The Partisan Politics of Counterterrorism: Reputations, Policy Transparency, and Electoral Outcomes

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Appendix

In this Appendix I will provide the proof of the results regarding the case of a *right-wing incumbent* (and hence a left-wing challenger). The proofs of the results regarding the case of a left-wing incumbents follow the exact same logic of the proofs provided below, and for the sake of completeness can be found in Appendix B.

Definition of Equilibrium

An equilibrium in the game is given by a tuple $\{\tilde{x}^*(\tilde{\theta}, \omega), x^*(\theta, \omega), r^*(x, T), \mu^*_{x,T}\}$ where, for the case of a right-wing incumbent, we have:

1. Optimal second-period choice of all politicians:

   - $\forall \tilde{\theta} \in \{l, h\}, \tilde{x}^*(\tilde{\theta}, un) \in \underset{x \in \{m,a\}}{\text{argmax}} \; -c \cdot \mathbb{1}_{\{x=a\}} - \alpha t(\theta, x)$
   - $\forall \tilde{\theta} \in \{l, h\}, \tilde{x}^*(\tilde{\theta}, ba) = a$
   - $\forall \tilde{\theta} \in \{l, h\}, \tilde{x}^*(\tilde{\theta}, bm) = m$

2. Optimal first-period choice of the Incumbent:

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- $\forall \theta \in \{l, h\}$,

$$x^*(\theta, un) \in \underset{x \in \{m,a\}}{\text{argmax}} \; -c \cdot \mathbb{1}_{\{x=a\}} - \alpha t(\theta, x) + \mathcal{B}\Big[\sum_{T \in \{0,1\}} r^*(x,T)\Big(\mathbb{1}_{\{T=0\}}(1-t(\theta,x)\Big) + \mathbb{1}_{\{T=1\}} t(\theta,x))\Big]$$

- $\forall \theta \in \{l, h\}, x^*(\theta, ba) = a$
- $\forall \theta \in \{l, h\}, x^*(\theta, bm) = m$

3. Optimal electoral choice of the Voter:

   $\forall (x, T) \in \{m, a\} \times \{0, 1\}$,

$$r^*(x,T) \in \underset{r(x,T) \in \{0,1\}}{\text{argmax}} \; -c \cdot \mathbb{1}_{\{x=a\}} - \alpha \cdot \mathbb{1}_{\{T=1\}} + r(x,T)\Big[\mu^*_{x,T} \cdot \gamma_{ba} + (1-\mu^*_{x,T})\gamma_{un}\Big] + \Big(1 - r(x,T)\Big)\Big[(1-p)\gamma_{bm} + p \cdot \gamma_{un}\Big]$$

4. Posterior beliefs $\mu^*_{x,T}$ satisfy Bayes' rule on the path of play, and off-the-path beliefs satisfy the Intuitive Criterion

Proofs of Main Results

To prove the results in the texts, let us first introduce the following Lemmata.

Lemma 1 *There does not exists a pooling equilibrium.*

Proof of Lemma 1: Suppose, by contradiction, there is a pooling equilibrium. Since the hawkish type always chooses $x = a$, $\forall \theta$ (she de facto has a dominant strategy to choose $x = a$), the only possible pooling equilibrium is the one in which both hawkish and unbiased types choose $x = a$, $\forall \theta$. On the path of play, the voter posterior assessment of the incumbent's type is equal to his prior assessment. As such, upon observing $x = a$, the voter reelects the incumbent if

$$(1-p)(\gamma_{ba} - \gamma_{bm}) \geq 0,$$

which is true whenever $\gamma_{ba} - \gamma_{bm} \geq 0$. We have that

$$\begin{aligned}\gamma_{ba} - \gamma_{bm} &= (1-\pi)(-c+\alpha t(h,a)) + \pi(-c+\alpha t(l,a)) - (1-\pi)(-\alpha t(h,m)) - \pi(-\alpha t(l,m)) \\ &= (1-\pi)(-c+\alpha\Delta^h) + \pi(-c+\alpha\Delta^l) \\ &= (-c+\alpha\Delta^h) - \pi\alpha(\Delta^h - \Delta^l)\end{aligned} \tag{1}$$

which is non negative whenever

$$\pi \leq \frac{-c+\Delta^h}{\alpha(\Delta^h - \Delta^l)} = \overline{\pi}_P.$$

If $\pi \leq \overline{\pi}_P$, the incumbent will be reelected, otherwise the voter will elect the challenger.

Upon seeing $x = m$, it has to be the case that $Pr(\omega = ba, \theta = l | x = m) > 0$. If it were zero, then the unbiased type facing a low threat would have an incentive to deviate to $x = m$, thereby obtaining reelection and choosing her preferred policy. Moreover, it has to be the case that the off-the-path beliefs about the incumbent being unbiased after observing $x = m$, denoted by ϵ, satisfy $\epsilon < p$. If this was not true, then from a reelection standpoint, choosing $x = m$ would be weakly better than choosing $x = a$, and it would be strictly better for a set of values of π. In fact, upon observing $x = m$ the voter reelects the incumbent if

$$(\epsilon - p)\gamma_{un} + (1-\epsilon)\gamma_{ba} - (1-p)\gamma_{bm} \geq 0.$$

Simple algebra shows that this is true whenever

$$\pi \leq \frac{(1-p)(-c+\alpha\Delta^h)}{(1-p)(-c+\alpha\Delta^h) + (1-\epsilon)(c-\alpha\Delta^l)} = \overline{\pi}'_P.$$

If $\pi \leq \overline{\pi}'_P$, the incumbent will be reelected, otherwise the voter will elect the challenger.

