Spencer et al. Model C

In[1]:=
```
μ1 =.; μ2 =.; T =.; t =.; τ =.; v =.; En =.; c1 =.; d =.; c2 =.; k1 =.; k2 = .;
```

In[2]:=
```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v))
   (En - (c1 t + c1 τ + k1 (2 k2 + τ) / (k2 + τ) + c2 (T - t - τ)))
```

Out[2]=
$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

In[3]:=
```
delwbydelt = D[w, t]
```

Out[3]=
$$(-c1+c2)\,e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)} + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

In[4]:=
```
delwbydelτ = D[w, τ]
```

Out[4]=
$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(-c1+c2-\frac{k1}{k2+\tau}+\frac{k1\,(2\,k2+\tau)}{(k2+\tau)^2}\right) + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

In[5]:=
```
sols = Solve[{delwbydelt == 0, delwbydelτ == 0}, {t, τ}]
```

Out[5]= { }

No general solution : must look on boundaries

Boundary 1 : t = 0

In[6]:=
```
w1 = Simplify[w /. t → 0]
```

Out[6]=
$$e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\left(En - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau} + c2\,(-T+\tau)\right)$$

In[7]:=
```
delw1bydelv = D[w1, v]
```

Out[7]=
$$e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\,(-\mu 1+\mu 2)\left(En - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau} + c2\,(-T+\tau)\right)$$

In[8]:=
```
vcrit1 = Simplify[Solve[delw1bydelv == 0, v]]
```

Out[8]= { }

No solutions when t = 0

Boundary 2 : $\tau = 0$

In[9]:=
```
w2 = Simplify[w /. τ → 0]
```

Out[9]=
$$e^{-t\,\mu 1-v\,\mu 1+t\,\mu 2-T\,\mu 2+v\,\mu 2}\,(En - 2\,k1 - c1\,t + c2\,t - c2\,T)$$

In[10]:=
```
delw2bydelt = D[w2, t]
```

Out[10]=
$$(-c1+c2)\,e^{-t\,\mu 1-v\,\mu 1+t\,\mu 2-T\,\mu 2+v\,\mu 2} + e^{-t\,\mu 1-v\,\mu 1+t\,\mu 2-T\,\mu 2+v\,\mu 2}\,(En - 2\,k1 - c1\,t + c2\,t - c2\,T)\,(-\mu 1+\mu 2)$$

In[11]:=
```
tcrit2 = Solve[delw2bydelt == 0, t]
```

Out[11]=
$$\{\{t \to (c1 - c2 + En\,\mu1 - 2\,k1\,\mu1 - c2\,T\,\mu1 - En\,\mu2 + 2\,k1\,\mu2 + c2\,T\,\mu2) / ((c1 - c2)\,(\mu1 - \mu2))\}\}$$

In[12]:=
```
tcrit2 /.
 {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, v → 16 - t}
```

Out[12]=
$$\{\{t \to 100.\,(-4.7 + 0.2\,k1)\}\}$$

No realistic solutions when $\tau = 0$

Boundary 3 : v = 0

In[13]:=
```
w3 = Simplify[w /. v → 0]
```

Out[13]=
$$\frac{1}{k2 + \tau} e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)}\,(En\,(k2 + \tau) - k1\,(2\,k2 + \tau) - (k2 + \tau)\,(c1\,(t + \tau) - c2\,(t - T + \tau)))$$

In[14]:=
```
delw3bydelt = D[w3, t]
```

Out[14]=
$$-(c1 - c2)\,e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)} + \frac{1}{k2 + \tau} e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)}\,(-\mu1 + \mu2)$$
$$(En\,(k2 + \tau) - k1\,(2\,k2 + \tau) - (k2 + \tau)\,(c1\,(t + \tau) - c2\,(t - T + \tau)))$$

