Spencer et al. Model E(A2)

τ and cd are fixed in this model; Since t + τ + v is fixed, this means we have only one free variable, say t.

But the cost of development (compared to Model A) is now cd1 t^2 + cd τ

```
μ1 =.; μ2 =.; T =.; t =.; τ =.; v =.; En =.; c1 =.; cd =.; cd1 =.; c2 =.;
```

```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v)) (En - (c1 t + cd1 t^2 + cd τ + c2 (T - t - τ)))
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(En - c1\,t - cd1\,t^2 - c2\,(-t+T-\tau) - cd\,\tau\right)$$

```
delwbydelt = D[w, t]
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-c1+c2-2\,cd1\,t) + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - cd1\,t^2 - c2\,(-t+T-\tau) - cd\,\tau\right)$$

```
solsint = Solve[delwbydelt == 0, t]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{t \to \left(2\,cd1 - c1\,\mu 1 + c2\,\mu 1 + c1\,\mu 2 - c2\,\mu 2 - \sqrt{\left(-2\,cd1 + c1\,\mu 1 - c2\,\mu 1 - c1\,\mu 2 + c2\,\mu 2\right)^2 - 4\,(cd1\,\mu 1 - cd1\,\mu 2)\,(-c1 + c2 - En\,\mu 1 + c2\,T\,\mu 1 + En\,\mu 2 - c2\,T\,\mu 2 - c2\,\mu 1\,\tau + cd\,\mu 1\,\tau + c2\,\mu 2\,\tau - cd\,\mu 2\,\tau)}\right) \Big/ (2\,(cd1\,\mu 1 - cd1\,\mu 2))\right\}, \left\{t \to \left(2\,cd1 - c1\,\mu 1 + c2\,\mu 1 + c1\,\mu 2 - c2\,\mu 2 + \sqrt{\left(-2\,cd1 + c1\,\mu 1 - c2\,\mu 1 - c1\,\mu 2 + c2\,\mu 2\right)^2 - 4\,(cd1\,\mu 1 - cd1\,\mu 2)\,(-c1 + c2 - En\,\mu 1 + c2\,T\,\mu 1 + En\,\mu 2 - c2\,T\,\mu 2 - c2\,\mu 1\,\tau + cd\,\mu 1\,\tau + c2\,\mu 2\,\tau - cd\,\mu 2\,\tau)}\right) \Big/ (2\,(cd1\,\mu 1 - cd1\,\mu 2))\right\}\right\}$$

```
wcrit = Simplify[w /. solsint[[1]]]
```

$$\frac{1}{(\mu 1 - \mu 2)^2}\, e^{\frac{-2\,cd1 + c1\,\mu 1 - c2\,\mu 1 - 2\,cd1\,v\,\mu 1 - c1\,\mu 2 + c2\,\mu 2 - 2\,cd1\,T\,\mu 2 + 2\,cd1\,v\,\mu 2 - 2\,cd1\,\mu 1\,\tau + 2\,cd1\,\mu 2\,\tau + \sqrt{4\,cd1^2 + (c1 - c2)^2\,(\mu 1 - \mu 2)^2 + 4\,cd1\,(\mu 1 - \mu 2)^2\,(En - cd\,\tau + c2\,(-T+\tau))}}{2\,cd1}}$$
$$\left(-2\,cd1 + \sqrt{\left(4\,cd1^2 + (c1 - c2)^2\,(\mu 1 - \mu 2)^2 + 4\,cd1\,(\mu 1 - \mu 2)^2\,(En - cd\,\tau + c2\,(-T+\tau))\right)}\right)$$

Must Also look on boundaries

Boundary 1 : t = 0

```
w1 = Simplify[w /. t → 0]
```

$$e^{-v\,\mu 1 - T\,\mu 2 + v\,\mu 2 - \mu 1\,\tau + \mu 2\,\tau}\,(En - cd\,\tau + c2\,(-T+\tau))$$

Boundary 2 : v = 0

```
w2 = Simplify[w /. v → 0]
```

$$e^{-t\,\mu 1 + t\,\mu 2 - T\,\mu 2 - \mu 1\,\tau + \mu 2\,\tau}\left(En - c1\,t + c2\,t - cd1\,t^2 - c2\,T + c2\,\tau - cd\,\tau\right)$$

Numerical Example with t + τ + v = 12 and τ = 4

```
wvout = w /. {v → 8 - t}
```

$$e^{-\mu 2\,(-8+T-\tau)-\mu 1\,(8+\tau)}\left(En - c1\,t - cd1\,t^2 - c2\,(-t+T-\tau) - cd\,\tau\right)$$

```
wcrit1 = Simplify[wvout /. solsint[[1]]]
```

$$\frac{1}{(\mu 1-\mu 2)^2}\,e^{-\mu 1\,(8+\tau)+\mu 2\,(8-T+\tau)}\left(-2\,cd1+\sqrt{\left((-2\,cd1+(c1-c2)\,(\mu 1-\mu 2)\right)^2 - 4\,cd1\,(\mu 1-\mu 2)\,(-c1-(\mu 1-\mu 2)\,(En-cd\,\tau)+c2\,(1+T\,\mu 1-T\,\mu 2-\mu 1\,\tau+\mu 2\,\tau))}\right)$$

```
wcrit1 /. {T → 16, τ → 4, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
0.0120308
```

```
solsint[[1]] /. {T → 16, τ → 4, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
{t → 1.44031}
```

```
w1 /. {T → 16, τ → 4, v → 8, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
0.0109295
```

```
w2 /. {T → 16, τ → 4, t → 8, En → 4, c1 → 0.1,
  c2 → 0.2, cd → 0.3, cd1 → 0.05, μ1 → 0.2, μ2 → 0.3}
```

```
-0.0546474
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0122957, {t → 1.}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{-0.0546474, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.2, c2 → 0.1, cd → 0.3, cd1 → 0.05,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.043718, {t → 0.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.15,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0113849, {t → 0.333333}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.15,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{-0.229519, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0327885, {t → 8.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.01,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0177604, {t → 5.}}
```

```
Maximize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.1,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{0.0116126, {t → 0.5}}
```

```
Minimize[{w /. {En → 4, c1 → 0.1, c2 → 0.2, cd → 0.3, cd1 → 0.1,
    μ1 → 0.2, μ2 → 0.3, τ → 4, v → 8 - t, T → 16}, 0 ≤ t, t ≤ 8}, {t}]
```

```
{-0.142083, {t → 8.}}
```