{t\to\infty} \widetilde{\gamma}^t \widetilde{X}_R + (1-\widetilde{\gamma}^t)\widetilde{C}_R = \widetilde{C}_R$$
$$\lim_{t\to\infty} \widetilde{\mu}_{Lt} = \lim_{t\to\infty} \widetilde{\gamma}^t \widetilde{X}_L + (1-\widetilde{\gamma}^t)\widetilde{C}_L = \widetilde{C}_L$$

As a result:

$$\lim_{t\to\infty} \hat{\Delta}_t = I_R - I_L - (\hat{C}_R - \hat{C}_L) = \tau$$
$$\lim_{t\to\infty} \widetilde{\Delta}_t = I_R - I_L - (\widetilde{C}_R - \widetilde{C}_L) = -\tau$$

Note that Result 3 holds whenever $\hat{\gamma}_L, \hat{\gamma}_R, \widetilde{\gamma}_L, \widetilde{\gamma}_L \in [0,1)$. This condition always holds in our setting because the incumbent reelection probability must be strictly less than 1 due to the assumption that $\epsilon_L$ and $\epsilon_R$ are continuous random variables with support over the real line. We do not require the assumption on the symmetric location of the incumbents to derive Result 3.

We now consider asymmetric incumbent locations and a linear deterministic component to the utility function. We assume throughout that $I_L < I_L + \frac{\tau}{2} < 0 < I_R - \frac{\tau}{2} < I_R$ to ensure that the left and right challengers are to the left and right of the voter's ideal policy. The incumbent reelection rates are now:

$$\hat{\gamma}_R = Pr(-|\hat{X}_R| + \epsilon_R > -|I_L + \frac{\tau}{2}| + \epsilon_L)$$
$$= Pr(-\hat{X}_R - I_L - \frac{\tau}{2} > \epsilon_L - \epsilon_R)$$
$$= F(-\hat{X}_R - I_L + \frac{\tau}{2}) = F(-\lambda I_R - (1-\lambda)(I_R - \frac{\tau}{2}) - I_L - \frac{\tau}{2}) = F(-I_R - I_L - \lambda\frac{\tau}{2})$$
$$\widetilde{\gamma}_R = F(-I_R - I_L + \lambda\frac{\tau}{2})$$
$$\hat{\gamma}_L = F(I_L + I_R - \lambda\frac{\tau}{2})$$
$$\widetilde{\gamma}_L = F(I_L + I_R + \lambda\frac{\tau}{2})$$

As $\tau > 0$ and $\lambda > 0$ we have $\hat{\gamma}_R < F(-I_R - I_L) < \widetilde{\gamma}_R$ and $\hat{\gamma}_L < F(I_R + I_L) < \widetilde{\gamma}_L$, where the middle terms are the baseline period incumbent reelection rates. These inequalities prove that Result 1 holds in the setting with asymmetric incumbent locations and a linear utility function. We now consider partisan polarization. The expected location of each party's legislators is:

$$\hat{\mu}_R = \frac{\hat{\gamma}_R}{\hat{\gamma}_R + 1 - \hat{\gamma}_L}\hat{X}_R + \frac{(1-\hat{\gamma}_L)}{\hat{\gamma}_R + 1 - \hat{\gamma}_L}\hat{C}_R$$
$$\hat{\mu}_L = \frac{\hat{\gamma}_L}{\hat{\gamma}_L + 1 - \hat{\gamma}_R}\hat{X}_L + \frac{(1-\hat{\gamma}_R)}{\hat{\gamma}_L + 1 - \hat{\gamma}_R}\hat{C}_L$$
$$\widetilde{\mu}_R = \frac{\widetilde{\gamma}_R}{\widetilde{\gamma}_R + 1 - \widetilde{\gamma}_L}\widetilde{X}_R + \frac{(1-\widetilde{\gamma}_L)}{\widetilde{\gamma}_R + 1 - \widetilde{\gamma}_L}\widetilde{C}_R$$
$$\widetilde{\mu}_L = \frac{\widetilde{\gamma}_L}{\widetilde{\gamma}_L + 1 - \widetilde{\gamma}_R}\widetilde{X}_L + \frac{(1-\widetilde{\gamma}_R)}{\widetilde{\gamma}_L + 1 - \widetilde{\gamma}_R}\widetilde{C}_L$$

Plugging in the expressions for the candidate locations, we have:

$$\hat{\Delta} = \hat{\mu}_R - \hat{\mu}_L = \frac{\tau(\lambda-2)}{2} + \frac{\lambda\tau\hat{\gamma}_R}{2(\hat{\gamma}_R - \hat{\gamma}_L + 1)} + \frac{\lambda\tau(\hat{\gamma}_R - 1)}{2(\hat{\gamma}_L - \hat{\gamma}_R + 1)}$$
$$\tilde{\Delta} = \widetilde{\mu}_R - \widetilde{\mu}_L = -\frac{\tau(\lambda-2)}{2} - \frac{\lambda\tau\widetilde{\gamma}_R}{2(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)} - \frac{\lambda\tau(\widetilde{\gamma}_R - 1)}{2(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)}$$

