# Modeling Related Processes with an Excess of Zeros: 

## Supplementary Information

## SI-1. JAGS MODEL

The JAGS model follows the paper's specification, and is quite general. It can handle any number of predictors, any number of dimensions, and any number of cut-offs within the different dimensions. It assumes that there is some observation-level data that can predict participation, and once participation is attained, it predicts the level of participation on all $d$ dimensions. The estimated variance-covariance matrix can be used to determine the correlations between dimensions.

```
model{
    #y is a matrix of outcomes of dimension NxD
    #N is the number of observations
    #D is the number of dimensions
    #Z is the matrix of first-stage predictors of dimension NxK
    #K is the number of first-stage predictors, including an intercept
    #X is an array of predictors for the second stage
    #n.cut is a scalar, the number of cut points
    #c is a vector, the second cut point
    #R is the inverse-Wishart prior matrix
    for(i in 1:N){
        #if outcomes have different number of levels, change n.cut to a vector
        #first stage
        S[i] <- max(min(S.star[i],1),0)
```

```
    probit(S.star[i]) <- s.star[i] #participation regime probability
    s.star[i] <- inprod(Z[i,], gamma[1:K])
    #second stage
    for(d in 1:D){
        Xnew[d,i] <- inprod(X[i,,d], beta[1:J,d])
    }
    y.star[i, 1:D] ~ dmnorm(Xnew[,i], invW[,])
    for(d in 1:D){
        for (i.cut in 1:n.cut){
        probit(Q[i,i.cut,d]) <- tau.unsorted[i.cut, d] - y.star[i,d]
        }
    }
    for(d in 1:D){
        p[i,d,1] <- (1-S[i]) + S[i]*max(min(Q[i,1,d],1),0)
        for(i.cut in 2:n.cut){
            p[i,d,i.cut] <- S[i]*(max(min(Q[i,i.cut,d],1),0) - max(min(Q[i,i.cut-1,d],1)
        }
        p[i,d,n.cut+1] <- S[i]*(1 - max(min(Q[i,2,d],1),0))
        y[i,d] ~ dcat(p[i,d,])
    }
}
for(j in 1:J){
    for(d in 1:D){
        beta[j,d] ~ dnorm(0,0.0001)
    }
}
for(k in 1:K){
    gamma[k] ~ dnorm(0,0.0001)
}
for(d in 1:D){
    tau.unsorted[1,d] <- 0
    tau.unsorted[2,d] <- c[d]
    #uncomment below for more than three outcomes
    #for(i.cut in 3:n.cut){
    # tau.unsorted[i.cut,d] <- tau[i.cut, d]
    # tau[i.cut, d] ~ dlnorm(0,0.025)
    #}
}
#can change D+1 to be anything greater than or equal to D
    #can pass this in as a parameter
invW[1:D,1:D] ~ dwish(R[,],D+1)
```

```
    Sigma[1:D,1:D] <- inverse(invW[,])
    for(d in 1:D){
    vars[d] <- Sigma[d,d]
    }
    for(l in 1:(D-1)){
    for(q in l:(D-1)){
        rho[q+1-1] <- Sigma[l,q+1]/sqrt(vars[l]*vars[q+1])
    }
    }
}
```


## SI-2. SIMULATION EXERCISE

Table SI-1 shows the true parameters for the data generating process for the first round of simulations. Table SI-2 shows the true parameters for the data generating process for the second round of simulations.

Table SI-1: True parameters for the first round of simulations.

| Simulation | $\gamma_{0}$ | $\gamma_{1}$ | $\beta_{10}$ | $\beta_{11}$ | $\beta_{20}$ | $\beta_{21}$ | $\beta_{30}$ | $\beta_{31}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1.5 | 4 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 2 | -1.5 | 5 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 3 | -1.5 | 6 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 4 | -1.5 | 7 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 5 | -1.5 | 8 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 6 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 7 | -1.5 | 10 | 0 | 2 | 0 | 2.5 | 0 | 2 |
| 8 | -1.5 | 11 | 0 | 2 | 0 | 2.5 | 0 | 2 |

Note: Each simulation was repeated 100 times. All correlation coefficients were set to zero. The first-stage parameters are in the second and third columns. The second-stage parameters are in the latter six columns. The subscripts on these parameters denote first the dimension, and second the indicator for the parameter. Zeros refer to intercepts, and ones refer to the variable of interest.

