

# A theoretical and numerical approach for selecting miniaturized antenna topologies on magneto-dielectric substrates

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**Abstract**—There is an increasing interest in developing miniaturized antennas for ultra-low power applications (tens of  $\mu\text{W}$ ) in the microwave range. However, to guarantee the system operations in such low-power conditions, radiation performances need to be preserved even if the adopted antennas dimensions are small compared to the wavelength. For this purpose, magneto-dielectric materials are currently exploited as promising substrates. In this paper we demonstrate by a theoretical approach that radiation efficiency can be preserved only by selected combinations of antenna topologies and substrate characteristics. Indeed, materials with relative permeability greater than unit, can be efficiently adopted only by antennas that may be represented as equivalent magnetic sources. Conversely, we demonstrate that if equivalent electric sources are involved, the antenna performance are significantly degraded. The analytical development is then confirmed by full-wave numerical simulations.

**Keywords**—*antenna miniaturization; magneto-dielectric substrates; full-wave simulation; antenna electric and magnetic equivalent currents.*

## I. INTRODUCTION

There are a variety of new application areas in which it is needed to achieve ultra-small wireless systems, such that they can be worn [1], and/or implanted under the tissues (e.g. in the case of biomedical devices). To accomplish this, it is required that the antenna system provides extremely reduced dimensions while preserving the best radiation performance to minimize the power consumption. This is a challenging task since it is well known that for best antenna operation its dimensions need to be proportionally related to the wavelength. In the UHF band, where the majority of these applications are developed, the latter is of the order of hundred millimeters. For this purpose, in the last few years, a significant research effort has been dedicated to the strategic use of design techniques and materials enabling the miniaturization of the components while maintaining the best possible performance. A highly exploited solution to build miniaturized and non-invasive antennas is the patch antenna mounted on dense dielectric substrate. The reduced effective wavelength  $\lambda_g$  is obtained at the expense of an increased patch-ground plane coupling, thus reducing the field *fringing effect*: in this way, the radiating properties of the

antenna may be degraded and the patch behaves more as a microstrip resonator. Indeed a significant reduction of the radiation resistance is observed, which causes a decay in radiation efficiency. At the same time a too low antenna impedance is obtained, which is difficult to be matched. An emerging alternative to dense dielectrics is provided by the recent realization of materials characterized by relative magnetic permeability  $\mu_r$  greater than unit [2]-[5]. They could overcome the above-mentioned limitations since, for a given relative permittivity and permeability  $\epsilon_r$  and  $\mu_r$ , they could simultaneously guarantee the required guided-wavelength reduction while controlling the medium impedance. A very interesting theoretical approach has been introduced in [6]. It is based on the antenna equivalent current sources, for establishing a set of design rules for a patch antenna on magneto-dielectric substrates.

In this paper we start from that approach and we develop an alternative theoretical design method, by considering representative antenna topologies on only magnetic (OM) and only dielectric (OD) materials. Then, based on the Love's principle of equivalence [7], we compute the associated equivalent current sources and we demonstrate that, depending on the antenna topology, high relative permittivity or permeability should be selected. The derived design rules are then confirmed by numerical simulations based on the full-wave analysis of the structure.

## II. THEORETICAL FORMULATION

Let us analyse the behaviour of resonant antennas, surrounded by different media, in terms of their equivalent current sources and volumes involved. Resonant antennas are chosen since they guarantee the best radiation efficiency, which is mandatory when only ultra-low power excitations are available. In this first step, for the sake of simplicity, we neglect both dielectric and magnetic losses.

### A. Computation of the equivalent antenna current sources

First let us consider an antenna immersed in empty space (ES) satisfying the resonance conditions. The magnetic energy stored at resonance can be computed in a defined volume  $V_{n-f}$ , which includes the whole near-field generated by the antenna:

$$\int_{V_{n-f}} \varepsilon_0 |\mathbf{E}(r, \theta, \phi)|^2 dV = \int_{V_{n-f}} \mu_0 |\mathbf{H}(r, \theta, \phi)|^2 dV \quad (1)$$

Let us now substitute the free-space with another isotropic medium: in this case we adopt an OM material with permeability  $\mu_r \mu_0$  and permittivity  $\varepsilon_0$ . We can reasonably suppose that the volume containing the OM material antenna near-field ( $V_{n-f}^{(\mu)}$ ) is reduced, with respect to the ES case, according to the rule relating the guided wavelength to the relative electromagnetic constants.

