## Online Appendix for Propaganda to Persuade Supplementary Material

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## **Appendix A: Proofs**

*Proof.* Lemma 1. The expected competence of the incumbent autocrat is  $\mu_s$ , which is  $\mu_s - 1/2$  greater than the expected competence of politician from its group. The difference between the incumbent's ideology and her opponent's ideology is d. Based on the trade-off between competence and ideology, the opponent retains the incumbent if the posterior belief about her competence  $\mu_s$  is above a threshold  $\mu_O \equiv d^2 + \frac{1}{2}$ .

*Proof.* Lemma 2. If  $\mu_s \ge \mu_O$ , the opposition keeps the incumbent if the incumbent is kept by the ally. Expecting this, the ally always supports the incumbent.

If  $\mu_s < \mu_O$ , her opponent will place its own candidate. The ally expects to make a payoff of  $E(u_A^0) \equiv \frac{1}{2} + (1 - (1 - e)\rho)(-d^2)$  from replacing the incumbent and a expect payoff of  $E(u_A^1) \equiv \rho\mu_s + (1 - \rho)\frac{1}{2} + (1 - \rho)(-d^2)$  from keeping the incumbent. If  $\mu_s \geq \mu_R \equiv -ed^2 + \frac{1}{2}$ ,  $E(u_A^1) \geq E(u_A^0)$ , and the ally thus supports the incumbent.

*Proof.* Proposition 1. When group *i* holds some belief  $\mu$ , it takes action  $\sigma_i$  according to their optimal decision derived in equation (1) and (2) and the incumbent makes an expected payoff, denoted by  $\hat{v}(\mu)$ , accordingly. An rule for propaganda disclosure  $\pi$  induces a distribution of posterior beliefs, denoted by  $\tau(\mu)$ . The incumbent's payoff from any rule for propaganda disclosure is thus the expectation of  $\hat{v}$  under  $\tau$ . Because the groups update beliefs following a bayesian rule, the expected posterior belief must equal the prior. The incumbent's problem is thus equivalent to choosing  $\tau(\mu)$  to solve the following optimization problem.

$$\max_{\tau} \quad E_{\tau} \hat{v}(\mu)$$
  
s.t. 
$$\sum_{Supp(\tau)} \mu d\tau(\mu) = \mu_0$$

To solve the above problem, first, we derive  $\hat{v}(\mu)$ , i.e. the expected payoff for the incumbent given some belief  $\mu$ . When  $\mu < \mu_A$ , the ally replaces the incumbent and so does

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the opposition. The incumbent is ousted for certain. When  $\mu \in [\mu_A, \mu_O)$ , the ally keeps the incumbent and the opposition ousts the incumbent. When  $\mu \geq \mu_O$ , the ally keeps the incumbent and so does the opponent. The incumbent stays in the office with certainty. In summary, we have

$$\hat{v}(\mu) = \begin{cases}
0 & \text{if } \mu < \mu_A \\
\rho & \text{if } \mu \in [\mu_A, \mu_O) \\
1 & \text{if } \mu \ge \mu_O
\end{cases}$$
(1)

where  $\mu_A \equiv -ed^2 + \frac{1}{2}$  and  $\mu_O \equiv d^2 + \frac{1}{2}$ .

I follow the concave-closure approach developed by Kamenica and Gentzkow (2011) to solve the optimization problem. Let V be the concave closure of  $\hat{v}$ :

$$V(\mu) \equiv \sup\{z | (\mu, z) \in co(\hat{v})\}$$

where  $co(\hat{v})$  denotes the convex hull of the graph of v.

 $V(\mu)$  is the largest payoff the incumbent can achieve with any rule for propaganda disclosure when the prior is  $\mu$ . If  $(\mu, z) \in co(\hat{v})$ , then there exists a distribution of posteriors  $\tau$  such that  $E_{\tau}\mu = \mu'$  and  $E_{\tau}\hat{v}(\mu) = z$ . Thus,  $co(\hat{v})$  is the set of  $(\mu, z)$  such that if the prior is  $\mu$ , there exists a rule for propaganda disclosure with value z. Hence,  $V(\mu)$  is the largest payoff she can achieve with any signal when the prior is  $\mu$ . The concave-closure approach shows that there are two formats of optimal rule for propaganda disclosure. When a certain condition is satisfied, the incumbent ruler chooses one as opposed to the other.

