Spencer et al. Model A

τ and cd are fixed in this model; Since t + τ + v is fixed, this means we have only one free variable, say t.

```
μ1 =.; μ2 =.; T =.; t =.; τ =.; v =.; En =.; c1 =.; cd =.; c2 =.;
```

```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v)) (En - (c1 t + cd τ + c2 (T - t - τ)))
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(En - c1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
delwbydelt = D[w, t]
```

$$(-c1+c2)\,e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)} + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\,(En - c1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
solsint = Solve[delwbydelt == 0, t]
```

$$\left\{\left\{t \to \frac{c1 - c2 + En\,\mu 1 - c2\,T\,\mu 1 - En\,\mu 2 + c2\,T\,\mu 2 + c2\,\mu 1\,\tau - cd\,\mu 1\,\tau - c2\,\mu 2\,\tau + cd\,\mu 2\,\tau}{(c1-c2)\,(\mu 1-\mu 2)}\right\}\right\}$$

```
wcrit = Simplify[w /. solsint[[1]]]
```

$$\frac{(c1-c2)\,e^{-\frac{-c2\,(1+T\,\mu 1+v\,\mu 1-v\,\mu 2)+(\mu 1-\mu 2)\,(En-cd\,\tau)+c1\,(1+v\,(\mu 1-\mu 2)+T\,\mu 2+\mu 1\,\tau-\mu 2\,\tau)}{c1-c2}}}{-\mu 1+\mu 2}$$

Must Also look on boundaries

Boundary 1 : t = 0

```
w1 = Simplify[w /. t → 0]
```

$$e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\,(En - cd\,\tau + c2\,(-T+\tau))$$

Boundary 2 : v = 0

```
w2 = Simplify[w /. v → 0]
```

$$e^{-t\,\mu 1+t\,\mu 2-T\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\,(En - c1\,t + c2\,t - c2\,T + c2\,\tau - cd\,\tau)$$

Numerical Example with t + τ + v = 12 and τ = 4

```
wvout = w /. {v → 8 - t}
```

$$e^{-\mu 2\,(-8+T-\tau)-\mu 1\,(8+\tau)}\,(En - c1\,t - c2\,(-t+T-\tau) - cd\,\tau)$$

```
wcrit1 = Simplify[wvout /. solsint[[1]]]
```

$$\frac{(c1-c2)\,e^{-\mu 1\,(8+\tau)+\mu 2\,(8-T+\tau)}}{-\mu 1+\mu 2}$$

```
wcrit1 /.
 {T → 16, τ → 4, En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, μ1 → 0.2, μ2 → 0.3}
```

```
-0.0273237
```

```
solsint[[1]] /.
 {T → 16, τ → 4, En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, μ1 → 0.2, μ2 → 0.3}
```

```
{t → -14.}
```

```
w1 /. {T → 16, τ → 4, v → 8, En → 4,
  c1 → 0.1, c2 → 0.2, cd → 0.3, μ1 → 0.2, μ2 → 0.3}
```

```
0.0109295
```

```
w2 /. {T → 16, τ → 4, t → 8, En → 4,
  c1 → 0.1, c2 → 0.2, cd → 0.3, μ1 → 0.2, μ2 → 0.3}
```

```
0.0327885
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0327885, {t → 8.}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0109295, {t → 0.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.2, c2 → 0.1, cd → 0.3,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.043718, {t → 0.}}
```