Accordingly, we assume that the same energy balance computed by (1) can be obtained in the reduced volume:

$$V_{n-f}^{(\mu)} \approx \frac{V_{n-f}}{\sqrt{\mu_r^3}} \quad (2)$$

The corresponding electric fields  $\mathbf{E}$  and  $\mathbf{E}_\mu$  on these boundaries could be supposed to be related as follows:

$$|\mathbf{E}(r, \theta, \phi)| = \mu_r^{-1/2} |\mathbf{E}_\mu(r_\mu, \theta, \phi)| \quad (3)$$

where

$$r_\mu = r \mu_r^{-1/2} \quad (4)$$

and  $\mathbf{E}_\mu$  represents the electric near-field vector associated with the antenna based on the OM material. These relationships are validated in the next section by the numerical simulation of two patch antennas.

Let us now adopt the variable substitution derived by (2) for  $dV$  and the hypothesis (3) to compute the energy stored inside the magnetic material:

$$\int_{V_{n-f}^{(\mu)}} \varepsilon_0 |\mathbf{E}_\mu(r_\mu, \theta, \phi)|^2 dV^\mu = \mu_r^{-3/2} \int_{V_{n-f}} \varepsilon_0 \mu_r |\mathbf{E}(r, \theta, \phi)|^2 dV \quad (5)$$

By using (1) and (5), the energy balance at the resonance frequency allows us to define the following equivalences among magnetic and electric stored energies:

$$\int_{V_{n-f}^{(\mu)}} \mu_0 \mu_r |\mathbf{H}_\mu(r_\mu, \theta, \phi)|^2 dV^\mu = \int_{V_{n-f}} \varepsilon_0 |\mathbf{E}_\mu(r_\mu, \theta, \phi)|^2 dV^\mu = \mu_r^{-1/2} \int_{V_{n-f}} \varepsilon_0 |\mathbf{E}(r, \theta, \phi)|^2 dV = \mu_r^{-1/2} \int_{V_{n-f}} \mu_0 |\mathbf{H}(r, \theta, \phi)|^2 dV \quad (6)$$

where  $\mathbf{H}_\mu$  is the magnetic near-field vector in the OM case.

Hence, by considering the first and the last terms in (6) and by adopting again the variable substitution (2), the following equation holds for the magnetic near-fields:

$$\int_{V_{n-f}} |\mathbf{H}_\mu(r_\mu, \theta, \phi)|^2 dV = \int_{V_{n-f}} |\mathbf{H}(r, \theta, \phi)|^2 dV \quad (7)$$

Let us develop the integrands (7) as product of functions:

$$\begin{aligned} |\mathbf{H}_\mu(r_\mu, \theta, \phi)| &= A_\mu(r_\mu) \cdot B(\theta, \phi) \cdot C_\mu \\ |\mathbf{H}(r, \theta, \phi)| &= A(r) \cdot B(\theta, \phi) \cdot C \end{aligned} \quad (8)$$

where we assume that the functions  $B$  and  $B_\mu$ , representing the dependence on the elevation and azimuth variables, are the same for the two magnetic fields, i.e. the two antennas share the same shape of the radiation functions, while the functions  $A$

and  $A_\mu$ , which depend only on the radial distance, are different in the two cases (according to (4)); the factors  $C$  and  $C_\mu$  simply depend on the material properties.

From (7) and (8), and taking into account the field dependence on  $r$ , it is possible to obtain:

$$|\mathbf{H}_\mu(r_\mu, \theta, \phi)| = |\mathbf{H}(r, \theta, \phi)| \quad (9)$$

This assumption is confirmed by the numerical simulation shown below.

We can now make use of the Love's field equivalence principle [7], and first compute the magnitude of the tangential components for the electromagnetic fields  $\mathbf{E}_\tau$ ,  $\mathbf{E}_\tau^{(\mu)}$  and  $\mathbf{H}_\tau$ ,  $\mathbf{H}_\tau^{(\mu)}$  at the volumes boundaries:

$$\begin{aligned} |\mathbf{E}_\tau| &= \mu_r^{-1/2} |\mathbf{E}_\tau^{(\mu)}| \\ |\mathbf{H}_\tau| &= |\mathbf{H}_\tau^{(\mu)}| \end{aligned} \quad (10)$$

The corresponding electric and magnetic magnitude of surface equivalent currents  $\mathbf{J}_s$ ,  $\mathbf{J}_s^{(\mu)}$  and  $\mathbf{M}_s$ ,  $\mathbf{M}_s^{(\mu)}$  can be related as:

$$\begin{aligned} |\mathbf{J}_s^{(\mu)}| &= |\mathbf{J}_s| = |\hat{\mathbf{n}} \times \mathbf{H}_\tau| \\ |\mathbf{M}_s^{(\mu)}| &= \sqrt{\mu_r} |\mathbf{M}_s| = |\mathbf{E}_\tau \times \hat{\mathbf{n}}| \end{aligned} \quad (11)$$

where in (11), (3) and (9) are used and the superscript  $(\mu)$  indicates the vectors in the OM case.