In[15]:=
```
tcrit3 = Simplify[Solve[delw3bydelt == 0, t]]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

Out[15]=
$$\{\{t \to (-c1\,(k2 + \tau)\,(-1 + \mu1\,\tau - \mu2\,\tau) + c2\,(k2 + \tau)\,(-1 + T\,(-\mu1 + \mu2) + \mu1\,\tau - \mu2\,\tau) + (\mu1 - \mu2)\,(En\,(k2 + \tau) - k1\,(2\,k2 + \tau))) / ((c1 - c2)\,(\mu1 - \mu2)\,(k2 + \tau))\}\}$$

In[16]:=
```
w3v = w3 /. {τ → 16 - t}
```

Out[16]=
$$\frac{1}{16 + k2 - t} e^{-16\,\mu1 + (16 - T)\,\mu2}\,(En\,(16 + k2 - t) - k1\,(16 + 2\,k2 - t) - (16 + k2 - t)\,(16\,c1 - c2\,(16 - T)))$$

In[17]:=
```
tcrit3v = tcrit3 /. {τ → 16 - t}
```

Out[17]=
$$\{\{t \to ((En\,(16 + k2 - t) - k1\,(16 + 2\,k2 - t))\,(\mu1 - \mu2) - c1\,(16 + k2 - t)\,(-1 + (16 - t)\,\mu1 - (16 - t)\,\mu2) + c2\,(16 + k2 - t)\,(-1 + (16 - t)\,\mu1 - (16 - t)\,\mu2 + T\,(-\mu1 + \mu2))) / ((c1 - c2)\,(16 + k2 - t)\,(\mu1 - \mu2))\}\}$$

In[18]:=
```
t3 = t /. tcrit3v[[1]]
```

Out[18]=
$$((En\,(16 + k2 - t) - k1\,(16 + 2\,k2 - t))\,(\mu1 - \mu2) - c1\,(16 + k2 - t)\,(-1 + (16 - t)\,\mu1 - (16 - t)\,\mu2) + c2\,(16 + k2 - t)\,(-1 + (16 - t)\,\mu1 - (16 - t)\,\mu2 + T\,(-\mu1 + \mu2))) / ((c1 - c2)\,(16 + k2 - t)\,(\mu1 - \mu2))$$

In[19]:=
```
solt = Solve[t == t3, t]
```

Out[19]=
$$\{\{t \to (-16\,c1 + 16\,c2 - c1\,k2 + c2\,k2 + 256\,c1\,\mu1 - 256\,c2\,\mu1 - 16\,En\,\mu1 + 16\,k1\,\mu1 + 16\,c1\,k2\,\mu1 - 16\,c2\,k2\,\mu1 - En\,k2\,\mu1 + 2\,k1\,k2\,\mu1 + 16\,c2\,T\,\mu1 + c2\,k2\,T\,\mu1 - 256\,c1\,\mu2 + 256\,c2\,\mu2 + 16\,En\,\mu2 - 16\,k1\,\mu2 - 16\,c1\,k2\,\mu2 + 16\,c2\,k2\,\mu2 + En\,k2\,\mu2 - 2\,k1\,k2\,\mu2 - 16\,c2\,T\,\mu2 - c2\,k2\,T\,\mu2) / (-c1 + c2 + 16\,c1\,\mu1 - 16\,c2\,\mu1 - En\,\mu1 + k1\,\mu1 + c2\,T\,\mu1 - 16\,c1\,\mu2 + 16\,c2\,\mu2 + En\,\mu2 - k1\,\mu2 - c2\,T\,\mu2)\}\}$$

In[39]:=
```
solt /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 0.2, k2 → 0.2}
```
Out[39]= {{t → 16.1992}}

In[40]:=
```
solt /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 2, k2 → 2}
```
Out[40]= {{t → 17.9142}}

Need to look in the corners, too

In[22]:=
```
w00 = Simplify[w /. {t → 0, τ → 0}]
```
Out[22]= $e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2}\ (En-2\,k1-c2\,T)$

In[41]:=
```
w00 /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, v → 16, T → 20, k1 → 1}
```
Out[41]= 0.0731085

In[24]:=
```
w0T = Simplify[w /. {t → 0, v → 0}]
```
Out[24]= $e^{-T\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\left(En-c1\,\tau-\frac{k1\ (2\,k2+\tau)}{k2+\tau}+c2\ (-T+\tau)\right)$

In[42]:=
```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 1, k2 → 1}
```
Out[42]= 0.0773308

In[43]:=
```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 1, k2 → 1}
```
Out[43]= 0.0773308

In[44]:=
```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 20, k2 → 20}
```
Out[44]= 0.0273972

In[28]:=
```
wT0 = Simplify[w /. {v → 0, τ → 0}]
```
Out[28]= $e^{-t\,\mu 1+t\,\mu 2-T\,\mu 2}\ (En-2\,k1-c1\,t+c2\,t-c2\,T)$

In[45]:=
```
wT0 /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, t → 16, T → 20, k1 → 1}
```
Out[45]= 0.075767

In[46]:=
```
Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 1, k2 → 1}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```
Out[46]= $\{0.0773308,\ \{t \to 1.70238\times 10^{-7},\ \tau \to 16.\}\}$

In[47]:=
```
Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 2, k2 → 2}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```
Out[47]= {0.0753978, {t → 0., τ → 16.}}

In[49]:=
```
Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 20, k2 → 0.2}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

Out[49]= $\{0.0454487, \{t \to 0., \tau \to 16.\}\}$

In[50]:=
```
Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 0.2, k2 → 20}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

Out[50]= $\left\{0.0785732, \left\{t \to 2.26423 \times 10^{-7}, \tau \to 16.\right\}\right\}$

In[51]:=
```
w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3,
  μ2 → 0.4, T → 20, v → 0,  t → 8, τ → 8, k1 → 1, k2 → 1}
```

Out[51]= 0.077244

In[52]:=
```
w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3,
  μ2 → 0.4, T → 20, v → 0,  t → 3, τ → 13, k1 → 1, k2 → 1}
```

Out[52]= 0.0773099