Model E(C)

```
μ1 =.; μ2 =.;
T =.; t =.; τ =.;
v =.; En =.; c1 =.; cd1 =.; c2 =.;
k1 =.; k2 = .;
```

```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v))
   (En - (c1 t + cd1 t + c1 τ + k1 (2 k2 + τ) / (k2 + τ) + c2 (T - t - τ)))
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

```
delwbydelt = D[w, t]
```

$$(-c1 + c2 - cd1)\,e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)} + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

```
delwbydelτ = D[w, τ]
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(-c1 + c2 - \frac{k1}{k2+\tau} + \frac{k1\,(2\,k2+\tau)}{(k2+\tau)^2}\right) + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - cd1\,t - c2\,(-t+T-\tau) - c1\,\tau - \frac{k1\,(2\,k2+\tau)}{k2+\tau}\right)$$

```
sols = Solve[{delwbydelt == 0, delwbydelτ == 0}, {t, τ}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{t \to \left(c1 - c2 + cd1 + En\,\mu1 - k1\,\mu1 - \frac{i\,c1\,\sqrt{k1}\,\sqrt{k2}\,\mu1}{\sqrt{cd1}} + \frac{i\,c2\,\sqrt{k1}\,\sqrt{k2}\,\mu1}{\sqrt{cd1}} + i\,\sqrt{cd1}\,\sqrt{k1}\,\sqrt{k2}\,\mu1 + c1\,k2\,\mu1 - c2\,k2\,\mu1 - c2\,T\,\mu1 - En\,\mu2 + k1\,\mu2 + \frac{i\,c1\,\sqrt{k1}\,\sqrt{k2}\,\mu2}{\sqrt{cd1}} - \frac{i\,c2\,\sqrt{k1}\,\sqrt{k2}\,\mu2}{\sqrt{cd1}} - i\,\sqrt{cd1}\,\sqrt{k1}\,\sqrt{k2}\,\mu2 - c1\,k2\,\mu2 + c2\,k2\,\mu2 + c2\,T\,\mu2\right) \Big/ (c1\,\mu1 - c2\,\mu1 + cd1\,\mu1 - c1\,\mu2 + c2\,\mu2 - cd1\,\mu2),\ \tau \to \frac{i\,\sqrt{k1}\,\sqrt{k2} - \sqrt{cd1}\,k2}{\sqrt{cd1}}\right\},$$

$$\left\{t \to \left(c1 - c2 + cd1 + En\,\mu1 - k1\,\mu1 + \frac{i\,c1\,\sqrt{k1}\,\sqrt{k2}\,\mu1}{\sqrt{cd1}} - \frac{i\,c2\,\sqrt{k1}\,\sqrt{k2}\,\mu1}{\sqrt{cd1}} - i\,\sqrt{cd1}\,\sqrt{k1}\,\sqrt{k2}\,\mu1 + c1\,k2\,\mu1 - c2\,k2\,\mu1 - c2\,T\,\mu1 - En\,\mu2 + k1\,\mu2 - \frac{i\,c1\,\sqrt{k1}\,\sqrt{k2}\,\mu2}{\sqrt{cd1}} + \frac{i\,c2\,\sqrt{k1}\,\sqrt{k2}\,\mu2}{\sqrt{cd1}} + i\,\sqrt{cd1}\,\sqrt{k1}\,\sqrt{k2}\,\mu2 - c1\,k2\,\mu2 + c2\,k2\,\mu2 + c2\,T\,\mu2\right) \Big/ (c1\,\mu1 - c2\,\mu1 + cd1\,\mu1 - c1\,\mu2 + c2\,\mu2 - cd1\,\mu2),\ \tau \to \frac{-i\,\sqrt{k1}\,\sqrt{k2} - \sqrt{cd1}\,k2}{\sqrt{cd1}}\right\}\right\}$$

No real general solution : must look on boundaries

Boundary 1 : t = 0

```
w1 = Simplify[w /. t → 0]
```

$$e^{-v\,\mu1 - T\,\mu2 + v\,\mu2 - \mu1\,\tau + \mu2\,\tau}\left(En - c1\,\tau - \frac{k1\,(2\,k2 + \tau)}{k2 + \tau} + c2\,(-T + \tau)\right)$$