Figure 1 shows the function  $\hat{v}$ , the concave closure V, and the optimal rule for propaganda disclosure when  $\frac{1}{\rho} \leq \frac{\mu_O}{\mu_A}$ .  $\mu$  denotes the probability that  $\theta = 1$ .  $\hat{v}$  is a step function: the incumbent's expected payoff is 0 whenever  $\mu < \mu_A$ ,  $\rho$  whenever  $\mu_A \leq \mu < \mu_O$ , and 1 whenever  $\mu \geq \mu_O$ . As panel C in Figure 1 shows, the signal induces two posterior values:  $\mu^l = 0$  and  $\mu^h = \mu_A$ .

Let the probability that the realized signal induces belief of  $\mu_R$  be  $\alpha$ . Because the distribution  $\tau$  is Bayes plausible, we must have

$$(1-\alpha) \times 0 + \alpha \times \mu_A = \mu^0$$

This implies that  $\alpha = \frac{\mu^0}{\mu_A}$ . Hence, the optimal  $\tau$  is that with probability  $\alpha = \frac{\mu^0}{\mu_A}$  the posterior belief is  $\mu_A$  and with probability  $1 - \alpha = 1 - \frac{\mu^0}{\mu_A}$  the posterior belief is 0. Now, we compute the signal that induces the optimal  $\tau$ . Denote the optimal rule for propaganda disclosure with a realization space  $\{s^-, s^+\}$  by  $\pi^*$ . If the realization is  $s^-$ , the ally replaces the incumbent,  $\sigma_A = 0$ . If the realization is  $s^+$ , the ally retains the incumbent,  $\sigma_A = 1$ , while the opponent replaces the incumbent,  $\sigma_O = 0$ . Let  $\pi_{\theta}^+ = \Pr[s^+|\theta]$ , i.e. the probability that the realized signal is  $s^+$  given the state of the world  $\theta$  and  $\pi_{\theta}^- = \Pr[s^-|\theta]$ , i.e. the probability that the



Figure 1: Design of Optimal Propaganda Disclosure  $\frac{1}{\rho} \leq \frac{\mu_O}{\mu_A}$ 

realized signal is s-given the state of the world  $\theta$ . We have

$$\pi_{\theta}^{+} = \begin{cases} 1 & \text{if } \theta = 1\\ \frac{\mu^{0}}{1-\mu^{0}} \frac{1-\mu_{A}}{\mu_{A}} & \text{if } \theta = 0 \end{cases}$$
(2)

and  $\pi_{\theta}^{-} = 1 - \pi_{\theta}^{+}$ .

Similarly, we could derive the optimal rule for propaganda disclosure in panel C Figure 2. Notice that when  $\frac{\rho}{\mu_A} = \frac{1}{\mu_O}$ . *R* is indifferent to the following rule for propaganda disclosures. The rule for propaganda disclosure induces posteriors which are 0,  $\mu_A$ , and  $\mu_O$ . The probability combination  $(1 - \alpha_A - \alpha_O, \alpha_A, \alpha_O)$  over the above posterior combination must satisfy the following Bayesian plausible requirement

$$(1 - \alpha_A - \alpha_O) \times 0 + \alpha_A \times \mu_A + \alpha_O \times \mu_O = \mu^0$$

where  $\alpha_A \in [0, 1]$  and  $\alpha_O \in [0, 1]$ . To simplify the discussion without loss of generality, I assume that among all the indifferent rules for propaganda disclosures, the incumbent chooses the one which assigns 0 probability to the posterior  $\alpha_O$ . Denote the optimal rule for propaganda disclosure with a realization space  $\{s^-, s^{++}\}$  by  $\pi^{**}$ . If the signal realization is  $s^-$ , the ally replaces the incumbent,  $\sigma_A = 0$ . If the signal realization is  $s^{++}$ , the ally retains the incumbent,  $\sigma_A = 1$ , and the opponent retains the incumbent,  $\sigma_O = 1$ . Let  $\pi_{\theta}^{++} = \Pr[s^{++}|\theta]$  and  $\pi_{\theta}^- = \Pr[s^-|\theta]$ . We have

$$\pi_{\theta}^{++} = \begin{cases} 1 & \text{if } \theta = 1 \\ \frac{\mu^0}{1-\mu^0} \frac{1-\mu_0}{\mu_0} & \text{if } \theta = 0 \end{cases}$$
(3)

and  $\pi_{\theta}^{-} = 1 - \pi_{\theta}^{++}$ .

## **Appendix B: Commitment**

The incumbent commits to a rule for propaganda disclosure by designing bureaucracies that gather and distribute information. First, I build a selection model as a micro foundation for the commitment assumption in the main model. Then I discuss how autocratic leaders in Maoist China and the Soviet Union designed bureaucracies to implement the rule for propaganda disclosure.