We now show that the change in polarization is negative when the reform is implemented:

$$2\hat{\Delta}(\hat{\gamma}_R - \hat{\gamma}_L + 1)(\hat{\gamma}_L - \hat{\gamma}_R + 1)$$
$$= \tau(\lambda-2)(\hat{\gamma}_R - \hat{\gamma}_L + 1)(\hat{\gamma}_L - \hat{\gamma}_R + 1) + \lambda\tau\hat{\gamma}_R(\hat{\gamma}_L - \hat{\gamma}_R + 1) + \lambda\tau(\hat{\gamma}_R - 1)(\hat{\gamma}_R - \hat{\gamma}_L + 1)$$
$$= \tau(\lambda-2)(\hat{\gamma}_R - \hat{\gamma}_L + 1)(\hat{\gamma}_L - \hat{\gamma}_R + 1) + \lambda\tau(\hat{\gamma}_L + \hat{\gamma}_R - 1)$$
$$= \tau(\lambda-2)(2\hat{\gamma}_R\hat{\gamma}_L - \hat{\gamma}_R^2 - \hat{\gamma}_L^2 + 1) + \lambda\tau(\hat{\gamma}_L + \hat{\gamma}_R - 1) < \tau(\lambda-2)(0) + \lambda\tau(0) = 0$$
$$\rightarrow \hat{\Delta} < 0$$

The inequality follows from the observation that the maximal value of the expression on the left hand side is achieved when either $\hat{\gamma}_L$ or $\hat{\gamma}_R$ is equal to 1 and the other quantity is equal to 0 and this value can never be achieved because each quantity is strictly in the unit interval. By the identical logic, we have:

$$2\widetilde{\Delta}(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)$$
$$= -\tau(\lambda - 2)(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1) - \lambda\tau\widetilde{\gamma}_R(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1) - \lambda\tau(\widetilde{\gamma}_R - 1)(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)$$
$$= -\tau(\lambda - 2)(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1) - \lambda\tau(\widetilde{\gamma}_L + \widetilde{\gamma}_R - 1)$$
$$= -\tau(\lambda - 2)(2\widetilde{\gamma}_R\widetilde{\gamma}_L - \widetilde{\gamma}_R^2 - \widetilde{\gamma}_L^2 + 1) - \lambda\tau(\widetilde{\gamma}_L + \widetilde{\gamma}_R - 1) > -\tau(\lambda - 2)(0) - \lambda\tau(0) = 0$$
$$\rightarrow \widetilde{\Delta} > 0$$

We now examine the magnitudes of the change in partisan polarization:

$$|\hat{\Delta}| = -\frac{\tau(\lambda - 2)}{2} - \frac{\lambda\tau\hat{\gamma}_R}{2(\hat{\gamma}_R - \hat{\gamma}_L + 1)} - \frac{\lambda\tau(\hat{\gamma}_R - 1)}{2(\hat{\gamma}_L - \hat{\gamma}_R + 1)}$$
$$|\tilde{\Delta}| = -\frac{\tau(\lambda - 2)}{2} - \frac{\lambda\tau\widetilde{\gamma}_R}{2(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)} - \frac{\lambda\tau(\widetilde{\gamma}_R - 1)}{2(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)}$$

We show that the difference in magnitudes is positive $|\hat{\Delta}| - |\tilde{\Delta}| > 0$.

$$|\hat{\Delta}| - |\tilde{\Delta}| = -\frac{\tau(\lambda - 2)}{2} - \frac{\lambda\tau\hat{\gamma}_R}{2(\hat{\gamma}_R - \hat{\gamma}_L + 1)} - \frac{\lambda\tau(\hat{\gamma}_R - 1)}{2(\hat{\gamma}_L - \hat{\gamma}_R + 1)}$$
$$+\frac{\tau(\lambda - 2)}{2} + \frac{\lambda\tau\widetilde{\gamma}_R}{2(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)} + \frac{\lambda\tau(\widetilde{\gamma}_R - 1)}{2(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)}$$
$$= \frac{\lambda\tau}{2}(-\frac{\hat{\gamma}_R}{(\hat{\gamma}_R - \hat{\gamma}_L + 1)} - \frac{\hat{\gamma}_R - 1}{(\hat{\gamma}_L - \hat{\gamma}_R + 1)} + \frac{\widetilde{\gamma}_R}{(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)} + \frac{\widetilde{\gamma}_R - 1}{(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)})$$