```
delw1bydelv = D[w1, v]
```

$$e^{-v\,\mu1 - T\,\mu2 + v\,\mu2 - \mu1\,\tau + \mu2\,\tau}\,(-\mu1 + \mu2)\left(En - c1\,\tau - \frac{k1\,(2\,k2 + \tau)}{k2 + \tau} + c2\,(-T + \tau)\right)$$

```
vcrit1 = Simplify[Solve[delw1bydelv == 0, v]]
```

$$\{\}$$

No solutions when t = 0

Boundary 2 : $\tau = 0$

```
w2 = Simplify[w /. τ → 0]
```

$$e^{-t\,\mu1 - v\,\mu1 + t\,\mu2 - T\,\mu2 + v\,\mu2}\,(En - 2\,k1 - c1\,t + c2\,t - cd1\,t - c2\,T)$$

```
delw2bydelt = D[w2, t]
```

$$(-c1 + c2 - cd1)\, e^{-t\,\mu1 - v\,\mu1 + t\,\mu2 - T\,\mu2 + v\,\mu2} + e^{-t\,\mu1 - v\,\mu1 + t\,\mu2 - T\,\mu2 + v\,\mu2}\,(En - 2\,k1 - c1\,t + c2\,t - cd1\,t - c2\,T)\,(-\mu1 + \mu2)$$

```
tcrit2 = Solve[delw2bydelt == 0, t]
```

$$\{\{t \to (c1 - c2 + cd1 + En\,\mu1 - 2\,k1\,\mu1 - c2\,T\,\mu1 - En\,\mu2 + 2\,k1\,\mu2 + c2\,T\,\mu2)\ /\ ((c1 - c2 + cd1)\,(\mu1 - \mu2))\}\}$$

```
tcrit2 /. {En → 50, c1 → 0.1, cd1 → 0.2, c2 → 0.2,
  d → 0.3, k1 → 0.1, μ1 → 0.3, μ2 → 0.4, T → 20, v → 16 - t}
```

$$\{\{t \to 448.\}\}$$

```
tcrit2 /. {En → 50, c1 → 0.1, cd1 → 0.05, c2 → 0.2,
  d → 0.3, k1 → 23.5, μ1 → 0.3, μ2 → 0.4, T → 20, v → 16 - t}
```

$$\{\{t \to 10.\}\}$$

```
w2 /. {En → 50, c1 → 0.1, cd1 → 0.05, c2 → 0.2, d → 0.3,
   k1 → 23.5, μ1 → 0.3, μ2 → 0.4, T → 20, v → 16 - t} /. t → 10
```

$$-0.000830779$$

Internal solutions possible when $\tau = 0$, but these not of interest, since w2 is at a minimum not a maximum, rapid development being expensive.

Boundary 3 : v = 0

```
w3 = Simplify[w /. v → 0]
```

$$\frac{1}{k2 + \tau}\, e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)}\,(En\,(k2 + \tau) - k1\,(2\,k2 + \tau) - (k2 + \tau)\,(cd1\,t + c1\,(t + \tau) - c2\,(t - T + \tau)))$$

```
delw3bydelt = D[w3, t]
```

$$-(c1 - c2 + cd1)\, e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)} + \frac{1}{k2 + \tau}\, e^{-\mu1\,(t+\tau) + \mu2\,(t - T + \tau)}\,(-\mu1 + \mu2)\,(En\,(k2 + \tau) - k1\,(2\,k2 + \tau) - (k2 + \tau)\,(cd1\,t + c1\,(t + \tau) - c2\,(t - T + \tau)))$$

```
tcrit3 = Simplify[Solve[delw3bydelt == 0, t]]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{t \to (cd1\,k2 + En\,k2\,\mu1 - 2\,k1\,k2\,\mu1 - En\,k2\,\mu2 + 2\,k1\,k2\,\mu2 + cd1\,\tau + En\,\mu1\,\tau - k1\,\mu1\,\tau - En\,\mu2\,\tau + k1\,\mu2\,\tau - c1\,(k2 + \tau)\,(-1 + \mu1\,\tau - \mu2\,\tau) + c2\,(k2 + \tau)\,(-1 - T\,\mu1 + T\,\mu2 + \mu1\,\tau - \mu2\,\tau))\ /\ ((c1 - c2 + cd1)\,(\mu1 - \mu2)\,(k2 + \tau))\}\}$$

```
w3v = w3 /. {τ → 16 - t}
```