It is easy to show that $\overline{\pi}_P > \overline{\pi}'_P$ whenever $p > \epsilon$. However, choosing $x = m$ is equilibrium dominated for $\omega = ba$, since it translates to a weakly worse electoral prospect and is the least preferred policy for a hawkish type. As such, the voter should attach probability zero to $\omega = ba$ after observing $x = m$, a contradiction. A similar argument establishes that there cannot be a pooling equilibrium in which all types choose $x = m$ $\forall\theta$ in the case of a left-wing incumbent. ■

Lemma 2 *There is no equilibrium where:*

(a) an unbiased right-wing incumbent chooses $x = a$ when $\theta = l$

(b) an unbiased left-wing incumbent chooses $x = m$ when $\theta = h$

Proof of Lemma 2: Part (a). Suppose, by contradiction, that there exists an equilibrium where an unbiased right-wing incumbent chooses $x = a$ when $\theta = l$. By Lemma 1, it cannot be that $\omega = un$ chooses $x = a$ $\forall\theta$. Therefore, it has to be that $\omega = un$ chooses $x = m$ when $\theta = h$. By Bayes' rule, upon observing $x = m$, the voter learns that the incumbent is unbiased and therefore reelects him no matter the realization of T. However, this implies that an unbiased incumbent has an incentive to deviate and choose $x = m$ even when $\theta = l$, since this is her preferred policy and it leads to reelection with probability one, thus establishing a contradiction.

Part (b). Suppose, by contradiction, that there exists an equilibrium where an unbiased left-wing incumbent chooses $x = m$ when $\theta = h$. By Lemma 1, it cannot be that $\omega = un$ chooses $x = m$ $\forall\theta$. Therefore, it has to be that $\omega = un$ chooses $x = a$ when $\theta = l$. By Bayes' rule, upon observing $x = a$, the voter learns that the incumbent is unbiased and therefore reelects him no matter the realization of T. However, this implies that an unbiased incumbent has an incentive to deviate and choose $x = a$ even when $\theta = h$, since this is her preferred policy and it leads to reelection with probability one, thus establishing a contradiction. ■

Let us now pin down the posterior beliefs of the voter about the incumbent's views on the security-liberty trade-off in each of the possible remaining strategies for the incumbent.

Lemma 3 *(a) If the unbiased incumbent optimally provides counterterrorism (OP), then the posterior beliefs of the voter satisfy the following:*

$$0 = \mu_{m,0} = \mu_{m,1} < (1-p) < \mu_{a,1} < \mu_{a,0} < 1.$$

(b) If the unbiased incumbent underprovides counterterrorism (UP), then the posterior beliefs of the voter satisfy the following:

$$0 = \mu'_{m,0} = \mu'_{m,1} < (1-p) < \mu'_{a,1} = \mu'_{a,0} = 1.$$

Proof of Lemma 3:

Part (a): Let us start from the posterior beliefs of the voter when the counterterrorism policy chosen by the incumbent politician is revealed and is $x = m$ (i.e. $x = m$). That is, let us pin down $\mu_{m,0}$ and $\mu_{m,1}$. Given that we assumed that a hawkish type always chooses the aggressive counterterrorism policy $x = a$ independently from level of the terrorist threat θ, we have

$$\mu_{m,0} = \mu_{m,1} = 0.$$

Now, consider the posterior beliefs of the voter when the counterterrorism policy chosen by the incumbent politician is revealed and is $x = a$ and a terrorist attack does not happen, i.e. $\mu_{a,0}$. By Bayes' rule, we have that

$$\begin{aligned}\mu_{a,0} &= \frac{(1-p)[(1-\pi)(1-t(h,a))+\pi(1-t(l,a))]}{(1-t(h,a))(1-\pi)+(1-t(l,a))(1-p)\pi} \\ &= \frac{1}{1+\frac{\pi(1-t(h,a))}{(1-\pi)(1-t(h,a))+\pi(1-t(l,a))}\frac{p}{1-p}}\end{aligned} \tag{2}$$

With very similar calculations, we obtain that

$$\mu_{a,1} = \frac{1}{1+\frac{\pi t(h,a)}{(1-\pi)t(h,a)+\pi t(l,a)}\frac{p}{1-p}} \tag{3}$$

We have now obtained the voter's posterior beliefs about the bias of the incumbent after observing the aggressive counterterrorism being enacted both in the case of occurrence of a terrorist attack and in the case of the lack thereof. We need to assess the relative ordering of these two posterior beliefs both with respect to the prior belief and with respect to each other. Simple algebra shows that

$$(1-p) < \mu_{a,1} < \mu_{a,0} < 1.$$

This, combined with the fact that $0 = \mu_{m,0} = \mu_{m,1}$ establishes the result.

Part (b): Let us start from the posterior beliefs of the voter when the counterterrorism policy chosen by the incumbent politician is revealed and is $x = m$. That is, let us pin down $\mu'_{m,0}$ and $\mu'_{m,1}$. Given that we assumed that a hawkish type always chooses the aggressive counterterrorism policy $x = a$ independently from level of the terrorist threat θ, we have

$$\mu'_{m,0} = \mu'_{m,1} = 0.$$

Now, consider the posterior beliefs of the voter when the counterterrorism policy chosen by the incumbent politician is revealed and is $x = a$ and a terrorist attack does not happen, i.e. $\mu_{a,0}$. Given that we assumed that an unbiased type always chooses the moderate counterterrorism policy $x = m$ independently from level of the terrorist threat θ, we have

$$\mu'_{a,0} = \mu'_{a,1} = 1.$$

This, combined with the fact that $0 = \mu'_{m,0} = \mu'_{m,1}$ establishes the result. ∎

Using Lemma 3, let me prove the following results.