Note that $\frac{\lambda\tau}{2} > 0$ so if we can show that the term in parentheses is positive then we will have proven the result. We separately show that both $-\frac{\hat{\gamma}_R}{(\hat{\gamma}_R - \hat{\gamma}_L + 1)} + \frac{\widetilde{\gamma}_R}{(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)}$

and $-\frac{\hat{\gamma}_R-1}{(\hat{\gamma}_L-\hat{\gamma}_R+1)}+\frac{\widetilde{\gamma}_R-1}{(\widetilde{\gamma}_L-\widetilde{\gamma}_R+1)}$ are positive. Recall from Result 1 that $\widetilde{\gamma}_R > \hat{\gamma}_R$ and $\widetilde{\gamma}_L > \hat{\gamma}_L$. Observe that:

$$0 < \widetilde{\gamma}_R(1-\hat{\gamma}_L) - \hat{\gamma}_R(1-\widetilde{\gamma}_L) = \widetilde{\gamma}_R - \widetilde{\gamma}_R\hat{\gamma}_L - \hat{\gamma}_R + \hat{\gamma}_R\widetilde{\gamma}_L$$

$$= -\hat{\gamma}_R\widetilde{\gamma}_R + \hat{\gamma}_R\widetilde{\gamma}_R + \widetilde{\gamma}_R - \widetilde{\gamma}_R\hat{\gamma}_L - \hat{\gamma}_R + \hat{\gamma}_R\widetilde{\gamma}_L = -\hat{\gamma}_R(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1) + \widetilde{\gamma}_R(\hat{\gamma}_R - \hat{\gamma}_L + 1)$$

$$\rightarrow 0 < -\frac{\hat{\gamma}_R}{(\hat{\gamma}_R - \hat{\gamma}_L + 1)} + \frac{\widetilde{\gamma}_R}{(\widetilde{\gamma}_R - \widetilde{\gamma}_L + 1)}$$

Also,

$$0 < \widetilde{\gamma}_L(1-\hat{\gamma}_R) - \hat{\gamma}_L(1-\widetilde{\gamma}_R) = \widetilde{\gamma}_L - \widetilde{\gamma}_L\hat{\gamma}_R - \hat{\gamma}_L + \hat{\gamma}_L\widetilde{\gamma}_R$$

$$= -(\hat{\gamma}_R\widetilde{\gamma}_L - \hat{\gamma}_R\widetilde{\gamma}_R + \hat{\gamma}_R - \widetilde{\gamma}_L + \widetilde{\gamma}_R - 1) + (\hat{\gamma}_L\widetilde{\gamma}_R - \hat{\gamma}_R\widetilde{\gamma}_R + \hat{\gamma}_R - \hat{\gamma}_L + \hat{\gamma}_R - 1)$$

$$= -(\hat{\gamma}_R - 1)(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1) + (\widetilde{\gamma}_R - 1)(\hat{\gamma}_L - \hat{\gamma}_R + 1)$$

$$\rightarrow 0 < -\frac{\hat{\gamma}_R - 1}{(\hat{\gamma}_L - \hat{\gamma}_R + 1)} + \frac{\widetilde{\gamma}_R - 1}{(\widetilde{\gamma}_L - \widetilde{\gamma}_R + 1)}$$

As both components are positive, their sum is positive and we have completed the proof of Result 2.

Finally, we consider the case where the deterministic component of the utility function is nonlinear. Recall that we need Result 1 to derive the second part of Result 2 on the magnitudes of the changes in polarization. Observe that Result 1 no longer holds when incumbent policy positions are asymmetric, the incumbent policy persistence parameter is less than 1, and the voter's utility function is non-linear. A counterexample illustrates this observation. Suppose that the voter has a quadratic utility function, $I_L = -1$, $I_R = 1.3$, $\lambda = 0.2$, $\tau = 0.1$, and that the difference in the stochastic shocks is a standard normal random variable. Then we have the counterexample:

$$\widetilde{\gamma}_R = \Phi(-(0.2 * 1.3 + 0.8 * (1.3 + 0.05)^2 + (-1 - 0.05)^2) \approx 0.2441 <$$
$$0.2466 \approx \Phi(-(0.2 * 1.3 + 0.8 * (1.3 - 0.05)^2 + (-1 + 0.05)^2) = \hat{\gamma}_R$$

This example shows that it is possible for the incumbent reelection rate to be greater when the reform is implemented relative to removed when the voter's utility function is nonlinear. While Results 1 and 2 no longer hold in this more general setting, we can place additional restrictions on $\lambda$ that ensure that the results hold. We first define the incumbent reelection rates for each scenario in the setting with a nonlinear utility function.

$$\widetilde{\gamma}_R = F(-g(\lambda I_R + (1-\lambda)(I_R + \frac{\tau}{2})) + g(I_L - \frac{\tau}{2})) = F(-g(I_R + \frac{\tau}{2}(1-\lambda)) + g(I_L - \frac{\tau}{2}))$$
$$\hat{\gamma}_R = F(-g(I_R - \frac{\tau}{2}(1-\lambda)) + g(I_L + \frac{\tau}{2}))$$
$$\widetilde{\gamma}_L = F(-g(I_L - \frac{\tau}{2}(1-\lambda)) + g(I_R + \frac{\tau}{2}))$$
$$\hat{\gamma}_L = F(-g(I_L + \frac{\tau}{2}(1-\lambda)) + g(I_R - \frac{\tau}{2}))$$