$$\frac{1}{16 + k2 - t}\, e^{-16\,\mu1 + (16 - T)\,\mu2}\,(En\,(16 + k2 - t) - k1\,(16 + 2\,k2 - t) - (16 + k2 - t)\,(16\,c1 + cd1\,t - c2\,(16 - T)))$$

```
tcrit3v = tcrit3 /. {τ → 16 - t}
```

```
{{t →
    (cd1 k2 + cd1 (16 - t) + En k2 μ1 - 2 k1 k2 μ1 + En (16 - t) μ1 - k1 (16 - t) μ1 - En k2 μ2 + 2 k1 k2
        μ2 - En (16 - t) μ2 + k1 (16 - t) μ2 - c1 (16 + k2 - t) (-1 + (16 - t) μ1 - (16 - t) μ2) +
      c2 (16 + k2 - t) (-1 + (16 - t) μ1 - T μ1 - (16 - t) μ2 + T μ2)) /
     ((c1 - c2 + cd1) (16 + k2 - t) (μ1 - μ2))}}
```

```
t3 = t /. tcrit3v[[1]]
```

```
(cd1 k2 + cd1 (16 - t) + En k2 μ1 - 2 k1 k2 μ1 + En (16 - t) μ1 - k1 (16 - t) μ1 - En k2 μ2 +
   2 k1 k2 μ2 - En (16 - t) μ2 + k1 (16 - t) μ2 - c1 (16 + k2 - t) (-1 + (16 - t) μ1 - (16 - t) μ2) +
   c2 (16 + k2 - t) (-1 + (16 - t) μ1 - T μ1 - (16 - t) μ2 + T μ2)) /
 ((c1 - c2 + cd1) (16 + k2 - t) (μ1 - μ2))
```

```
solt = Solve[t == t3, t]
```

```
{{t → (c1 - c2 + cd1 - 16 c1 μ1 + 16 c2 μ1 + 16 cd1 μ1 + En μ1 - k1 μ1 + cd1 k2 μ1 -
       c2 T μ1 + 16 c1 μ2 - 16 c2 μ2 - 16 cd1 μ2 - En μ2 + k1 μ2 - cd1 k2 μ2 + c2 T μ2 -
       √((-c1 + c2 - cd1 + 16 c1 μ1 - 16 c2 μ1 - 16 cd1 μ1 - En μ1 + k1 μ1 - cd1 k2 μ1 + c2 T μ1 -
             16 c1 μ2 + 16 c2 μ2 + 16 cd1 μ2 + En μ2 - k1 μ2 + cd1 k2 μ2 - c2 T μ2)^2 -
          4 (cd1 μ1 - cd1 μ2) (16 c1 - 16 c2 + 16 cd1 + c1 k2 - c2 k2 + cd1 k2 - 256 c1 μ1 +
             256 c2 μ1 + 16 En μ1 - 16 k1 μ1 - 16 c1 k2 μ1 + 16 c2 k2 μ1 + En k2 μ1 -
             2 k1 k2 μ1 - 16 c2 T μ1 - c2 k2 T μ1 + 256 c1 μ2 - 256 c2 μ2 - 16 En μ2 + 16 k1 μ2 +
             16 c1 k2 μ2 - 16 c2 k2 μ2 - En k2 μ2 + 2 k1 k2 μ2 + 16 c2 T μ2 + c2 k2 T μ2))) /
     (2 (cd1 μ1 - cd1 μ2))},
  {t → (c1 - c2 + cd1 - 16 c1 μ1 + 16 c2 μ1 + 16 cd1 μ1 +
       En μ1 - k1 μ1 + cd1 k2 μ1 - c2 T μ1 + 16 c1 μ2 - 16 c2 μ2 -
       16 cd1 μ2 - En μ2 + k1 μ2 - cd1 k2 μ2 + c2 T μ2 +
       √((-c1 + c2 - cd1 + 16 c1 μ1 - 16 c2 μ1 - 16 cd1 μ1 - En μ1 + k1 μ1 - cd1 k2 μ1 + c2 T μ1 -
             16 c1 μ2 + 16 c2 μ2 + 16 cd1 μ2 + En μ2 - k1 μ2 + cd1 k2 μ2 - c2 T μ2)^2 -
          4 (cd1 μ1 - cd1 μ2) (16 c1 - 16 c2 + 16 cd1 + c1 k2 - c2 k2 + cd1 k2 - 256 c1 μ1 +
             256 c2 μ1 + 16 En μ1 - 16 k1 μ1 - 16 c1 k2 μ1 + 16 c2 k2 μ1 + En k2 μ1 - 2 k1 k2 μ1 - 16
              c2 T μ1 - c2 k2 T μ1 + 256 c1 μ2 - 256 c2 μ2 - 16 En μ2 + 16 k1 μ2 + 16 c1 k2 μ2 - 16 c2
              k2 μ2 - En k2 μ2 + 2 k1 k2 μ2 + 16 c2 T μ2 + c2 k2 T μ2))) / (2 (cd1 μ1 - cd1 μ2))}}
```