Lemma 4 *Assume an unbiased incumbent always chooses the optimal counterterrorism policy given the level of the terrorist threat. Then the voter has the following reelection rule:*

- *if* $\pi > \overline{\pi}_{a,1}$*, he elects* C *unless he observes* $x = m$*;*
- *if* $\overline{\pi}_{a,0} < \pi \leq \overline{\pi}_{a,1}$*, he reelects* I *unless he observes* $(a, 0)$*;*
- *if* $\pi \leq \overline{\pi}_{a,0}$*, he always reelects* I*.*

Proof of Lemma 4:

- Case 1: $(a, 1)$

  We know that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] = (1 - \mu_{a,1} - p)\gamma_{un} + \mu_{a,1}\gamma_{ba} - (1 - p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

  $$\pi \leq \overline{\pi}_{a,1} = \frac{(1-p)(-c + \alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1 - \mu_{a,1} - p)(c - \alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}_{a,1}$.

- Case 2: $(a, 0)$

  We know that $\mathbb{E}[u_V(I)]_{(a,0)} - \mathbb{E}[u_V(C)] = (1 - \mu_{a,0} - p)\gamma_{un} + \mu_{a,0}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,0)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{a,0} = \frac{(1-p)(-c + \alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1 - \mu_{a,0} - p)(c - \alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}_{a,0}$.

- Case 3: (m, T)

  We know that only unbiased incumbents choose a moderate counterterrorism policy. Therefore, upon observing a moderate policy, the voter knows the incumbent is unbiased and therefore reelects her.

Now, we can rank the thresholds that characterize the reelection rule in any scenario observed by the voter. In particular we have

$$\overline{\pi}_{a,1} > \overline{\pi}_{a,0}$$

and so we can easily obtain the result by combining the cases above. ■

Lemma 5 *Assume an unbiased incumbent always chooses the moderate counterterrorism policy regardless of the level of the terrorist threat. Then the voter has the following reelection rule:*

- *if* $\pi > \overline{\pi}'_{a,T}$, *he elects* C *unless he observes* $x = m$*;*

- *if* $\pi \leq \overline{\pi}'_{a,T}$, *he always reelects* I.

Proof of Lemma 5:

- Case 1': $(a, 1)$

  We know that only biased incumbents choose an aggressive counterterrorism policy.

Therefore, upon observing an aggressive policy, the voter knows the incumbent is biased. We know that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] = \gamma_{ba} - p\gamma_{un} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}'_{a,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu'_{a,1} - p)(c - \alpha\Delta^l)]}$$

whereas the voter elects the challenger if $\pi > \overline{\pi}'_{a,1}$.

- Case 2': $(a, 0)$

  We know that only biased incumbents choose an aggressive counterterrorism policy. Therefore, upon observing an aggressive policy, the voter knows the incumbent is biased. Given that the posterior beliefs of the voter are identical to the case of $(a, 1)$, also the lower bound on π for the unconditional reelection of the incumbent is the same as above.

- Case 3': (m, T)

  We know that only unbiased incumbents choose a moderate counterterrorism policy. Therefore, upon observing a moderate policy, the voter knows the incumbent is unbiased and therefore reelects her.

Combining these cases above establishes the result. ■

We can now prove all the results in the text.

Proof of Proposition 1:

Part (a). We focus on the case of a right-wing incumbent. Consider the reelection rule specified in Lemma 4. Let us consider separately each case depending on the value of π.

Case 1 : $\pi > \overline{\pi}_{a,1}$

In this case we have that when the threat level is high ($\theta = h$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h, a) \tag{4}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h, m) + \mathcal{B} \tag{5}$$

Instead, when the threat level is low ($\theta = l$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l, a) \tag{6}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l, m) + \mathcal{B} \tag{7}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(4) - (5) \geq 0 \text{ and } (6) - (7) \leq 0$$

The second inequality is always true, while the first inequality is satisfied if $\Delta^h \geq \frac{c+\mathcal{B}}{\alpha}$.

Case 2 $: \overline{\pi}_{a,0} < \pi \leq \overline{\pi}_{a,1}$

In this case we have that when the threat level is high ($\theta = h$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h, a) + t(h, a)\mathcal{B} \tag{8}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \mathcal{B} \tag{9}$$

Instead, when the threat level is low ($\theta = l$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + t(l,a)\mathcal{B} \tag{10}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \mathcal{B} \tag{11}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(8) - (9) \geq 0 \text{ and } (10) - (11) \leq 0$$

The second inequality is always true, while the first inequality is satisfied if $\Delta^h \geq \frac{c+(1-t(h,a))\mathcal{B}}{\alpha}$.

Case 3 : $\overline{\pi}_{a,0} > \pi$

In this case the incumbent is elected unconditionally and therefore chooses the counterterrorism policy she prefers (i.e. the optimal one for the voter) as stated in the result.

In both cases 1 and 2, we see how the unbiased incumbent will optimally provide counterterrorism if the effectiveness of aggressive policies at curbing the probability of a successful attack is high enough, thus proving the result.

Part (b). Consider the reelection rule specified in Lemma 5. Let us consider separately each case depending on the value of π.

<u>*Case 1* : $\pi > \overline{\pi}'_{a,T}$</u>

In this case, when the threat level is high ($\theta = h$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h, a) \tag{12}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h, m) + \mathcal{B} \tag{13}$$

Instead, when the threat level is low ($\theta = l$), the incumbent's expected utility from choosing $x = a$ is,

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l, a) \tag{14}$$

while her expected utility from choosing $x = m$ is,

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l, m) + \mathcal{B} \tag{15}$$

Now the unbiased incumbent will underprovide counterterrorism iff:

$$(12) - (13) \leq 0 \text{ and } (14) - (15) \leq 0$$

The second inequality is always true, while the first inequality is satisfied if $\Delta^h \leq \frac{c+\mathcal{B}}{\alpha}$.

