Appendix

# I. Descriptive Statistics of the Variables

To measure citizens’ freedom to discuss political issues and formulate opinions, we rely on the V-Dem’s “Freedom of expression index” (v2x\_freexp\_thick), which combines the freedom of discussion and academic expression, no print, broadcast, and internet censorship, no media self-censorship, and no harassment of journalists.

To measure citizens’ access to alternative sources of information, we rely on V-Dem’s “Alternative source information index” (v2xme\_altinf), which measures the extent to which there is a media bias against the opposition, whether media criticize the government, and whether media represent a wide range of political perspectives.

To capture the quality of competitive elections, we rely on V-Dem’s “Electoral component index” (v2x\_EDcomp\_thick). Consistent with the features of polyarchy that focus on the electoral mechanism, this index combines the following elements: whether suffrage is extensive; political and civil society organizations can operate freely; elections are clean and not marred by fraud; and the chief executive is selected through elections.

We here describe these variables in turn.

## Freedom of Expression Index (v2x\_freexp\_thick)

Table 1. Descriptive statistics of the Freedom of expression index in both continuous and categorized formats.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ***Valid N*** | ***Mean*** | ***Minimum*** | ***Maximum*** | ***Std.Dev.*** |
| ***v2x\_freexp\_thick*** | 15941 | 0.492 | 0.022 | 0.992 | 0.302 |
| ***v2x\_freexp5C*** | 15969 | 0.487 | 0 | 1 | 0.365 |



Figure 1. Histograms of the Freedom of expression index in both continuous and categorized formats.

## Alternative Source Information Index (v2xme\_altinf)

Table 2. Descriptive statistics of the Alternative source information index in both continuous and categorized formats.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Valid N** | **Mean** | **Minimum** | **Maximum** | **Std.Dev.** |
| **v2xme\_altinf** | 15986 | 0.493 | 0.033 | 0.989 | 0.305 |
| **v2xme\_altinf5C** | 15986 | 0.490 | 0 | 1 | 0.381 |



Figure 2. Histograms of the Alternative source information index in both continuous and categorized formats.

## Electoral Component Index (v2x\_EDcomp\_thick)

Table 3. Descriptive statistics of the Electoral component index in both continuous and categorized formats.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Valid N** | **Mean** | **Minimum** | **Maximum** | **Std.Dev.** |
| **v2x\_EDcomp\_thick** | 16007 | 0.343 | 0.008 | 0.970 | 0.292 |
| **v2x\_EDcomp\_thick5C** | 16007 | 0.318 | 0 | 1 | 0.351 |



Figure 3. Histograms of the Electoral component index in both continuous and categorized formats.

# II. Illustrative Results

We here present further analyses of our illustrative application using dependency tables and Bayesian dynamical systems.

## Dependency Tables

Table 4 below exemplifies the resulting type of aggregate summary of some 2,000 individual analyses following the dependency tables approach outlined above, over 22 variables included in the V-Dem indices for electoral and liberal democracy. Here we present the number of requisite conditions for each of these 22 variables reaching their highest state (the top category).

One should naturally not draw any strong conclusions from small differences in the number of dependencies (requisite conditions), found in such a table. But we can be more certain about sequential relationships (not causality) from large differences. For example, several indicators of civil liberties have very few dependencies. Inspecting which these dependencies are (source data available upon request), we can confirm that they consist exclusively of conditions requiring only one of the lowest levels on other variables. They are thus “weak” dependencies.

**Table 4**. Number of conditions required to the reach highest state (Category 5).

|  |  |
| --- | --- |
|  | **# Requisite conditions****(max = 188)\*** |
| Share with suffrage | 14 |
| Freedom of movement for men | 20 |
| Election voter registry | 24 |
| Property rights for men | 26 |
| Government intimidation | 26 |
| Freedom of movement for women | 30 |
| Party ban | 31 |
| Barriers to parties | 37 |
| Property rights for women | 47 |
| Opposition parties autonomy | 48 |
| Media self-censorship | 50 |
| Civil society organizations repression | 56 |
| Civil society organizations entry and exit | 56 |
| Media bias | 57 |
| Legislature opposition parties | 58 |
| EMB autonomy | 60 |
| Compliance with judiciary | 63 |
| High court independence | 67 |
| Harassment of journalists | 69 |
| Access to justice for men | 71 |
| Access to justice for women | 71 |
| Executive oversight | 72 |

\*For this example, we have included 21 variables, summarizing to a total of 84 if all variables were observed in all instances being 4 when the target variable was 4. The sum is 83 as some of the variables had other than 4 states. We here present only a subsection of our variable choice for illustrative purposes.