All of the expressions are continuous functions in $\lambda$. $\widetilde{\gamma}_L$ and $\widetilde{\gamma}_R$ are monotone increasing functions in $\lambda$ while $\hat{\gamma}_L$ and $\hat{\gamma}_R$ are monotone decreasing in $\lambda$. Note that when $\lambda = 1$, Result 1 continues to hold for nonlinear utility functions.

$$\widetilde{\gamma}_R = F(-g(I_R) + g(I_L - \frac{\tau}{2})) > F(-g(I_R) + g(I_L + \frac{\tau}{2})) = \hat{\gamma}_R$$
$$\widetilde{\gamma}_L = F(-g(I_L) + g(I_R + \frac{\tau}{2})) > F(-g(I_L) + g(I_R - \frac{\tau}{2})) = \hat{\gamma}_L$$

Due to the monotonicity properties of the functions, starting from a value of $\lambda = 1$, Result 1 will always hold until the value of $\lambda$ becomes sufficiently small. In some configurations of the parameter values, Result 1 will continue to hold at $\lambda = 0$. In these cases, we do not need to place additional restrictions on $\lambda$. We can now formally define the necessary restrictions on $\lambda$, such that Result 1 and Result 2 hold. Letting $\lambda_L \in (max(\underline{\lambda}_L, 0), 1]$ and $\lambda_R \in (max(\underline{\lambda}_R, 0), 1]$ where $\underline{\lambda}_L$ and $\underline{\lambda}_R$* are defined such that:

$$\begin{aligned}\widetilde{\gamma}_R &= F(-g(I_R + \frac{\tau}{2}(1 - \underline{\lambda}_R)) + g(I_L - \frac{\tau}{2})) \\ &= F(-g(I_R - \frac{\tau}{2}(1 - \underline{\lambda}_R)) + g(I_L + \frac{\tau}{2})) = \hat{\gamma}_R \\ \widetilde{\gamma}_L &= F(-g(I_L - \frac{\tau}{2}(1 - \underline{\lambda}_L)) + g(I_R + \frac{\tau}{2})) \\ &= F(-g(I_L + \frac{\tau}{2}(1 - \underline{\lambda}_L)) + g(I_R - \frac{\tau}{2})) = \hat{\gamma}_L\end{aligned}$$

These conditions on $\lambda$ ensure that $\widetilde{\gamma}_R > \hat{\gamma}_R$ and $\widetilde{\gamma}_L > \hat{\gamma}_L$ whenever $\lambda_L \in (max(\underline{\lambda}_L, 0), 1]$ and $\lambda_R \in (max(\underline{\lambda}_R, 0), 1]$ hold. When incumbent policy persistence is sufficiently high, Results 1 and 2 will continue to hold even in the richer setting of asymmetric

*For simplicity, we allow the values of $\lambda$ to differ across parties. We could also restrict the value of $\lambda$ to be equal across parties. In that case, the binding lower bound on $\lambda$ would be determined by the minimum of $\lambda_L$ and $\lambda_R$.

incumbent ideological locations and a nonlinear voter utility function.

## Additional Empirical Results

Table A.1: Removal and Implementation of Unlimited PAC Contributions by State

| Treatment Variable | State | Years in Treatment |
| --- | --- | --- |
| Unlimited PAC Contributions Removed | California | 2001-2012 |
| | Colorado | 1999-2012 |
| | Georgia | 1991-2012 |
| | Idaho | 1997-2012 |
| | Illinois | 2011-2012 |
| | Massachusetts | 1993-2012 |
| | Maryland | 1991-2012 |
| | Missouri | 1995-1998; 2000-2001; 2003-2006; 2008 |
| | Nebraska | 1993-2012 |
| | New Jersey | 1993-2012 |
| | New Mexico | 2011-2012 |
| | Nevada | 1992-2012 |
| | New York | 1993-2012 |
| | Ohio | 1996-2012 |
| | Oregon | 1995-1996 |
| | South Carolina | 1992-2012 |
| | Tennessee | 1995-2012 |
| | Washington | 1993-2012 |
| Unlimited PAC Contributions Implemented | California | 1992-2000 |
| | Missouri | 1999; 2002; 2007; 2009-2012 |
| | Oregon | 1997-2012 |

Table A.2: Polarization (Barber Controls)

| | (1)<br>Abs(SM Score) | (2)<br>Abs(SM Score) | (3)<br>Abs(SM Score) | (4)<br>Abs(SM Score) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0492* | | | |
| | (0.0227) | | | |
| Remove Unlimited PAC Contributions | | 0.0272 | -0.000119 | -0.00112 |
| | | (0.0276) | (0.0128) | (0.0141) |
| Implement Unlimited PAC Contributions | | -0.0379 | -0.0423+ | -0.0433* |
| | | (0.0436) | (0.0211) | (0.0214) |
| Observations | 113393 | 113393 | 113393 | 113215 |
| $R^2$ | 0.217 | 0.217 | 0.222 | 0.230 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Barber Controls | Yes | Yes | Yes | Yes |
| Diff. in Coeffs | | -.065+ | -.042* | -.042* |
| SE Diff. in Coeffs | | (.0328) | (.0166) | (.0165) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses

$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.3: Polarization (Controls for Unlimited Individual and Party Contributions)

| | (1)<br>Abs(SM Score) | (2)<br>Abs(SM Score) | (3)<br>Abs(SM Score) | (4)<br>Abs(SM Score) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0548*** | | | |
| | (0.0106) | | | |
| Unlimited Indiv. Contributions | 0.0202 | 0.0231 | 0.00525 | 0.00862 |
| | (0.0194) | (0.0161) | (0.0111) | (0.0107) |
| Unlimited Party Contributions | 0.0144 | 0.0139 | 0.0123 | 0.00955 |
| | (0.0112) | (0.00997) | (0.00842) | (0.00874) |
| Remove Unlimited PAC Contributions | | 0.0310 | 0.00960 | 0.0101 |
| | | (0.0261) | (0.0110) | (0.0129) |
| Implement Unlimited PAC Contributions | | -0.0498 | -0.0450* | -0.0459* |
| | | (0.0389) | (0.0212) | (0.0220) |
| Observations | 122021 | 122021 | 122021 | 121709 |
| $R^2$ | 0.216 | 0.216 | 0.222 | 0.229 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.081* | -.055*** | -.056*** |
| SE Diff. in Coeffs | | (.0314) | (.013) | (.0123) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.4: Probability Incumbent Runs for Reelection (Lower Chambers Only)

| | (1) Incumbent Runs | (2) Incumbent Runs | (3) Incumbent Runs | (4) Incumbent Runs |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0222 | | | |
| | (0.0266) | | | |
| Remove Unlimited PAC Contributions | | -0.0373 | -0.0338 | -0.0353* |
| | | (0.0230) | (0.0203) | (0.0148) |
| Implement Unlimited PAC Contributions | | -0.147+ | -0.255*** | -0.221*** |
| | | (0.0858) | (0.0397) | (0.0339) |
| Observations | 45034 | 41311 | 41311 | 41311 |
| $R^2$ | 0.034 | 0.038 | 0.045 | 0.249 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.11 | -.221*** | -.185*** |
| SE Diff. in Coeffs | | (.0853) | (.0378) | (.0328) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.5: Probability Incumbent Wins Reelection (Lower Chambers Only)

| | (1) Incumbent Wins | (2) Incumbent Wins | (3) Incumbent Wins | (4) Incumbent Wins |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0281 | | | |
| | (0.0282) | | | |
| Remove Unlimited PAC Contributions | | -0.0385+ | -0.0456* | -0.0267+ |
| | | (0.0229) | (0.0185) | (0.0139) |
| Implement Unlimited PAC Contributions | | -0.155* | -0.145* | -0.221*** |
| | | (0.0769) | (0.0590) | (0.0367) |
| Observations | 45034 | 41311 | 41311 | 41311 |
| $R^2$ | 0.036 | 0.040 | 0.200 | 0.204 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| Dem Incumbent x Year | No | No | Yes | Yes |
| State Time Trends | No | No | No | Yes |
| Diff. in Coeffs | | -.117 | -.099 | -.194*** |
| SE Diff. in Coeffs | | (.0778) | (.0658) | (.0374) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.6: Polarization (Lower Chambers Only)

| | (1) Abs(SM Score) | (2) Abs(SM Score) | (3) Abs(SM Score) | (4) Abs(SM Score) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0436* | | | |
| | (0.0188) | | | |
| Remove Unlimited PAC Contributions | | 0.0327 | 0.0221 | 0.0215 |
| | | (0.0259) | (0.0243) | (0.0250) |
| Implement Unlimited PAC Contributions | | -0.0201 | -0.0233 | -0.0241 |
| | | (0.0345) | (0.0295) | (0.0289) |
| Observations | 89645 | 89645 | 89645 | 89491 |
| $R^2$ | 0.227 | 0.228 | 0.234 | 0.240 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.053* | -.045** | -.046** |
| SE Diff. in Coeffs | | (.0242) | (.0169) | (.0156) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.7: Polarization and Term Limit Interactions (Lower Chambers Only)