<u>*Case 2* : $\pi \leq \overline{\pi}'_{a,T}$</u>

In this case we have that the incumbent will be reelected for sure. As a consequence, she will choose her preferred counterterrorism policy, which is the optimal one from the point of view of the voter. As a consequence, a necessary condition for the existence of an equilibrium in which the incumbent underprovides counterterrorism is that $\pi > \overline{\pi}'_{a,T}$, as stated in the result.

In case 1, we see how the unbiased incumbent will underprovide counterterrorism if the effectiveness of aggressive policies at curbing the probability of a terrorist attack under a high threat level is low enough, thus proving the result.

For the second part of the result, it suffices to combine Lemma 1 and Lemma 2. ■

Proof of Proposition 2: The result follows directly from Lemma 3.
To see part (a), start from the case of a right-wing incumbent. Consider that if the unbiased incumbent optimally provides counterterrorism (OP), then

$$0 = \mu_{m,0} = \mu_{m,1},$$

and the same holds if the unbiased incumbent underprovides counterterrorism (UP). Therefore, in the case of a right-wing incumbent a terrorist attack does not affect the voter's posterior beliefs. This proves part (a). A similar logic is used to prove part (b). ■

Proof of Proposition 3: A quick inspection of Lemma 4 and Lemma 5 shows how any increase from one threshold level of π to the next is associated with a smaller number of policy-outcome pairs (x, T) after which the incumbent is granted reelection. This immediately implies that this number is weakly decreasing in π.

For maximal restrictiveness, I need to show that there exist thresholds, one for the optimal provision equilibrium and one for the underprovision equilibrium, such that, if π is above the relevant threshold, then a right-wing incumbent gets reelected only if the voter believes she is unbiased with probability 1. Call these thresholds $\underline{\pi}^{OP}$ and $\underline{\pi}^{UP}$ for the optimal provision equilibrium and one for the underprovision equilibrium, respectively. By Lemma 4 and Lemma 5 these thresholds are respectively

$$\underline{\pi}^{OP} = \overline{\pi}_{a,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu_{a,1}-p)(c-\alpha\Delta^l)]}$$

and

$$\underline{\pi}^{UP} = \overline{\pi}'_{a,T} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu'_{a,1}-p)(c-\alpha\Delta^l)]}.$$

For maximal permissiveness, I need to show that there exist thresholds, one for the optimal provision equilibrium and one for the underprovision equilibrium, such that, if π is below the relevant threshold, then a right-wing incumbent gets reelected no matter what the policy-outcome pair ends up being. Call these thresholds $\overline{\pi}^{OP}$ and $\overline{\pi}^{UP}$ for the optimal provision equilibrium and one for the underprovision equilibrium, respectively. By Lemma 4 and Lemma 5 these thresholds are respectively

$$\overline{\pi}^{OP} = \overline{\pi}_{a,0} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu_{a,0}-p)(c-\alpha\Delta^l)]}$$

and

$$\overline{\pi}^{UP} = \overline{\pi}'_{a,T} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu'_{a,1}-p)(c-\alpha\Delta^l)]}.$$

This concludes the argument. ■

Unobservable Counterterrorism Policies

To avoid confusion, when the policy is not observed by the voter I write $x = \phi$. To prove the results in the extension where counterterrorism policies might remain unobservable, let us first introduce the following Lemmata.

Lemma 6 *(a) If the unbiased incumbent optimally provides counterterrorism (OP), then the posterior beliefs of the voter satisfy the following:*

$$0 = \mu_{m,0} = \mu_{m,1} < \mu_{\phi,1} < (1-p) < \mu_{\phi,0} < \mu_{a,1} < \mu_{a,0} < 1.$$

(b) If the unbiased incumbent underprovides counterterrorism (UP), then the posterior beliefs of the voter satisfy the following:

$$0 = \mu'_{m,0} = \mu'_{m,1} < \mu'_{\phi,1} < (1-p) < \mu'_{\phi,0} < \mu'_{a,1} = \mu'_{a,0} = 1.$$

Proof of Lemma 6:

Part (a): Recall from the proof of Lemma 3 that

$$\mu_{a,0} = \frac{1}{1 + \frac{\pi(1-t(h,a))}{(1-\pi)(1-t(h,a))+\pi(1-t(l,a))}\frac{p}{1-p}}$$

$$\mu_{a,1} = \frac{1}{1 + \frac{\pi t(h,a)}{(1-\pi)t(h,a)+\pi t(l,a)}\frac{p}{1-p}}$$

and

$$\mu_{m,0} = \mu_{m,1} = 0.$$

Additionally, from the proof of Lemma 3 we know that

$$(1-p) < \mu_{a,1} < \mu_{a,0} < 1.$$

We now need to determine the voter's posterior beliefs about the bias of the incumbent when the features of the counterterrorism policy enacted are not revealed before the election stage. Let us start with $\mu_{\phi,0}$. By Bayes' Rule,

$$\mu_{\phi,0} = \frac{\Pr(T=0, x=\phi|\omega=ba)\Pr(\omega=ba)}{Pr(T=0, x=\phi|\omega=ba)\Pr(\omega=ba)+\Pr(T=0, x=\phi|\omega=un)\Pr(\omega=un)}$$

Hence we have

$$\mu_{\phi,0} = \frac{[(1-t(h,a))(1-\pi)+(1-t(l,a))\pi](1-p)}{[(1-t(h,a))(1-\pi)+(1-t(l,a))\pi](1-p)+[(1-t(h,a))(1-\pi)+(1-t(l,m))\pi]p}$$

$$= \frac{1}{1+\frac{(1-t(h,a))(1-\pi)+(1-t(l,m))\pi}{(1-t(h,a))(1-\pi)+(1-t(l,a))\pi}\frac{p}{1-p}} \tag{16}$$

Analogously,

$$\mu_{\phi,1} = \frac{[t(h,a)(1-\pi)+t(l,a)\pi](1-p)}{[t(h,a)(1-\pi)+t(l,a)\pi](1-p)+[t(h,a)(1-\pi)+t(l,m)\pi]p}$$
$$= \frac{1}{1+\frac{t(h,a)(1-\pi)+t(l,m)\pi}{t(h,a)(1-\pi)+t(l,a)\pi}\frac{p}{1-p}} \tag{17}$$

Simple algebra shows that

$$\mu_{\phi,1} < (1-p) < \mu_{\phi,0} < \mu_{a,1} < \mu_{a,0} < 1.$$

This, combined with the fact that $0 = \mu_{m,0} = \mu_{m,1}$ establishes the result.

