Online Appendix for "The Distributive Effects of Corruption"

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Proof of proposition 1: Write the value functions of the government in recursive form (the Bellman equations) for the equilibrium path of the cooperative situation described by the strategies above, as well as for the best one-stage deviation (OSD) by the government:

$$V^g_{COOP}(M_{t,i}) = R_{t,i} - c + \delta\left[(1-p)E\left[V^g_{COOP}(M_{t+1,o})\right] + pE\left[V^g_{COOP}(M_{t+1,i\prime})\right]\right] \quad (1)$$

$$V^g_{OSD}(M_{t,i}) = R_{t,i} + \delta\left[(1-p)E\left[V^g_{PUN}(M_{t+1,o})\right] + pE\left[V^g_{PUN}(M_{t+1,i\prime})\right]\right] \quad (2)$$

Here $V^g_{COOP}(M_{t,i})$ is the value function of the government on the equilibrium path in the cooperative equilibrium described by the strategies above, in stages where the government is matched with an insider player, $V^g(M_{t,o})$ is the value function of the government when matched with an outsider player, $V^g_{OSD}(M_{t,i})$ is the value of the best one-stage deviation by the government, and $V^g_{PUN}(M_{t,i})$ is the value function of the government when play has switched to the punishment prescribed by the strategies described above. The reason future instances of the value function are written in expectation is because these values might depend on the α parameters drawn in future stages, and those parameters are random. The subgame perfect equilibrium condition is that no profitable one shot deviations exist for the government:

$$\forall t, \forall i \quad V^g_{COOP}(M_{t,i}) \geq V^g_{OSD}(M_{t,i}) \quad (3)$$

This condition will have to hold with equality, as there is no need for citizens to pay more

than enough rents to the government according to condition *ii)* (the remaining income is incorporated in citizens' utilities as the private good). To see what values of the rent parameter lead to the condition holding, compute V^g_{COOP} : First, take the expected value of both Bellman equations in (1-2). Doing this will lead to equilibrium conditions involving the expected value of the rent payment, $E[R_{i,t}]$, so individual behavior by the citizens will have to correspond to a probability law that leads to a certain expected value for the rent, If it is further assumed that citizens use symmetric strategies, it is also possible to denote all i players by a single subscript, rather than using i and $i\prime$. Using the fact that $E[E[X]] = E[X]$ and the linear property of the expected value, the Bellman equations now become:

$$\begin{aligned} E\left[V^g_{COOP}(M_{t,i})\right] &= E\left[R_{t,i}\right] - c + \delta\left[(1-p)E\left[V^G_{COOP}(M_{t+1,o})\right] + pE\left[V^G_{COOP}(M_{t+1,i})\right]\right] \\ E\left[V^G_{OSD}(M_{t,i})\right] &= E\left[R_{t,i}\right] + \delta\left[(1-p)E\left[V^G_{PUNISH}(M_{t+1,o})\right] + pE\left[V^G_{PUNISH}(M_{t+1,i})\right]\right] \end{aligned}$$

Then from the first equation:

$$E\left[V^g_{COOP}(M_{t,i})\right](1-\delta p) = E\left[R_{t,i}\right] - c + \delta(1-p)E\left[V^G_{COOP}(M_{t+1,o})\right] \tag{4}$$

The value function when being paired with an outsider player in the cooperative equilibrium can be derived from its own set of two Bellman equations, again written in expectation, and again assuming symmetric strategies by O players:

$$\begin{aligned} E\left[V^g_{COOP}(M_{t+1,o})\right] &= E\left[R_{t+1,o}\right] - c + \\ &\quad +\delta\left[(1-p)E\left[V^g_{COOP}(M_{t+2,o})\right] + pE\left[V^g_{COOP}(M_{t+2,i})\right]\right] \qquad (5) \\ E\left[V^g_{OSD}(M_{t+1,o})\right] &= E\left[R_{t+1,o}\right] + \\ &\quad +\delta\left[(1-p)E\left[V^g(M_{t+2,o})\right] + pE\left[V^g(M_{t+2,i})\right]\right] \qquad (6) \end{aligned}$$

The value functions following a deviation by the government here do not carry any subscript regarding the strategy used by players who are matched in those periods, because once stage

$t+2$ has been reached following a defection by the government, for the citizens this event lies in the same information set as reaching stage $t+2$ following cooperative behavior by the government. Therefore, the citizens have to behave identically in the two histories, as they do not know which one they are in. The government will have unique best responses to whatever behaviors the citizens engage in, so it always has to be that $E\left[V^g_{COOP}(M_{t+2,o})\right] = E\left[V^g(M_{t+2,o})\right]$ and $E\left[V^g_{COOP}(M_{t+2,i})\right] = E\left[V^g(M_{t+2,i})\right]$.