| | (1) Abs(SM Score) | (2) Abs(SM Score) | (3) Abs(SM Score) | (4) Abs(SM Score) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0374* | | | |
| | (0.0179) | | | |
| Remove Unlimited PAC Contributions | | 0.00997 | 0.0146 | 0.0138 |
| | | (0.0198) | (0.0288) | (0.0294) |
| Remove Unlimited × Term Limit | | 0.0497* | 0.0466 | 0.0490 |
| | | (0.0239) | (0.0381) | (0.0394) |
| Implement Unlimited PAC Contributions | | 0.0383* | -0.00427 | -0.00602 |
| | | (0.0173) | (0.0395) | (0.0406) |
| Implement Unlimited × Term Limit | | -0.0530+ | 0.0134 | 0.0169 |
| | | (0.0307) | (0.0252) | (0.0283) |
| Observations | 89645 | 89645 | 89645 | 89491 |
| $R^2$ | 0.228 | 0.228 | 0.234 | 0.240 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs w/o Term Limits | | .028* | -.019 | -.02 |
| SE Diff. in Coeffs w/o Term Limits | | (.0112) | (.0267) | (.0273) |
| Removed with Term Limit | | .06* | .061 | .063 |
| SE Removed with Term Limit | | (.0275) | (.0481) | (.0499) |
| Implemented with Term Limit | | -.015 | .009 | .011 |
| SE Implemented with Term Limit | | (.036) | (.0475) | (.0493) |
| Diff. in Coeffs w/ Term Limits | | -.074** | -.052** | -.052** |
| SE Diff. in Coeffs w/ Term Limits | | (.025) | (.0185) | (.0173) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.8: Incumbent Reelection Dynamic Effects

| | (1) Incumbent Wins | (2) Incumbent Wins |
|---|---|---|
| Remove Unlimited This Term | 0.0265 | 0.0265 |
| | (0.0253) | (0.0255) |
| Implement Unlimited This Term | -0.266*** | -0.294*** |
| | (0.0196) | (0.0470) |
| Implement Unlimited One Period Previous | -0.0327 | -0.0326 |
| | (0.0400) | (0.0273) |
| Implement Unlimited Two Periods Previous | -0.00826 | -0.0341 |
| | (0.0272) | (0.0351) |
| Implement Unlimited Three Periods Previous | -0.00396 | -0.0242 |
| | (0.0245) | (0.0301) |
| Remove Unlimited One Period Previous | 0.0740*** | 0.0635* |
| | (0.0133) | (0.0266) |
| Remove Unlimited Two Periods Previous | -0.0276 | -0.0578+ |
| | (0.0214) | (0.0314) |
| Remove Unlimited Three Periods Previous | 0.00501 | -0.00289 |
| | (0.0146) | (0.0146) |
| Observations | 37101 | 37101 |
| $R^2$ | 0.205 | 0.207 |
| State Fixed Effects | Yes | Yes |
| Year Fixed Effects | Yes | Yes |
| State Time Trends | Yes | Yes |
| Dem Incumbent x Year | No | Yes |

Heteroskedasticity robust standard errors clustered at the state level in parentheses.
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.9: Challenger Polarization

| | (1) Abs(CF Score Dyn) | (2) Abs(CF Score Dyn) | (3) Abs(CF Score Dyn) | (4) Abs(CF Score Dyn) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0390$^{+}$ | | | |
| | (0.0216) | | | |
| Remove Unlimited PAC Contributions | | 0.0119 | 0.00980 | 0.0151 |
| | | (0.0222) | (0.0197) | (0.0214) |
| Implement Unlimited PAC Contributions | | -0.0526 | -0.0389 | -0.0381 |
| | | (0.0455) | (0.0675) | (0.0643) |
| Observations | 39133 | 38875 | 38875 | 38875 |
| $R^2$ | 0.115 | 0.115 | 0.119 | 0.142 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.064 | -.049 | -.053 |
| SE Diff. in Coeffs | | (.0415) | (.0718) | (.0678) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.10: Polarization (Reform This Term)

| | (1) Abs(SM Score) | (2) Abs(SM Score) | (3) Abs(SM Score) |
|---|---|---|---|
| Remove Unlimited This Term | -0.0144 | 0.00323 | 0.00126 |
| | (0.0119) | (0.00798) | (0.00905) |
| Implement Unlimited This Term | -0.0372$^{**}$ | -0.0301$^{**}$ | -0.0326$^{**}$ |
| | (0.0108) | (0.00950) | (0.0103) |
| Observations | 122021 | 122021 | 121709 |
| $R^2$ | 0.216 | 0.222 | 0.229 |
| State Fixed Effects | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes |
| State Time Trends | No | Yes | Yes |
| Dem Incumbent x Year | No | No | Yes |
| Diff. in Coeffs | -.023$^{*}$ | -.033$^{***}$ | -.034$^{***}$ |
| SE Diff. in Coeffs | (.0107) | (.008) | (.0082) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.11: Polarization (Ideology Multiplied by -1 for Dems)