Part (b): Recall from the proof of Lemma 3 that

$$\mu'_{m,0} = \mu'_{m,1} = 0,$$

and

$$\mu'_{a,0} = \mu'_{a,1} = 1.$$

We need to determine the voter's posterior beliefs about the bias of the incumbent when the counterterrorism policy enacted is not revealed before the election stage. Let us start with $\mu'_{\phi,0}$.

By Bayes' Rule

$$\mu'_{\phi,0} = \frac{\Pr(T=0, x=\phi|\omega=ba)\Pr(\omega=ba)}{\Pr(T=0, x=\phi|\omega=ba)Pr(\omega=ba) + \Pr(T=0, x=\phi|\omega=un)\Pr(\omega=un)}.$$

Hence we have

$$\mu'_{\phi,0} = \frac{[(1-t(h,a))(1-\pi)+(1-t(l,a))\pi](1-p)}{[(1-t(h,a))(1-\pi)+(1-t(l,a))\pi](1-p)+[(1-t(h,m))(1-\pi)+(1-t(l,m))\pi]p}$$

$$= \frac{1}{1+\frac{(1-t(h,m))(1-\pi)+(1-t(l,m))\pi}{(1-t(h,a))(1-\pi)+(1-t(l,a))\pi}\frac{p}{1-p}}$$

Analogously,

$$\mu'_{\phi,1} = \frac{[t(h,a)(1-\pi)+t(l,a)\pi](1-p)}{[t(h,a)(1-\pi)+t(l,a)\pi](1-p)+[t(h,m)(1-\pi)+t(l,m)\pi]p}$$

$$= \frac{1}{1+\frac{t(h,m)(1-\pi)+t(l,m)\pi}{t(h,a)(1-\pi)+t(l,a)\pi}\frac{p}{1-p}}$$

Simple algebra shows that

$$0 = \mu'_{m,0} = \mu'_{m,1} < \mu'_{\phi,1} < (1-p) < \mu'_{\phi,0} < \mu'_{a,1} = \mu'_{a,0} = 1,$$

establishes the result. ■

Using Lemma 6, let me prove the following results.

Lemma 7 *Assume an unbiased incumbent always chooses the optimal counterterrorism policy given the level of the terrorist threat. Then the voter has the following reelection rule:*

- *if* $\pi > \overline{\pi}_{\phi,1}$, *he elects* C *unless he observes* $x = m$*;*
- *if* $\overline{\pi}_{\phi,0} < \pi \leq \overline{\pi}_{\phi,1}$, *he elects* C *unless he observes* $x = m$ *or* $(\phi, 1)$*;*
- *if* $\overline{\pi}_{a,1} < \pi \leq \overline{\pi}_{\phi,0}$, *he reelects* I *unless he observes* $x = a$*;*
- *if* $\overline{\pi}_{a,0} < \pi \leq \overline{\pi}_{a,1}$, *he reelects* I *unless he observes* $(a, 0)$*;*
- *if* $\pi \leq \overline{\pi}_{a,0}$, *he always reelects* I*.*

Proof of Lemma 7:

- Case 1: $(a, 1)$

  We know that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] = (1-\mu_{a,1}-p)\gamma_{un} + \mu_{a,1}\gamma_{ba} - (1-p)\gamma_{bm}$.

Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{a,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu_{a,1}-p)(c-\alpha\Delta^l)]}$$

whereas the voter elects the challenger if $\pi > \overline{\pi}_{a,1}$.

- Case 2: $(a, 0)$

  We know that $\mathbb{E}[u_V(I)]_{(a,0)} - \mathbb{E}[u_V(C)] = (1-\mu_{a,0}-p)\gamma_{un} + \mu_{a,0}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,0)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{a,0} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu_{a,0}-p)(c-\alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}_{a,0}$.

- Case 3: $(\phi, 0)$

  We know that $\mathbb{E}[u_V(I)]_{(\phi,0)} - \mathbb{E}[u_V(C)] = (1-\mu_{\phi,0}-p)\gamma_{un} + \mu_{\phi,0}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(\phi,0)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{\phi,0} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu_{\phi,0}-p)(c-\alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}_{\phi,0}$.

- Case 4: $(\phi, 1)$

  We know that $\mathbb{E}[u_V(I)]_{(\phi,1)} - \mathbb{E}[u_V(C)] = (1-\mu_{\phi,1}-p)\gamma_{un} + \mu_{\phi,1}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(\phi,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{\phi,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu_{\phi,1}-p)(c-\alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi \geq \overline{\pi}_{\phi,1}$.

- Case 5: (m, T)

  We know that only unbiased incumbents choose a moderate counterterrorism policy. Therefore, upon observing a moderate policy, the voter knows the incumbent is unbiased and therefore reelects her.

