Spencer et al. Model E(A1)

τ and cd are fixed in this model; Since t + τ + v is fixed, this means we have only one free variable, say t.
But the cost of development (compared to Model A) is now cd1 t + cd τ

```
μ1 =.; μ2 =.; T =.; t =.; τ =.; v =.; En =.; c1 =.; cd =.; cd1 = .; c2 =.;
```

```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v)) (En - (c1 t + cd1 t + cd τ + c2 (T - t - τ)))
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
delwbydelt = D[w, t]
```

$$(-c1 + c2 - cd1)\,e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)} + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1 + \mu 2)\,(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
solsint = Solve[delwbydelt == 0, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{t \to (c1 - c2 + cd1 + En\,\mu 1 - c2\,T\,\mu 1 - En\,\mu 2 + c2\,T\,\mu 2 + c2\,\mu 1\,\tau - cd\,\mu 1\,\tau - c2\,\mu 2\,\tau + cd\,\mu 2\,\tau)\,/\,((c1 - c2 + cd1)\,(\mu 1 - \mu 2))\}\}$$

```
wcrit = Simplify[w /. solsint[[1]]]
```

$$\frac{1}{-\mu 1 + \mu 2}\,(c1 - c2 + cd1)\,e^{-\frac{cd1+En\,\mu 1+cd1\,v\,\mu 1-En\,\mu 2+cd1\,T\,\mu 2-cd1\,v\,\mu 2-c2\,(1+T\,\mu 1+v\,\mu 1-v\,\mu 2)-cd\,\mu 1\,\tau+cd1\,\mu 1\,\tau+cd\,\mu 2\,\tau-cd1\,\mu 2\,\tau+c1\,(1+v\,(\mu 1-\mu 2)+T\,\mu 2+\mu 1\,\tau-\mu 2\,\tau)}{c1-c2+cd1}}$$

Must Also look on boundaries

Boundary 1 : t = 0

```
w1 = Simplify[w /. t → 0]
```

$$e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\,(En - cd\,\tau + c2\,(-T+\tau))$$

Boundary 2 : v = 0

```
w2 = Simplify[w /. v → 0]
```

$$e^{-t\,\mu 1+t\,\mu 2-T\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\,(En - c1\,t + c2\,t - cd1\,t - c2\,T + c2\,\tau - cd\,\tau)$$

Numerical Example with t + τ + v = 12 and τ = 4

```
wvout = w /. {v → 8 - t}
```

$$e^{-\mu 2\,(-8+T-\tau)-\mu 1\,(8+\tau)}\,(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
wcrit1 = Simplify[wvout /. solsint[[1]]]
```

$$\left((c1 - c2 + cd1)\,e^{-\mu 1\,(8+\tau)+\mu 2\,(8-T+\tau)}\right) \Big/ (-\mu 1 + \mu 2)$$

```
wcrit1 /. {T → 16, τ → 4, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
-0.0136619
```

```
solsint[[1]] /. {T → 16, τ → 4, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
{t → -18.}
```

```
w1 /. {T → 16, τ → 4, v → 8, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
0.0109295
```

```
w2 /. {T → 16, τ → 4, t → 8, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
0.021859
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.021859, {t → 8.}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0109295, {t → 0.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.2, c2 → 0.1, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.043718, {t → 0.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.15,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0109295, {t → 0.}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.15,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{-1.21342 × 10^-17, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0327885, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.01,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0306026, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.1,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0109295, {t → 0.}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.1,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
{0.0109295, {t → 0.}}
```