So the condition that $V^g_{COOP}(M_{t+1,o}) \geq V^g_{OSD}(M_{t+1,o})$ leads to $E\left[V^g_{COOP}(M_{t+1,o})\right] \geq E\left[V^g_{OSD}(M_{t+1,o})\right]$, which then leads to:

$$E\left[R_{t+1,o}\right] - c \geq E\left[R_{t+1,o}\right] \tag{7}$$

which is never true as long as $c > 0$. Therefore cooperation cannot be sustained in stages where the government meets outside players, so the value function of the government when matched with an outside player is:

$$E\left[V^g_{COOP}(M_{t+1,o})\right] = \frac{\delta p}{1-\delta p} E\left[V^g_{COOP}(M_{t+2,i})\right] \tag{8}$$

Putting this back in the value function for cooperating with an insider:

$$E[V^g_{COOP}(M_{t,i})] = \frac{1-\delta(1-p)}{1-\delta}\left(E\left[R_{t,i}\right] - c\right) \tag{9}$$

The value function for the best one stage deviation is given by capturing the entire rent payment now, which triggers the punishment future matches with insider players. Therefore a stream of zero payments follows defection by the government, and

$$E\left[V^g_{OSD}(M_{t,i})\right] = E\left[R_{t,i}\right] \tag{10}$$

The no-deviation condition has to hold: $E\left[V^g_{COOP}(M_{t,i})\right] = E\left[V^g_{OSD}(M_{t,i})\right]$, so it follows that:

$$E\left[R_{t,i}\right] = \frac{1-\delta(1-p)}{1-\delta}\left(E\left[R_{t,i}\right]-c\right) \tag{11}$$

$$\implies E[R_{t,i}] = \frac{c(1-\delta(1-p))}{\delta p} \tag{12}$$

Using symmetric strategies for all players, it follows that a constant rent payment by any insider player keeps the government in the cooperative equilibrium, and that payment is:

$$R_{t,i} = \frac{c(1-\delta(1-p))}{\delta p} \tag{13}$$

For citizens, the value functions in the cooperative equilibrium are simpler:

$$V^i_{COOP}(M_{t,i}) = \alpha_i G_i - R_{t,i} + w + \delta V^i(\neg(M_{t+1,i})) \tag{14}$$

$$V^i_{OSD}(M_{t,i}) = w + \delta V^i(\neg(M_{t+1,i})) \tag{15}$$

Future periods, in which i is not matched, have the value function $V_i(\neg(M_{t+1,i}))$. So the cooperation condition $V^i_{COOP} \geq V^i_{OSD}$ reduces to:

$$\alpha_i \geq \frac{c(1-\delta(1-p))}{\delta p} \tag{16}$$

Proof of proposition 2: Write the value functions of the government on the equilibrium path and the best one stage deviation for the government:

$$V^g_{COOP}(M_{t,i}) = R_{i,t} - c + \delta\left[(1-q)E\left[V^G_{COOP}(M_{t+1,o})\right] + qE\left[V^G_{COOP}(M_{t+1,i})\right]\right] \tag{17}$$

$$V^G_{OSD}(M_{t,i}) = R_{i,t} + \delta\left[(1-q)E\left[V^G_{PUN}(M_{t+1,o})\right] + qpE\left[V^G_{PUN}(M_{t+1,i})\right]\right] \tag{18}$$

The government's conditions for maximizing income while being in this cooperative equilibrium are

$$(t,i) = \left(t, \arg\max_j V^g_{COOP}(C_{t,j}),\ \text{s.t. } j \in \mathfrak{M}_t\right) \tag{19}$$

$$V^g_{COOP}(M_{t,i}) \geq V^g_{OSD}(M_{t,i}) \tag{20}$$

(The maximization problem is set up here only with respect to i players, as in equilibrium an o player is never chosen, by the same logic as in the previous section.) The optimization condition for the government is similar to that in case 1 but with the addition of the problem of choosing among the citizens which have been matched, that is of choosing at each t the one i (or o) for which the value function conditions have to hold. Since the forms of the two relevant value functions are identical to those in case 1, except for replacing p with q, the same derivation leads to the following cooperative equilibrium condition for the government:

$$E\left[R_{t,i}\right] \geq \frac{c(1-\delta(1-q))}{\delta q} \tag{21}$$

The condition is not yet reduced to equality, because there might be more than one insider player willing to pay a bribe. To see which i player is chosen, write the value functions for the citizens:

$$V^i_{COOP}(C_{t,i}) = \alpha_i G_i - R_{t,i} + w + \delta V^i(\neg(M_{t+1,i})) \tag{22}$$

$$V^i_{OSD}(C_{t,i}) = w + \delta V^i(\neg(M_{t+1,i})) \tag{23}$$

$$V^i(\neg C_{t,i}) = V^i_{OSD}(C_{t,i}) \tag{24}$$

The set of relevant value functions now includes the last one, for the situation in which player i is not chosen at time t. Then the cooperation condition for citizens (which is the same as the condition for being willing to participate in a cooperative equilibrium altogether by

being chosen as partner by the government) is:

$$\alpha_i \geq R_i \tag{25}$$

In case more than one citizen is matched, it is clear that the one with the highest α_i level will become the partner of the government official, and will offer a rent payment (just slightly above) the maximum rent payment of the second-ranked drawn citizen.

With the given strategies, the government's problem of choosing the player to cooperate with in each period to maximize its value function for cooperation (equation 19) is reduced to choosing the player who can pay the highest rent each period, as there is nothing to be gained by choosing a player who can pay less.

The citizens in I then solve:

$$\begin{aligned} & \arg\max_{R_i} \alpha_i - R_i \\ \text{subject to} \quad & \alpha_i \geq R_i \text{ and } R_i \geq R_j \quad \forall j \in \mathfrak{M}_t \end{aligned} \tag{26}$$

This leads for the player who has $\alpha^{(1)}$ at time t to set:

$$R_{t,i} = \alpha^{(2)} \tag{27}$$