Now, we can rank the thresholds that characterize the reelection rule in any scenario observed by the voter. In particular we have

$$\overline{\pi}_{\phi,1} > \overline{\pi}_{\phi,0} > \overline{\pi}_{a,1} > \overline{\pi}_{a,0}$$

and so we can easily obtain the result stated above. ■

Lemma 8 *Assume an unbiased incumbent always chooses the moderate counterterrorism policy regardless of the level of the terrorist threat. Then the voter has the following reelection rule:*

- *if* $\pi > \overline{\pi}'_{\phi,1}$*, he elects* C *unless he observes* $x = m$*;*
- *if* $\overline{\pi}'_{\phi,0} < \pi \leq \overline{\pi}'_{\phi,1}$*, he elects* C *unless he observes* $x = m$ *or* $(\phi, 1)$*;*
- *if* $\overline{\pi}'_{a,T} < \pi \leq \overline{\pi}'_{\phi,0}$*, he reelects* I *unless he observes* $x = a$*;*
- *if* $\pi \leq \overline{\pi}'_{a,T}$*, he always reelects* I*.*

Proof of Lemma 8:

- Case 1': $(a, 1)$

  We know that only biased incumbents choose an aggressive counterterrorism policy. Therefore, upon observing an aggressive policy, the voter knows the incumbent is biased. We know that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] = \gamma_{ba} - p\gamma_{un} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(a,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}'_{a,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu'_{a,1} - p)(c - \alpha\Delta^l)]}$$

whereas the voter elects the challenger if $\pi > \overline{\pi}'_{a,1}$.

- Case 2': $(a, 0)$

  We know that only biased incumbents choose an aggressive counterterrorism policy. Therefore, upon observing an aggressive policy, the voter knows the incumbent is biased. Given that the posterior beliefs of the voter are identical to the case of $(a, 1)$, also the lower bound on π for the unconditional reelection of the incumbent is the same as above.

- Case 3': $(\phi, 0)$

  We know that $\mathbb{E}[u_V(I)]_{(\phi,0)} - \mathbb{E}[u_V(C)] = (1 - \mu'_{\phi,0} - p)\gamma_{un} + \mu'_{\phi,0}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(\phi,0)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}'_{\phi,0} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu'_{\phi,0} - p)(c - \alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}'_{\phi,0}$.

- Case 4': $(\phi, 1)$

  We know that $\mathbb{E}[u_V(I)]_{(\phi,1)} - \mathbb{E}[u_V(C)] = (1 - \mu'_{\phi,1} - p)\gamma_{un} + \mu'_{\phi,1}\gamma_{ba} - (1-p)\gamma_{bm}$. Simplifying and rearranging we get that $\mathbb{E}[u_V(I)]_{(\phi,1)} - \mathbb{E}[u_V(C)] \geq 0$ if

$$\pi \leq \overline{\pi}_{\phi,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h - \Delta^l) - (1-\mu'_{\phi,1} - p)(c - \alpha\Delta^l)]}$$

  whereas the voter elects the challenger if $\pi > \overline{\pi}_{\phi,1}$.

- Case 5': (m, T)

We know that only unbiased incumbents choose a moderate counterterrorism policy. Therefore, upon observing a moderate policy, the voter knows the incumbent is unbiased and therefore reelects her.

Now, we can rank the thresholds that characterize the reelection rule in any scenario observed by the voter. In particular we have

$$\overline{\pi}'_{\phi,1} > \overline{\pi}'_{\phi,0} > \overline{\pi}'_{a,T}$$

and so we can easily obtain the result stated above. ■

We can now prove the following results referenced in the Observable vs Unobservable Counterterrorism Policies section of the main text.

Proposition A.1 *For any prior on the threat level,*[1]

(a) there exists an equilibrium in which an unbiased incumbent optimally provides *counterterrorism iff the effectiveness of aggressive policies is high enough or the probability of a low threat is low enough.*

(b) there exists an equilibrium in which an unbiased incumbent underprovides *counterterrorism iff the effectiveness of aggressive policies is low enough and the probability of a low threat is low enough.*

Proof of Proposition A.1:

Part (a). Consider the reelection rule specified in Lemma 7. Let us consider separately each

[1] The statement of this Proposition is identical to the one in Proposition 1, although the bounds on the effectiveness of aggressive policies and on the probability of a low threat are different.

case depending on the value of π.

Case 1 $: \pi > \overline{\pi}_{\phi,1}$

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + 0 \cdot \mathcal{B} \tag{18}$$

where the 0 reflects the impossibility of being confirmed in office after choosing $x = a$, given that under the reelection rule considered here the voter reelects the incumbent only when observing $x = m$.

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \tau\mathcal{B} \tag{19}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + 0 \cdot \mathcal{B} \tag{20}$$

where the reelection probability is equal to 0 for the same reason mentioned above.

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \tau\mathcal{B} \tag{21}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(18) - (19) \geq 0 \text{ and } (20) - (21) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \geq \underline{\Delta}_1^h = \frac{c+\tau\mathcal{B}}{\alpha}$.

Case 2 $: \overline{\pi}_{\phi,0} < \pi \leq \overline{\pi}_{\phi,1}$

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + (1-\tau)\mathcal{B}t(h,a) \tag{22}$$

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + [\tau\mathcal{B} + (1-\tau)\mathcal{B}t(h,m)] \tag{23}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + (1-\tau)\mathcal{B}t(l,a) \tag{24}$$

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + [\tau\mathcal{B} + (1-\tau)\mathcal{B}t(l,m)] \tag{25}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(22) - (23) \geq 0 \text{ and } (24) - (25) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \geq \underline{\Delta}_2^h = \frac{c+\tau\mathcal{B}}{\alpha-(1-\tau)\mathcal{B}}$.