| | (1)<br>Trans.(SM Score) | (2)<br>Trans.(SM Score) | (3)<br>Trans.(SM Score) | (4)<br>Trans.(SM Score) |
|---|---|---|---|---|
| Unlimited PAC Contributions | -0.0314 | | | |
| | (0.0262) | | | |
| Remove Unlimited PAC Contributions | | 0.000958 | 0.00322 | 0.00383 |
| | | (0.0293) | (0.0200) | (0.0208) |
| Implement Unlimited PAC Contributions | | -0.0582 | -0.0453 | -0.0445 |
| | | (0.0433) | (0.0284) | (0.0280) |
| Observations | 122021 | 122021 | 122021 | 121709 |
| $R^2$ | 0.219 | 0.219 | 0.225 | 0.231 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.059$^+$ | -.049* | -.048* |
| SE Diff. in Coeffs | | (.0334) | (.0199) | (.0192) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^+$ $p < 0.10$, $^*$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.12: Polarization (Alternative Coding)

| | (1)<br>Abs(SM Score-0.5) | (2)<br>Abs(SM Score-0.25) | (3)<br>Abs(SM Score+0.25) | (4)<br>Abs(SM Score+0.5) |
|---|---|---|---|---|
| Remove Unlimited PAC Contributions | -0.0201 | -0.00725 | 0.000299 | -0.00651 |
| | (0.0126) | (0.0140) | (0.0158) | (0.0138) |
| Implement Unlimited PAC Contributions | -0.0613** | -0.0505** | -0.0434* | -0.0446* |
| | (0.0176) | (0.0186) | (0.0200) | (0.0187) |
| Observations | 121709 | 121709 | 121709 | 121709 |
| $R^2$ | 0.674 | 0.444 | 0.334 | 0.589 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | Yes | Yes | Yes | Yes |
| Dem Incumbent x Year | Yes | Yes | Yes | Yes |
| Diff. in Coeffs | -.041** | -.043** | -.044** | -.038** |
| SE Diff. in Coeffs | (.0123) | (.0127) | (.0127) | (.0126) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^+$ $p < 0.10$, $^*$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

## The Impact of Party and Individual Contribution Limits

We conduct some preliminary tests of the implications of our theory with the implementation and removal of two additional campaign finance reforms: individual and party contribution limits. Previous work suggests that unlimited party contributions will lead to moderation in legislatures (La Raja and Schaffner 2015) while unlimited individual contributions will cause increased polarization (Barber 2016). We examine the impact of implementing and removing these reforms on legislators' ideological extremism using the same models as in the main text of the paper.

In Table A.13 we show the impact of party contribution limits on the absolute value of legislators' Shor-McCarty scores. Column 1 indicates that the coefficient on the unlimited party contributions variable is not statistically significant. In fact, it is positive which is the opposite direction of what La Raja and Schaffner (2015) find. That being said, we find some evidence in support of our theory in Column 2. Implementing unlimited party contributions causes a marginally significant decrease in polarization. However, these results are not robust to the inclusion of time trends in Column 3 and Column 4. A potential explanation for these weak findings is that the effects of party contribution limits appear to be limited to more professionalized legislatures (La Raja and Schaffner 2015).

Table A.14 displays the results with the implementation and removal of individual contribution limits as the independent variables. In Column 1 we do not find evidence consistent with this theory that unlimited individual contributions will cause an increase in polarization. In fact, the point estimate is in the opposite direction of what is predicted by Barber (2016). As a result, it is not surprising that we do not find support for the expectations derived from our theoretical model in Columns 1 to 3 of Table A.14. There are several potential reasons as to why we do not find support for our theoretical predictions. We use a slightly different sample than Barber (2016) and as shown in Column 1 of Table A.14 find no impact of individual contribution limits in general. We look at both upper and lower legislative chambers while Barber only examines lower chambers. In addition, although Barber (2016) finds that the substantive magnitude of the effects of individual limits is larger than the effect of PAC contribution limits, the impact of individual limits lose their statistical significance when examining alternative dependent variables like the difference in the average ideology of the parties.

Table A.13: Polarization (Party Contribution Limits)

| | (1)<br>Abs(SM Score) | (2)<br>Abs(SM Score) | (3)<br>Abs(SM Score) | (4)<br>Abs(SM Score) |
|---|---|---|---|---|
| Unlimited Party Contributions | 0.00749 | | | |
| | (0.00813) | | | |
| Remove Unlimited Party Contributions | | -0.0159 | -0.00177 | 0.00169 |
| | | (0.0131) | (0.00930) | (0.0101) |
| Implement Unlimited Party Contributions | | -0.0247$^{+}$ | 0.00326 | 0.00651 |
| | | (0.0133) | (0.0151) | (0.0171) |
| Observations | 122021 | 122021 | 122021 | 121709 |
| $R^2$ | 0.216 | 0.216 | 0.222 | 0.229 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.009 | .005 | .005 |
| SE Diff. in Coeffs | | (.0065) | (.0138) | (.0137) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table A.14: Polarization (Individual Contribution Limits)

