Spencer et al. Model D

```
μ1 =.; μ2 =.; T =.; t =.; τ =.; v =.; En =.; c1 =.; c2 =.; k1 =.; k2 =.; j =.;
```

```
w = e^(-μ1 (t + τ + v)) e^(-μ2 (T - t - τ - v)) (En - (c1 t + c1 τ + k1 (τ - k2)^2 + c2 (T - t - τ)))
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - k1\,(-k2+\tau)^2\right)$$

```
delwbydelt = D[w, t]
```

$$(-c1+c2)\,e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)} + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - k1\,(-k2+\tau)^2\right)$$

```
delwbydelτ = D[w, τ]
```

$$e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-c1+c2-2\,k1\,(-k2+\tau)) + e^{-\mu 2\,(-t+T-v-\tau)-\mu 1\,(t+v+\tau)}\,(-\mu 1+\mu 2)\left(En - c1\,t - c2\,(-t+T-\tau) - c1\,\tau - k1\,(-k2+\tau)^2\right)$$

```
sols = Simplify[Solve[{delwbydelt == 0, delwbydelτ == 0}, {t, τ}]]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\{\{t \to (En\,(\mu 1-\mu 2)+c2\,(-1+k2\,\mu 1-T\,\mu 1-k2\,\mu 2+T\,\mu 2)+c1\,(1+k2\,(-\mu 1+\mu 2)))/((c1-c2)\,(\mu 1-\mu 2)),\ \tau \to k2\}\}$$

```
Simplify[w /. sols[[1]]]
```

$$\frac{1}{-\mu 1+\mu 2}(c1-c2)\,e^{\frac{En\,(-\mu 1+\mu 2)-c1\,(1+v\,(\mu 1-\mu 2)+T\,\mu 2)+c2\,(1+T\,\mu 1+v\,\mu 1-v\,\mu 2)}{c1-c2}}$$

Negative (since c1 < c2 and $\mu 1 < \mu 2$)

```
sols[[1]] /. {En → 50, c1 → 0.1, c2 → 0.2,
  μ1 → 0.3, μ2 → 0.4, T → 20, v → 16 - t - τ, k1 → 1, k2 → 1}
```

$$\{t \to -471.,\ \tau \to 1\}$$

```
w /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 1, k2 → 1} /. {t → -471.00000000000006`, τ → 1}
```

$$-0.00166156$$

No fully internal solution : must look on boundaries

Boundary 1 : t = 0

```
w1 = Simplify[w /. {t → 0, τ → j - v}]
```

$$e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\left(En + c1\,(-j+v) - k1\,(-j+k2+v)^2 - c2\,(-j+T+v)\right)$$

```
delw1bydelv = D[w1, v]
```

$$e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\,(c1-c2-2\,k1\,(-j+k2+v))$$

```
vcrit1 = Simplify[Solve[delw1bydelv == 0, v]]
```

$$\left\{\left\{v \to \frac{c1-c2+2\,j\,k1-2\,k1\,k2}{2\,k1}\right\}\right\}$$

```
Simplify[w1 /. vcrit1[[1]]]
```

$$\frac{1}{4\,k1}\,e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\left(c1^2+c2^2+4\,En\,k1-2\,c1\,(c2+2\,k1\,k2)+4\,c2\,k1\,(k2-T)\right)$$

```
Maximize[{w1 /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3,
    μ2 → 0.4, T → 20, j → 16, k1 → 1, k2 → 1}, 0 ≤ v, v ≤ 16}, {v}]
```

```
{0.0766019, {v → 14.95}}
```

```
Maximize[{w1 /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3,
    μ2 → 0.4, T → 20, j → 16, k1 → 2, k2 → 1}, 0 ≤ v, v ≤ 16}, {v}]
```

```
{0.0765999, {v → 14.975}}
```

One edge solution when t = 0

Boundary 2 : $\tau = 0$

```
w2 = Simplify[w /. {τ → 0, v → j - t}]
```

$$e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\left(En-k1\,k2^2-c1\,t+c2\,t-c2\,T\right)$$

```
delw2bydelt = D[w2, t]
```

$$(-c1+c2)\,e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}$$

```
tcrit2 = Solve[delw2bydelt == 0, t]
```

```
{{}}
```

```
Maximize[{w2 /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3,
    μ2 → 0.4, T → 20, j → 16, k1 → 1, k2 → 1}, 0 ≤ t, t ≤ 16}, {t}]
```

```
{0.0774286, {t → 16.}}
```

One corner solution when $\tau = 0$

Boundary 3 : v = 0

```
w3 = Simplify[w /. {v → 0, τ → j - t}]
```

$$e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\left(En-c1\,t+c1\,(-j+t)-k1\,(-j+k2+t)^2+c2\,(j-T)\right)$$

```
delw3bydelt = D[w3, t]
```

$$-2\,e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\,k1\,(-j+k2+t)$$

```
tcrit3 = Simplify[Solve[delw3bydelt == 0, t]]
```

```
{{t → j - k2}}
```

```
Simplify[w3 /. tcrit3[[1]]]
```

$$e^{-j\,\mu 1+j\,\mu 2-T\,\mu 2}\,(En-c1\,j+c2\,(j-T))$$

```
Maximize[{w3 /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3,
    μ2 → 0.4, T → 20, j → 16, k1 → 1, k2 → 1}, 0 ≤ t, t ≤ 16}, {t}]
```

```
{0.0790901, {t → 15.}}
```

```
Maximize[{w3 /. {En → 50, c1 → 0.1, c2 → 0.2, μ1 → 0.3,
    μ2 → 0.4, T → 20, j → 16, k1 → 2, k2 → 1}, 0 ≤ t, t ≤ 16}, {t}]
{0.0790901, {t → 15.}}
```

One edge solution when v = 0

Need to look in the corners, too

```
w00 = Simplify[w /. {t → 0, τ → 0}]
e^(-v μ1-T μ2+v μ2) (En - k1 k2^2 - c2 T)

w00 /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, v → 16, T → 20, k1 → 1, k2 → 1}
0.0747701

w0T = Simplify[w /. {t → 0, v → 0}]
e^(-T μ2-μ1 τ+μ2 τ) (En - c2 T - k1 (k2 - τ)^2 - c1 τ + c2 τ)

w0T /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, τ → 16, T → 20, k1 → 1, k2 → 1}
-0.29476

wT0 = Simplify[w /. {v → 0, τ → 0}]
e^(-t μ1+t μ2-T μ2) (En - k1 k2^2 - c1 t + c2 t - c2 T)

wT0 /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3,
  μ1 → 0.3, μ2 → 0.4, t → 16, T → 20, k1 → 1, k2 → 1}
0.0774286

Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 1, k2 → 1}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
{0.0790901, {t → 15., τ → 1.00001}}

Maximize[
 {w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3, μ2 → 0.4, T → 20,
    v → 16 - t - τ, k1 → 1, k2 → 2}, 0 ≤ t, 0 ≤ τ, τ ≤ 16 - t}, {t, τ}]
{0.0790901, {t → 14., τ → 2.}}

w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3,
  μ2 → 0.4, T → 20, v → 0,  t → 8, τ → 8, k1 → 1, k2 → 1}
-0.00232618

w /. {En → 50, c1 → 0.1, c2 → 0.2, d → 0.3, μ1 → 0.3,
  μ2 → 0.4, T → 20, v → 0,  t → 3, τ → 13, k1 → 1, k2 → 1}
-0.160174
```