<u>*Case 3* : $\overline{\pi}_{a,1} < \pi \leq \overline{\pi}_{\phi,0}$</u>

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + (1-\tau)\mathcal{B} \tag{26}$$

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \mathcal{B} \tag{27}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + (1-\tau)\mathcal{B} \tag{28}$$

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \mathcal{B} \tag{29}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(26) - (27) \geq 0 \text{ and } (28) - (29) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \geq \underline{\Delta}_3^h = \frac{c+\tau\mathcal{B}}{\alpha}$.

<u>*Case 4* : $\overline{\pi}_{a,0} < \pi \leq \overline{\pi}_{a,1}$</u>

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + (1-\tau+\tau t(h,a))\mathcal{B} \tag{30}$$

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \mathcal{B} \tag{31}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + (1 - \tau + \tau t(l,a))\mathcal{B} \tag{32}$$

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \mathcal{B} \tag{33}$$

Now the unbiased incumbent will optimally provide counterterrorism iff:

$$(30) - (31) \geq 0 \text{ and } (32) - (33) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \geq \underline{\Delta}_4^h = \frac{c+\tau\mathcal{B}(1-t(h,a))}{\alpha}$.

<u>*Case 5* $: \overline{\pi}_{a,0} > \pi$</u>

In this case the incumbent is elected unconditionally and therefore chooses the counterterrorism policy she prefers, that is the optimal one for the voter, regardless of the level of transparency, as stated in the result.

In each of the cases from 1 to 4, we see how the unbiased incumbent will optimally provide counterterrorism if the probability of policy revelation is low enough, thus proving the result.

Part (b). Consider the reelection rule specified in Lemma 8. Let us consider separately each case depending on the value of π.

<u>*Case 1* $: \pi > \overline{\pi}'_{\phi,1}$</u>

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + 0 \cdot \mathcal{B} \tag{34}$$

where the 0 reflects the impossibility of being confirmed in office after choosing $x = a$, given that under the reelection rule considered here the voter reelects the incumbent only when observing $x = m$.

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \tau\mathcal{B} \tag{35}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + 0 \cdot \mathcal{B} \tag{36}$$

where the reelection probability is equal to 0 for the same reason mentioned above.

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \tau\mathcal{B} \tag{37}$$

Now the unbiased incumbent will underprovide counterterrorism iff:

$$(34) - (35) \leq 0 \text{ and } (36) - (37) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \leq \overline{\Delta}_1^h = \frac{c+\tau\mathcal{B}}{\alpha}$.

<u>*Case 2* $: \overline{\pi}'_{\phi,0} < \pi \leq \overline{\pi}'_{\phi,1}$</u>

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + (1-\tau)\mathcal{B}t(h,a) \tag{38}$$

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + [\tau\mathcal{B} + (1-\tau)\mathcal{B}t(h,m)] \tag{39}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + (1-\tau)\mathcal{B}t(h,a) \tag{40}$$

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + [\tau\mathcal{B} + (1-\tau)\mathcal{B}t(l,m)] \tag{41}$$

Now the unbiased incumbent will underprovide counterterrorism iff:

$$(38) - (39) \leq 0 \text{ and } (40) - (41) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \leq \overline{\Delta}_2^h = \frac{c+\tau\mathcal{B}}{\alpha-(1-\tau)\mathcal{B}}$.

<u>*Case 3* $: \overline{\pi}'_{a,T} < \pi \leq \overline{\pi}'_{\phi,0}$</u>

In this case we have:

$$\mathbb{E}[u_{un}(a|h)] = -c - \alpha t(h,a) + (1-\tau)\mathcal{B} \tag{42}$$

$$\mathbb{E}[u_{un}(m|h)] = -\alpha t(h,m) + \mathcal{B} \tag{43}$$

$$\mathbb{E}[u_{un}(a|l)] = -c - \alpha t(l,a) + (1-\tau)\mathcal{B} \tag{44}$$

$$\mathbb{E}[u_{un}(m|l)] = -\alpha t(l,m) + \mathcal{B} \tag{45}$$

Now the unbiased incumbent will underprovide counterterrorism iff:

$$(42) - (43) \leq 0 \text{ and } (44) - (45) \leq 0$$

The second inequality is always true. The first inequality is satisfied if $\Delta^h \leq \overline{\Delta}_3^h = \frac{c+\tau\mathcal{B}}{\alpha}$.

<u>*Case 4* $: \pi \leq \overline{\pi}'_{a,T}$</u>

In this case we have that the incumbent will be reelected for sure. As a consequence, she will choose her preferred counterterrorism policy, which is the optimal one from the point of view of the voter. As a consequence, a necessary condition for the existence of an equilibrium in which the incumbent underprovides counterterrorism is that $\pi > \overline{\pi}'_{a,T}$, as stated in the result.

In each of the cases from 1 to 3, we see how the unbiased incumbent will underprovide counterterrorism if the probability of policy revelation is high enough, thus proving the result. ■

Proof of Proposition 4: In order to prove the result I will show that, for each level of π, an increase in the level of transparency (i.e τ) moves each relevant threshold on the effectiveness of aggressive policies in a direction that makes the set of parameter values that sustains optimal provision as an equilibrium *smaller* and that makes the set of parameter values that sustains suboptimal provision as an equilibrium *larger*.

Let us go through each threshold, starting with the case of a right-wing incumbent optimally providing counterterrorism. Given that for an optimal provision equilibrium to exists when a right-wing politician is in office Δ^h must be high enough, to prove the result we need to show that the thresholds for existence of such an equilibrium becomes *larger* when τ increases, so to make the threshold less likely to be cleared.