| | (1) Abs(SM Score) | (2) Abs(SM Score) | (3) Abs(SM Score) | (4) Abs(SM Score) |
|---|---|---|---|---|
| Unlimited Indiv. Contributions | -0.0248 | | | |
| | (0.0277) | | | |
| Remove Unlimited Indiv. Contributions | | -0.0121 | 0.00223 | 0.000997 |
| | | (0.0228) | (0.0181) | (0.0195) |
| Implement Unlimited Indiv. Contributions | | -0.0659$^{+}$ | -0.0263 | -0.0264 |
| | | (0.0393) | (0.0218) | (0.0225) |
| Observations | 122021 | 122021 | 122021 | 121709 |
| $R^2$ | 0.216 | 0.216 | 0.222 | 0.229 |
| State Fixed Effects | Yes | Yes | Yes | Yes |
| Year Fixed Effects | Yes | Yes | Yes | Yes |
| State Time Trends | No | No | Yes | Yes |
| Dem Incumbent x Year | No | No | No | Yes |
| Diff. in Coeffs | | -.054 | -.029 | -.027 |
| SE Diff. in Coeffs | | (.0331) | (.0185) | (.0181) |

Heteroskedasticity robust standard errors clustered at the state level in parentheses
$^{+}$ $p < 0.10$, $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

## Counterfactual Details

The counterfactuals are determined by estimating a series of regressions and using predicted values from the regressions to form polarization levels and incumbent reelection rates under the alternative scenarios for the average state year in the sample.

The period 0 incumbent share is 1 by construction. The period 1 incumbent share is determined by first estimating the linear probability model:

$$\begin{aligned} IncWon_{lst} = \alpha_s + \gamma_t + \beta_1 ImpPACUnlimit_{st} + \beta_2 RemPACUnlimit_{st} \\ + \delta_1 AnyTermLimit_{st} + \delta_2 AnyTermLimit_{st} \times RemPACUnlimit_{st} \\ + \delta_3 AnyTermLimit_{st} \times ImpPACUnlimit_{st} + \epsilon_{lst} \end{aligned}$$

The above regression is estimated on the full set of incumbents so incumbents who choose not to run for reelection or are defeated in the primary are included in the sample. We also include an indicator for the presence of term limits and interactions of this indicator with the campaign finance regulation treatments in order to estimate the incumbent reelection rate when incumbents are not constrained from running due to term limit restrictions. The period 1 incumbent share in the case where the reform is implemented is then $IncShareImp(t = 1) = \bar{\hat{\alpha}}_s + \bar{\hat{\gamma}}_t + \hat{\beta}_1$ and the period 1 incumbent share in the case where the reform is removed is $IncShareRem(t = 1) = \bar{\hat{\alpha}}_s + \bar{\hat{\gamma}}_t + \hat{\beta}_2$. Instead of separately estimating this quantity in each election period we impose the restriction that the reelection outcomes are independent across periods so that after $t$ elections the incumbent shares are $IncShareRem(t) = (IncShareRem(t = 1))^t$ and $IncShareImp(t) = (IncShareImp(t = 1))^t$. In our term limit counterfactuals, we assume that there is a two-term limit and that 1/2 of the legislators are term limited each election. Then in period 1 we have $IncShareImpTL(t = 1) = (IncShareImp * 1/2)$, in period 2 $IncShareImpTL(t = 2) = (IncShareImp * 1/2)^2$, and in election periods 3 and greater we have $IncShareImpTL = IncShareRemTL = 0$.

The baseline level of polarization in election period 0 is determined by first estimating the regression on the sample of first term legislators:

$$AbsSMScore_{lst} = \alpha_s + \gamma_t + \beta_1 ImpPACUnlimit_{st} + \beta_2 RemPACUnlimit_{st} + \epsilon_{lst}$$

The period 0 right party location is then $AbsS\hat{M}Score = \bar{\hat{\alpha}}_s + \bar{\hat{\gamma}}_t$ and the period 0 left party location is then $-AbsS\hat{M}Score$. Period 0 polarization is then $2AbsS\hat{M}Score$.

The long-run new entrant positions are then estimated as: $AbsSM\hat{S}coreImp = \bar{\hat{\alpha}}_s + \bar{\hat{\gamma}}_t + \hat{\beta}_1$ and $AbsSM\hat{S}coreRem = \bar{\hat{\alpha}}_s + \bar{\hat{\gamma}}_t + \hat{\beta}_2$. The expected new entrant

Democratic candidate location is the negative of this quantity. We then combine the expected incumbent share with the expected new entrant ideological locations to calculate the expected level of polarization. Without term limits, the polarization level when the reform is implemented is $IncShareImp(t) * 2AbsS\hat{M}Score + (1 - IncShareImp(t))2AbsSM\hat{S}coreImp$ and when it is removed is $IncShareRem(t) * 2AbsS\hat{M}Score + (1 - IncShareRem(t))2AbsSM\hat{S}coreRem$. For counterfactuals with term limits, we use the incumbent share with term limits calculation described above.

## References

Barber, Michael J. 2016. "Ideological Donors, Contribution Limits, and the Polarization of American Legislatures." *Journal of Politics* 78(1):296–310.

La Raja, Raymond J. and Brian F. Schaffner. 2015. *Campaign Finance and Political Polarization: When Purists Prevail*. University of Michigan Press.