On the other hand, we note that several variables related to rule of law and oversight of the executive, as well as government influence on the media, have a very high number of dependencies. This indicates that not only are they dependent on many other variables before they can reach high (more democratic) levels, but also that for many such variables, they must reach high levels first, before the executive oversight, high court independence, or government media censorship can become really democratic.

The table above is illustrative only. It builds on an actual description of V-Dem data but should not be taken as a substantive result. We will use the latest version of the data (v6) produced with a new and more reliable measurement model and additional data to do that, and we also need to run several tests of robustness and consistency checks before we can speak of results. Nonetheless, the table above illustrates the kind of nuanced and detailed sequences that can be established empirically with the method proposed here.

## Bayesian Dynamical Systems

The dynamical system with the largest Bayes’ factor allowing for a specified number of terms can be computed using the R package *bdynsys* (see Ranganathan et al. 2014). Using continuous versions of the variables, the computed model with the largest Bayes’ factor (and thus suggested to be the best fit to the data) when allowing for up to four terms in an equation for the variables Freedom of expression (*x*), Alternative source information (*y*) and Electoral component index (*z*) is:

The equations quantify the direction and rate of change for the variables *x*, *y*, and *z*, respectively, depending on current values of *x*, *y*, and *z*. There is an interaction between all the variables, since all of them occur in all equations. For fixed values of *x* and *y*, the rate of change of *y* is linear to the value of *z*, and *x* is close to it, with an added cubic term. The rate that is amplified is positive for *y* unless *y* is much larger than *x*, that is, alternative source information cannot remain at high values when there is little freedom of expression. The equation for *dx* is a bit more complicated, but is positive as long as not both *x* and *y* are large. This means that for freedom of expression to increase when already high, the electoral component index also needs to be high.

Finally, *z* can grow on its own merits up to 0.28 (since the polynomial in the parenthesis is positive up until that point), after which the variable becomes self-limiting. Thus, for the electoral component index to grow beyond a third, both freedom of expression and alternative source information are needed.

The dynamical system largely verifies and quantifies our previous hints from the transition graphs: for most of the transitions process, the Electoral component index is dependent on the other two variables before it can grow, and more so than in the other direction, except at the onset of the transition process (potentially up to one third), where the Electoral component index grows first. The first two equations do suggest that Freedom of expression and Alternative source information have an intertwined transition process, as before, but also that some electoral rights must be implemented before the variables can reach their highest values.



**Figure 4**. Phase portrait showing the expected direction of change given different values of Freedom of expression and Alternative source information. Longer arrows indicate more rapid change. The figure also includes the actual trajectories of six arbitrarily chosen countries.

Looking at systems of only two variables at a time enables us to inspect changes graphically in phase portraits, as in Figure 4. The phase portrait shows arrows over the domain of values that have historically been taken on by the two variables. For any point in this domain, the closest arrow shows the predicted direction of change for the pair of variables, and the relative length of the arrow depicts the relative rate of change. Given any starting point, we can follow the arrows to find the predicted trajectory of transitions from there. As a comparison, the actual historical trajectories of six arbitrarily chosen countries are also depicted in the figure, where the dot marks the first year, and the curves stretch as far as we have data.

Freedom of expression and alternative source information grow largely together. However, while the latter can still grow when the former is small, the former will rarely grow as long as it is larger than the latter. Starting at a point where the variables take on equals values, similar to the transition graphs, the variables are predicted to keep on growing together, but will often end up slightly below their maximum value. Contrasting to assumptions in Figure 1 (main document) of convex or concave growth, the model also predicts S-curves if Freedom of expression is larger than Alternative source information, but the frequency tables reveal that this happens rarely.

Figure 5 shows the phase portrait for Freedom of expression and the Electoral component index. The system including Alternative source information instead of Freedom of expression is similar. Similar to the transition graph, for the main part of the transition process, the variables grow together, roughly linearly, where Freedom of expression has a higher rate of change in absolute values. Following the arrows close to the origin, we can see that the Electoral component index grows most at first, after which freedom of expression will mainly develop. In cases where the electoral index is larger than freedom of expression, not only is freedom of expression predicted to grow, but the electoral index is also predicted to decrease towards having a slightly lower value than freedom of expression, before it can grow again (arrows in the upper triangle point both right and down). The electoral index is thus more restricted by freedom of expression, which is mostly not predicted to decrease (arrows pointing left) except when being almost at maximum value, than vice versa.

Again, the assumptions in Figure 1 (main document) are not generally satisfied, but states off the main path of transition, where Freedom of expression is slightly larger than the Electoral component index, are rare, and are predicted to transition rapidly into states along the main path.



**Figure 5**. Phase portrait showing the expected direction of change given different values of Freedom of expression and Electoral component index. Longer arrows indicate more rapid change. The figure also includes the actual trajectories of six arbitrarily chosen countries.