Going through the different thresholds depending on the value of π, we have

$$\frac{\partial \underline{\Delta}_1^h}{\partial \tau} = \frac{\mathcal{B}}{\alpha} > 0$$

$$\frac{\partial \underline{\Delta}_2^h}{\partial \tau} = \frac{\mathcal{B}[\alpha - \mathcal{B}(1-\tau)] - \mathcal{B}(\tau\mathcal{B} + c)}{[\alpha - (1-\tau)\mathcal{B}]^2} = \frac{\mathcal{B}(\alpha - \mathcal{B} - c)}{[\alpha - (1-\tau)\mathcal{B}]^2} > 0$$

where the last inequality follows from the fact that $\alpha > \mathcal{B} + c$ by assumption.

$$\frac{\partial \underline{\Delta}_3^h}{\partial \tau} = \frac{\mathcal{B}}{\alpha} > 0$$

$$\frac{\partial \underline{\Delta}_4^h}{\partial \tau} = \frac{\mathcal{B}(1 - t(h,a))}{\alpha} > 0$$

Let us now consider the case of a right-wing incumbent underproviding counterterrorism. Given that for an underprovision equilibrium to exists when a right-wing politician is in office Δ^h must be low enough, to prove the result we need to show that the thresholds for existence of such an equilibrium becomes *larger* when τ increases, so to make the threshold more likely to be cleared.

Going through the different thresholds depending on the value of π, we have,

$$\frac{\partial \overline{\Delta}_1^h}{\partial \tau} = \frac{\mathcal{B}}{\alpha} > 0$$

$$\frac{\partial \overline{\Delta}_2^h}{\partial \tau} = \frac{\mathcal{B}[\alpha - \mathcal{B}(1-\tau)] - \mathcal{B}(\tau\mathcal{B} + c)}{[\alpha - (1-\tau)\mathcal{B}]^2} = \frac{\mathcal{B}(\alpha - \mathcal{B} - c)}{[\alpha - (1-\tau)\mathcal{B}]^2} > 0$$

where the last inequality follows from the fact that $\alpha > \mathcal{B} + c$ by assumption.

$$\frac{\partial \overline{\Delta}_3^h}{\partial \tau} = \frac{\mathcal{B}}{\alpha} > 0$$

which proves the result. ■

Proof of Proposition 5: The result follows directly from Lemma 6.

If an unbiased incumbent optimally provides counterterrorism (OP), then

$$\mu_{\phi,0} = \frac{1}{1 + \frac{(1-t(h,a))(1-\pi)+(1-t(l,m))\pi}{(1-t(h,a))(1-\pi)+(1-t(l,a))\pi}\frac{p}{1-p}}$$

and

$$\mu_{\phi,1} = \frac{1}{1 + \frac{t(h,a)(1-\pi)+t(l,m)\pi}{t(h,a)(1-\pi)+t(l,a)\pi} \frac{p}{1-p}}$$

Simple algebra shows that,

$$\mu_{\phi,0} > (1-p) > \mu_{\phi,1}$$

If an unbiased incumbent underprovides counterterrorism (UP), then

$$\mu'_{\phi,0} = \frac{1}{1 + \frac{(1-t(h,m))(1-\pi)+(1-t(l,m))\pi}{(1-t(h,a))(1-\pi)+(1-t(l,a))\pi} \frac{p}{1-p}}$$

and

$$\mu'_{\phi,1} = \frac{1}{1 + \frac{t(h,m)(1-\pi)+t(l,m)\pi}{t(h,a)(1-\pi)+t(l,a)\pi} \frac{p}{1-p}}$$

Simple algebra shows again that,

$$\mu'_{\phi,0} > (1-p) > \mu'_{\phi,1}$$

thus proving the result. ■

Proposition A.3 (**Effect of terrorist threat on electoral prospects**) *In any equilibrium, the voter retention rule is:*

(a) weakly more (less) restrictive as the probability of facing a low threat increases;

(b) maximally restrictive (permissive) if the probability of facing a low threat is high enough;

(c) maximally permissive (restrictive) if the probability of facing a low threat is low enough.

Proof of Proposition A.3:

(a) A quick inspection of Lemma 7 and Lemma 8 shows how any increase from one threshold level of π to the next is associated with a smaller number of policy-outcome pairs (x, T) after which the incumbent is granted reelection. This immediately implies that this number is weakly decreasing in π.

(b) To establish this part of the result, I need to show that there exist thresholds, one for the optimal provision equilibrium and one for the underprovision equilibrium, such that, if π is above the relevant threshold, then a right-wing incumbent gets reelected only if the voter believes she is unbiased with probability 1. Call these thresholds $\underline{\pi}^{OP}$ and $\underline{\pi}^{UP}$ for the optimal provision equilibrium and one for the underprovision equilibrium, respectively. By Lemma 7 and Lemma 8 these thresholds are respectively $\underline{\pi}^{OP} = \overline{\pi}_{\phi,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu_{\phi,1}-p)(c-\alpha\Delta^l)]}$ and $\underline{\pi}^{UP} = \overline{\pi}'_{\phi,1} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu'_{\phi,1}-p)(c-\alpha\Delta^l)]}$.

(c) To establish part (c), I need to show that there exist thresholds, one for the optimal provision equilibrium and one for the underprovision equilibrium, such that, if π is below the relevant threshold, then a right-wing incumbent gets reelected no matter what the policy-outcome pair ends up being. Call these thresholds $\overline{\pi}^{OP}$ and $\overline{\pi}^{UP}$ for the optimal provision equilibrium and one for the underprovision equilibrium, respectively. By Lemma 7 and Lemma 8 these thresholds are respectively $\overline{\pi}^{OP} = \overline{\pi}_{a,0} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu_{a,1}-p)(c-\alpha\Delta^l)]}$ and $\overline{\pi}^{UP} = \overline{\pi}'_{a,T} = \frac{(1-p)(-c+\alpha\Delta^h)}{[(1-p)\alpha(\Delta^h-\Delta^l)-(1-\mu'_{a,1}-p)(c-\alpha\Delta^l)]}$.

■