Figure 2: Design of Optimal Propaganda Disclosure  $\frac{1}{\rho} > \frac{\mu_0}{\mu_A}$ 

#### **Micro Foundation**

Consider that the incumbent wants to commit to a rule for propaganda disclosure such that with probability q that her incompetence will be communicated as propaganda. To implement such rule for propaganda disclosure, the autocrat could staff the bureaucracy such that 1-q proportion of the bureaucrats are honest and the rest q proportion corrupted. Both honest and corrupted bureaucrats generate propaganda when the competence is high. When the competence is low, an honest bureaucrat generates an unfavorable message while a corrupted bureaucrat generates propaganda. A random bureaucrat is picked and his/her message is the message distributed by the bureaucracy.

#### The Maoist China Case

In the late 1950s, Mao, the then leadership of the Chinese government, adopted the Great Leap Forward policy. This policy caused one of the greatest famines in human history. However, when the famine was spreading over the country, reports that demonstrate the effectiveness of the Great Leap Forward Policy were abundant. Mao didn't not directly engaging in gathering and reporting information about the outcome of the Great leap forward policy. Instead, Mao delegated to the statistical report system and the local governments. By implementing structure features in the statistical report system and local governments, Mao shaped the rule for propaganda disclosure. Mao advocated that politics should take command over the statistical report system. Data collected by party cadres assisted by the masses were supposed to be more accurate than the bureaucrats in the statistical system. The result was a gross exaggeration of production figures in 1958 and the breakdown of the statistical reporting for several years(Banister, 1991). Besides, Mao waged political campaigns to cultivate low-level officials' radical ideology and to shape their career incentives accordingly. Motivated by radical ideology and career incentives, lower-level officials tended to over-report grain production (Kung and Chen, 2011).

#### The Soviet Union Case<sup>1</sup>

The communist party in the Soviet Union exercised control over the media through its propaganda department. The propaganda department decided the appointment of the chief editors in news agencies. The chief editor was the key decision-maker in the editorial

<sup>&</sup>lt;sup>1</sup>See McNair (2006), for example, for studies on Soviet media.

board which exercised the daily decision over what information to gather and how to report it. Chief editors who were loyal to the communist party used their newspaper as a platform for communist propaganda. Influenced by professionalism, some chief editors made decisions according to true journalism. By choosing the composition of chief editors, the party affects the rule for propaganda disclosure.

# Appendix C: Results if $\mu^0 > -ed^2 + \frac{1}{2}$ .

If the prior  $\mu^0 > d^2 + 1/2$ , both groups support the incumbent. The incumbent babbles in equilibrium. If the prior  $-ed^2 + 1/2 < \mu^0 \le d^2 + 1/2$ , the incumbent chooses between the following rules for propaganda disclosure. The incumbent could babble which leads to the support from her ally. With babbling, the incumbent stays in the office with probability  $\rho$ . The incumbent could also use propaganda to persuade the opponent. With such rule for propaganda disclosure, the incumbent stays in the office with probability  $\frac{\mu^0}{\mu_0}$ . If  $\rho > \frac{\mu^0}{\mu_0}$ , the incumbent babbles; otherwise, the incumbent designs a rule for propaganda disclosure to persuade the opposition. I summarize the results in the following proposition.

to persuade the opposition. I summarize the results in the following proposition. **Proposition 1.** If  $\rho > \frac{\mu^0}{\mu_O}$  or  $\mu^0 > d^2 + 1/2$ , the optimal rule for propaganda disclosure is babbling. If  $\rho \leq \frac{\mu^0}{\mu_O}$  and  $-ed^2 + 1/2 < \mu^0 \leq d^2 + 1/2$ , the optimal rule for propaganda disclosure  $\pi_1^{++}$  has support on  $\{s^-, s^{++}\}$ , where given realization  $s^-$ ,  $\sigma_R = 0$  and  $\sigma_O = 0$ and given realization  $s^{++}$ ,  $\sigma_R = 1$  and  $\sigma_O = 1$ . Let  $\pi_{\theta}^{++} \equiv \Pr[s^{++}|\theta]$ , then

$$\pi_{\theta}^{++} = \begin{cases} 1 & \text{if } \theta = 1\\ \frac{\mu^0}{1-\mu^0} \frac{1-\mu_O}{\mu_O} & \text{if } \theta = 0. \end{cases}$$
(4)

 $\mu_O = d^2 + \frac{1}{2}.$ 

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