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 0.2, k2 → 0.2}
```

```
{{t → 312.668}, {t → 16.1991}}
```

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 2, k2 → 2}
```

```
{{t → 300.761}, {t → 17.9057}}
```

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 2, k2 → 20}
```

```
{{t → 301.67}, {t → 34.9963}}
```

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 20, k2 → 20}
```

```
{{t → 197.208}, {t → 19.4583}}
```

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 20, k2 → 2}
```

$$\{\{t \to 182.29\}, \{t \to 16.3769\}\}$$

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.05, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 0.2, k2 → 0.2}
```

$$\{\{t \to 958.001\}, \{t \to 16.1992\}\}$$

```
solt /. {En → 50, c1 → 0.1, cd1 → 0.25, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20, k1 → 0.2, k2 → 0.2}
```

$$\{\{t \to 183.601\}, \{t \to 16.199\}\}$$

But t <= 16, so none of these solutions work either

Need to look in the corners, too

```
w00 = Simplify[w /. {t → 0, τ → 0}]
```

$$e^{-v\,\mu 1-T\,\mu 2+v\,\mu 2}\ (En - 2\,k1 - c2\,T)$$

```
w00 /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, v → 16, T → 20, k1 → 1}
```

$$0.0731085$$

```
w0T = Simplify[w /. {t → 0, v → 0}]
```

$$e^{-T\,\mu 2-\mu 1\,\tau+\mu 2\,\tau}\left(En - c1\,\tau - \frac{k1\ (2\,k2+\tau)}{k2+\tau} + c2\ (-T+\tau)\right)$$

```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 1, k2 → 1}
```

$$0.0773308$$

```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 1, k2 → 1}
```

$$0.0773308$$

```
w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 20, k2 → 20}
```

$$0.0273972$$

```
wT0 = Simplify[w /. {v → 0, τ → 0}]
```

$$e^{-t\,\mu 1+t\,\mu 2-T\,\mu 2}\ (En - 2\,k1 - c1\,t + c2\,t - cd1\,t - c2\,T)$$

```
wT0 /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, t → 16, T → 20, k1 → 1}
```

$$0.0717793$$

```
Maximize[
 {w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4,
    T → 20, v → 16 - t - τ, k1 → 1, k2 → 1}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

$$\{0.0773308, \{t \to 3.39025 \times 10^{-9}, \tau \to 16.\}\}$$

```
Maximize[
 {w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4,
    T → 20, v → 16 - t - τ, k1 → 2, k2 → 2}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

$$\{0.0753978, \{t \to 6.91589 \times 10^{-9}, \tau \to 16.\}\}$$

```
Maximize[
 {w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4,
    T → 20, v → 16 - t - τ, k1 → 20, k2 → 0.2}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

$$\{0.0454487, \{t \to 5.64628 \times 10^{-7}, \tau \to 16.\}\}$$

```
Maximize[
 {w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4,
    T → 20, v → 16 - t - τ, k1 → 0.2, k2 → 20}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

$$\{0.0785732, \{t \to 3.23792 \times 10^{-9}, \tau \to 16.\}\}$$

```
w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, T → 20, v → 0,  t → 8, τ → 8, k1 → 1, k2 → 1}
```

0.0752501

```
w /. {En → 50, c1 → 0.1, cd1 → 0.15, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, T → 20, v → 0,  t → 3, τ → 13, k1 → 1, k2 → 1}
```

0.0765622

Maximum appears to be when {t → 0, v → 0}

```
Maximize[
 {w /. {En → 50, c1 → 0.1, cd1 → 0.5, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4,
    T → 20, v → 16 - t - τ, k1 → 0.2, k2 → 20}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
```

$$\{0.0785732, \{t \to 0., \tau \to 16.\}\}$$