Table SI-2: True parameters for the second round of simulations.

| Simulation | $\gamma_{0}$ | $\gamma_{1}$ | $\beta_{10}$ | $\beta_{11}$ | $\beta_{20}$ | $\beta_{21}$ | $\beta_{30}$ | $\beta_{31}$ | $\rho_{12}$ | $\rho_{13}$ | $\rho_{23}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.8 | 0 | 0 |
| 2 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.6 | 0 | 0 |
| 3 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.4 | 0 | 0 |
| 4 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.2 | 0 | 0 |
| 5 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | 0 | 0 | 0 |
| 6 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .2 | 0 | 0 |
| 7 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .4 | 0 | 0 |
| 8 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .6 | 0 | 0 |
| 9 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .8 | 0 | 0 |
| 10 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.8 | .64 | -.51 |
| 11 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.6 | .36 | -.22 |
| 12 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.4 | .16 | -.06 |
| 13 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.2 | .04 | -.01 |
| 14 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | 0 | 0 | 0 |
| 15 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .2 | .04 | .01 |
| 16 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .4 | .16 | .06 |
| 17 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .6 | .36 | .22 |
| 18 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .8 | .64 | .51 |
| 19 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.8 | -.8 | .64 |
| 20 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.6 | -.6 | .36 |
| 21 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.4 | -.4 | .16 |
| 22 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | -.2 | -.2 | .04 |
| 23 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | 0 | 0 | 0 |
| 24 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .2 | .2 | .04 |
| 25 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .4 | .4 | .16 |
| 26 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .6 | .6 | .36 |
| 27 | -1.5 | 9 | 0 | 2 | 0 | 2.5 | 0 | 2 | .8 | .8 | .64 |

Note: Each simulation was repeated 100 times. The first-stage parameters are in the second and third columns. The second-stage parameters are in the fourth through tenth columns. For the first- and second-stage parameters, the subscripts denote first the dimension, and second the indicator for the parameter. Zeros refer to intercepts, and ones refer to the variable of interest. The final columns are the correlation coefficients of the second stage. The subscripts denote the dimensions (e.g. a subscript of 12 refers to the correlation between the residuals of the first and second dimensions). The first nine set the second and third correlations, $\rho_{13}$ and $\rho_{23}$, to zero, and $\rho_{12}$ varies from -0.8 to 0.8 by 0.2 . The second nine keep the same $\rho_{12}$ shift, setting $\rho_{13}$ to $\rho_{12}^{2}$ and $\rho_{23}$ to $\rho_{12}^{3}$. The final nine again maintain the same $\rho_{12}$ shift and set $\rho_{13}$ to $\rho_{12}$ and $\rho_{23}=\rho_{12}^{2}$. This choice stemmed partly from the need to generate positive definite matrices.

## SI-3. PRESIDENTIAL CAMPAIGNS IN MEXICO

To better assess how our inferences would be changed if the main application in the paper did not allow for correlations, I present a model that restricts the correlation parameters to be zero. This is similar to the simulation exercises comparing ZIMVOP to an uncorrelated multivariate probit. The first-step estimates are nearly identical between models, but the second-step estimates vary fairly substantially, with some estimates reliable in one model and not in the other. A main parameter of interest, Previous vote, though not reliable at the $95 \%$ level for the PRI, is not even reliable at the $90 \%$ in the second model. The results, presented next to each other, are shown in Figure SI-1. Not only would we be losing the information provided by the correlation estimates, the simulations suggest that we should trust the posteriors from ZIMVOP.

Figure SI-1: Comparing ZIMVOP to a model without correlations on the main application.


Note: The second-step results from ZIMVOP are presented on the left panel, and the results not allowing correlations is on the right panel. As can be seen, the posteriors are fairly different. Not only would we be losing the information provided by the correlation estimates, the simulations suggest that we should trust the posteriors from ZIMVOP.

