

Spencer et al. Model B

$\mu 1 = .; \mu 2 = .; T = .; t = .; \tau = .; v = .; En = .; c1 = .; d = .; c2 = .;$

$$w = e^{-\mu 1 (t + \tau + v)} e^{-\mu 2 (T - t - \tau - v)} (En - (c1 t + c1 \tau + d + c2 (T - t - \tau)))$$

$$e^{-\mu 2 (-t + T - v - \tau) - \mu 1 (t + v + \tau)} (-d + En - c1 t - c2 (-t + T - \tau) - c1 \tau)$$

$delwbydelt = D[w, t]$

$$(-c1 + c2) e^{-\mu 2 (-t + T - v - \tau) - \mu 1 (t + v + \tau)} +$$

$$e^{-\mu 2 (-t + T - v - \tau) - \mu 1 (t + v + \tau)} (-\mu 1 + \mu 2) (-d + En - c1 t - c2 (-t + T - \tau) - c1 \tau)$$

$delwbydel\tau = D[w, \tau]$

$$(-c1 + c2) e^{-\mu 2 (-t + T - v - \tau) - \mu 1 (t + v + \tau)} +$$

$$e^{-\mu 2 (-t + T - v - \tau) - \mu 1 (t + v + \tau)} (-\mu 1 + \mu 2) (-d + En - c1 t - c2 (-t + T - \tau) - c1 \tau)$$

$sols = \text{Solve}[\{delwbydelt == 0, delwbydel\tau == 0\}, \{t, \tau\}]$

Solve::ifun : Inverse functions are being used by Solve, so

some solutions may not be found; use Reduce for complete solution information. >>

Solve::svars : Equations may not give solutions for all "solve" variables. >>

$$\left\{ \left\{ \tau \rightarrow -t - \frac{-c1 + c2 + d \mu 1 - En \mu 1 + c2 T \mu 1 - d \mu 2 + En \mu 2 - c2 T \mu 2}{(c1 - c2) (\mu 1 - \mu 2)} \right\} \right\}$$

No general solution : must look on boundaries

Boundary I : $t = 0$

$w1 = \text{Simplify}[w /. t \rightarrow 0]$

$$-e^{-v \mu 1 - T \mu 2 + v \mu 2 - \mu 1 \tau + \mu 2 \tau} (d - En + c2 T + c1 \tau - c2 \tau)$$

$delw1bydel\tau = D[w1, \tau]$

$$-(c1 - c2) e^{-v \mu 1 - T \mu 2 + v \mu 2 - \mu 1 \tau + \mu 2 \tau} - e^{-v \mu 1 - T \mu 2 + v \mu 2 - \mu 1 \tau + \mu 2 \tau} (-\mu 1 + \mu 2) (d - En + c2 T + c1 \tau - c2 \tau)$$

$\tau crit1 = \text{Solve}[delw1bydel\tau == 0, \tau]$

$$\{\{ \tau \rightarrow (c1 - c2 - d \mu 1 + En \mu 1 - c2 T \mu 1 + d \mu 2 - En \mu 2 + c2 T \mu 2) / ((c1 - c2) (\mu 1 - \mu 2)) \}\}$$

$w1v = w1 /. \{v \rightarrow 16 - \tau\}$

$$-e^{-T \mu 2 - \mu 1 (16 - \tau) + \mu 2 (16 - \tau) - \mu 1 \tau + \mu 2 \tau} (d - En + c2 T + c1 \tau - c2 \tau)$$

$wmax1 = \text{Simplify}[w1v /. \tau crit1[[1]]]$

$$\frac{(c1 - c2) e^{-16 \mu 1 - (-16 + T) \mu 2}}{-\mu 1 + \mu 2}$$

$$wmax1 /. \{En \rightarrow 50, c1 \rightarrow 0.1, c2 \rightarrow 0.2, d \rightarrow 0.3, \mu 1 \rightarrow 0.3, \mu 2 \rightarrow 0.4, T \rightarrow 20\}$$

$$-0.00166156$$

Boundary 2 : $\tau = 0$

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w2 = Simplify[w /.  $\tau \rightarrow 0$ ]
 $-e^{-(t+v) \mu_1 + (t-T+v) \mu_2} (d - En + c_1 t - c_2 t + c_2 T)$ 

delw2bydelt = D[w2, t]
 $-(c_1 - c_2) e^{-(t+v) \mu_1 + (t-T+v) \mu_2} - e^{-(t+v) \mu_1 + (t-T+v) \mu_2} (d - En + c_1 t - c_2 t + c_2 T) (-\mu_1 + \mu_2)$ 

tcrit2 = Solve[delw2bydelt == 0, t]
 $\{\{t \rightarrow (c_1 - c_2 - d \mu_1 + En \mu_1 - c_2 T \mu_1 + d \mu_2 - En \mu_2 + c_2 T \mu_2) / ((c_1 - c_2) (\mu_1 - \mu_2))\}\}$ 