It is noteworthy that the equivalent electric sources are not affected by the introduction of a magnetic material, while the equivalent magnetic sources are increased by a factor  $\sqrt{\mu_r}$ , approximately. Thus a suitable exploitation of a high permittivity substrate (OM) is suggested for those antennas based mainly on equivalent magnetic sources. In this case we do not expect significant degradation of the antenna performances whereas its dimensions are reduced according to (4).

By means of the duality principle an only-dielectric (OD) material can be described following the same steps, thus providing results similar to (11), with the superscript  $(\varepsilon)$  indicating the vectors in the OD case:

$$\begin{aligned} |\mathbf{E}_\tau| &= |\mathbf{E}_\tau^{(\varepsilon)}| \quad ; \quad |\mathbf{H}_\tau| = \varepsilon_r^{-1/2} |\mathbf{H}_\tau^{(\varepsilon)}| \\ |\mathbf{J}_s^{(\varepsilon)}| &= \sqrt{\varepsilon_r} |\mathbf{J}_s| = |\hat{\mathbf{n}} \times \mathbf{H}_\tau| \\ |\mathbf{M}_s^{(\varepsilon)}| &= |\mathbf{M}_s| = |\mathbf{E}_\tau \times \hat{\mathbf{n}}| \end{aligned} \quad (12)$$

## B. Computation of the far-field power density

As stated in [9], the near-field and source properties provided by (3), (9), (11) can also be adopted for the far-field vectors, thus we can evaluate the active power radiated by antennas exploiting an OM substrate.

If the antenna may be mainly represented by its equivalent electric current  $J_S$ , the relationship between the radiated power density by the antenna realized in the ES medium ( $P_{J_S}$ ) and the radiated power density by the antenna realized on the OM medium ( $P_{J_S}^{(\mu)}$ ) may be approximately related as:

$$P_{J_S}^{(\mu)} \approx \mu_r^{-1} P_{J_S} \quad (13)$$

According to the results described in the previous subsection it is clear that miniaturization by an OM material is inconvenient in case of an antenna exploiting electrical equivalent currents.

Conversely, for antennas exploiting magnetic equivalent currents  $M_S$ , an OM substrate allows a suitable antenna dimensions reduction since it preserves the radiation performances, but behaves like an empty space from the point of view of active radiated power density:

$$P_{M_S}^{(\mu)} \approx P_{M_S} \quad (14)$$

The obtained power density (14), allows to straightforwardly predict the corresponding radiation efficiency and to confirm the advantage in using an OM material in presence of magnetic equivalent currents. It is worth mentioning that in the evaluation of the flux of (14) the relation (4) has to be considered.

Starting from (12) dual relationships hold for an antenna on an OD material.

### III. NUMERICAL VALIDATION

The conclusions of the previous section are now validated by full-wave numerical simulation. CST Microwave Studio [8] is adopted. For demonstration purposes, as a first example of application we analyse a rectangular patch, designed to operate at 2.4 GHz. Two different patch configurations are compared: a) the space between the patch and the ground plane is filled by empty space (ES); b) the space between the patch and the ground plane is filled by between an OM material (with  $\epsilon_r=1$ ,  $\mu_r=4$ ). The resulting dimensions are:  $L_a=54$  mm,  $W_a=80$  mm; ii) and:  $L_b=32$  mm,  $W_b=40$  mm (approximately obeying to (4)). (Fig. 1).

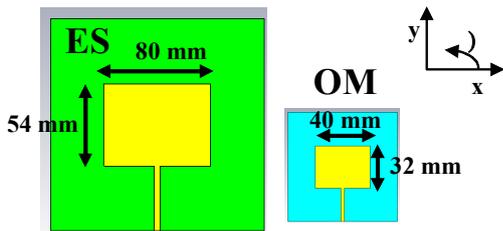


Fig. 1. Top views of the analysed patch antennas: these antenna types are described mainly by magnetic current sources.

First of all, let us discuss the numerical results obtained with the two configurations in terms of the near-field regions of the antennas. In Fig. 2 the simulated electric and magnetic

near-fields of the two patch antennas are shown. The field values are taken on a cut plane in the middle of the substrate brick. These numerical results validate the relationships (3) and (9). The E-field of the patch exploiting the OM-substrate is approximately twice the one on the ES material, while the H-fields are approximately the same, considering for both fields a size compression by a factor of two.