Exactly the same as Solution for  $\tau$  when  $t = 0$ 

w2v = w2 /. {v  $\rightarrow 16 - t$ }
 $-e^{-16 \mu_1 + (16-T) \mu_2} (d - En + c_1 t - c_2 t + c_2 T)$ 

wmax2 = Simplify[w2v /. tcrit2[[1]]]

$$\frac{(-c_1 + c_2) e^{-16 \mu_1 - (-16+T) \mu_2}}{\mu_1 - \mu_2}$$


wmax2 /. {En  $\rightarrow 50$ , c1  $\rightarrow 0.1$ , c2  $\rightarrow 0.2$ , d  $\rightarrow 0.3$ ,  $\mu_1 \rightarrow 0.3$ ,  $\mu_2 \rightarrow 0.4$ , T  $\rightarrow 20$ }
-0.00166156

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Boundary 3 : $v = 0$

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w3 = Simplify[w /. v  $\rightarrow 0$ ]
 $-e^{-\mu_1 (t+\tau) + \mu_2 (t-T+\tau)} (d - En + c_1 t - c_2 t + c_2 T + c_1 \tau - c_2 \tau)$ 

delw3bydelt = D[w3, t]
 $-(c_1 - c_2) e^{-\mu_1 (t+\tau) + \mu_2 (t-T+\tau)} - e^{-\mu_1 (t+\tau) + \mu_2 (t-T+\tau)} (-\mu_1 + \mu_2) (d - En + c_1 t - c_2 t + c_2 T + c_1 \tau - c_2 \tau)$ 

tcrit3 = Simplify[Solve[delw3bydelt == 0, t]]
 $\{\{t \rightarrow (- (d - En) (\mu_1 - \mu_2) + c_2 (-1 - T \mu_1 + T \mu_2 + \mu_1 \tau - \mu_2 \tau) + c_1 (1 - \mu_1 \tau + \mu_2 \tau)) / ((c_1 - c_2) (\mu_1 - \mu_2))\}\}$ 

w3v = w3 /. { $\tau \rightarrow 16 - t$ }
 $-e^{-16 \mu_1 + (16-T) \mu_2} (d - En + c_1 (16 - t) - c_2 (16 - t) + c_1 t - c_2 t + c_2 T)$ 

wmax3 = Simplify[w3v /. tcrit3[[1]]]
 $-e^{-16 \mu_1 - (-16+T) \mu_2} (16 c_1 + d - En + c_2 (-16 + T))$ 

wmax3 /. {En  $\rightarrow 50$ , c1  $\rightarrow 0.1$ , c2  $\rightarrow 0.2$ , d  $\rightarrow 0.3$ ,  $\mu_1 \rightarrow 0.3$ ,  $\mu_2 \rightarrow 0.4$ , T  $\rightarrow 20$ }
0.0785917

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Need to look in the corners, too

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w00 = Simplify[w /. {t  $\rightarrow 0$ ,  $\tau \rightarrow 0$ }]
 $-e^{-v \mu_1 - T \mu_2 + v \mu_2} (d - En + c_2 T)$ 

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```
w00 /. {En → 50, c1 → 0.1, c2 → 0.2,
  d → 0.3, μ1 → 0.3, μ2 → 0.4, v → 16, T → 20}
0.0759332
```

```
w0T = Simplify[w /. {t → 0, v → 0}]
 $-e^{-T\mu_2 - \mu_1\tau + \mu_2\tau} (d - En + c_2T + c_1\tau - c_2\tau)$ 
```

```
w0T /.
  {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20}
0.0785917
```

```
wT0 = Simplify[w /. {v → 0, τ → 0}]
 $-e^{-t\mu_1 + t\mu_2 - T\mu_2} (d - En + c_1t - c_2t + c_2T)$ 
```

```
wT0 /.
  {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, t → 16, T → 20}
0.0785917
```

```
{tcrit1[[1]], tcrit2[[1]]} /.
  {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20}
{{τ → -467.}, {t → -467.}}
```

Not biological solutions

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{wmax1, wmax2, w00, w0T, wT0} /.
  {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20}
{-0.00166156, -0.00166156, 45.7 e-8.+0.1v,
  -e-8.+0.1τ (-45.7 - 0.1 τ), -e-8.+0.1t (-45.7 - 0.1 t)}
```

```
Maximize[{w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3,
  μ2 → 0.4, T → 20, v → 16 - t - τ}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
{0.0785917, {t → 16., τ → 0.}}
```

```
{wmax1, wmax2, w00, w0T, wT0} /. {L → 0.9, c1 → 0.01,
  c2 → 0.2, cd → 0.3, μ1 → 0.3, μ2 → 0.2, μd → 0.25, T → 20}
{0.00702594, 0.00702594, -e-4.-0.1v (4. + d - En),
  -e-4.-0.1τ (4. + d - En - 0.19 τ), -e-4.-0.1t (4. + d - En - 0.19 t)}
```

```
{wmax1, wmax2, w00, w0T, wT0} /. {L → 0.9, c1 → 0.01,
  c2 → 0.2, cd → 0.21, μ1 → 0.3, μ2 → 0.2, μd → 0.29, T → 20}
{0.00702594, 0.00702594, -e-4.-0.1v (4. + d - En),
  -e-4.-0.1τ (4. + d - En - 0.19 τ), -e-4.-0.1t (4. + d - En - 0.19 t)}
```

```
w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, T → 20, v → 0, t → 8, τ → 8}
0.0785917
```

```
w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, T → 20, v → 0, t → 3, τ → 13}
0.0785917
```