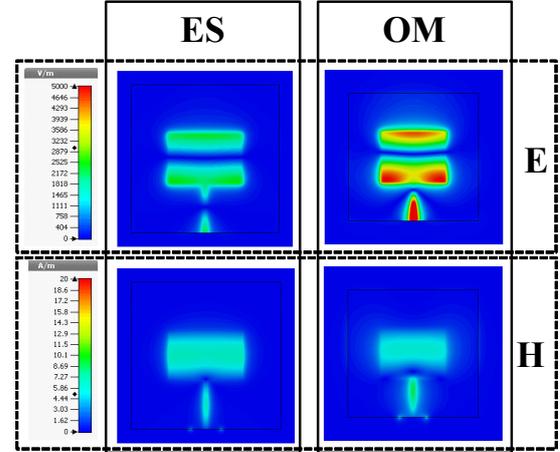


Fig. 2. E- and H- near-field numerical simulation results of the two patches: the E-field for the antenna on the OM substrate is much higher than that on the empty space.

As a further test we compare different antenna layouts: the same miniaturized patch antenna (exploiting magnetic equivalent surface currents) and a single full-wavelength loop antenna (exploiting electric equivalent surface currents) embedded in the substrate. In these cases two substrates are considered: the same OM as before and the dual only-dielectric (OD) substrate ( $\epsilon_r=4$ ,  $\mu_r=1$ ), in order to validate the previous theoretical results. Fig. 3 reports the loop antenna layouts, whose dimensions are identical since the guided wavelength  $\lambda_g$  is the same for both the OM and OD cases: the line width is 1 mm, while the inner radius length provides a full-wavelength behaviour at 2.4 GHz.

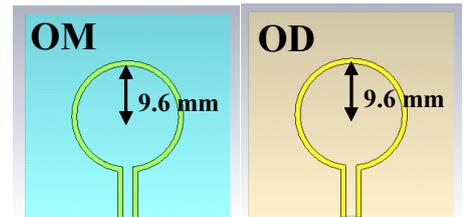


Fig. 3. Loop antennas on an only-dielectric and an only-magnetic materials with dual electromagnetic properties: this antenna topology is mainly equivalent to electric current sources.

In Fig. 4 the far-field power densities of the considered topologies are compared, while Fig. 5 shows similar comparisons in terms of radiated power patterns at a distance of 1 m from the phase centre of the antennas.

These figures confirm again the design guidelines derived in the previous section. When the antenna equivalent current sources are of the electric type, as is the case of the loop antenna, the increased material permeability significantly degrades the radiated performance and should not be chosen for antenna miniaturization. Indeed, from the simulation results it is evident the reduction of both the radiated power and the far-field for the loop antenna on an OM substrate. On the contrary miniaturization on dielectric substrates can be efficiently exploited by these antenna topologies.

Of course dual considerations hold for the patch antenna, which is equivalently represented by surface magnetic currents.

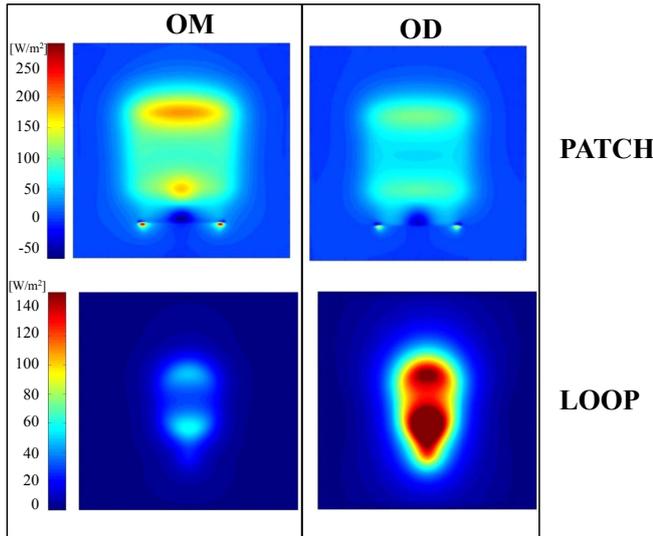


Fig. 4. Far-field power density for the patch and loop antennas, on OM and OD substrates.

#### IV. CONCLUSION

We have proposed a method for defining the antenna topology suitable for exploiting high permeability and/or high permittivity substrates for miniaturization purposes. We have first provided an estimation of the region containing the near-field generated by the antenna. On its surface we have computed the equivalent magnetic and electric current sources and we have shown that the radiation performances of an antenna, which is mainly based on magnetic sources, is not affected by the introduction of high permeability materials. Dually the radiation performances of an antenna which is mainly based on electric sources is not affected by the introduction of high permittivity materials. These results have been validated by full-wave numerical simulations of some reference topologies. Therefore magneto-dielectric materials should be properly synthesized for an efficient miniaturization of a chosen antenna topology.

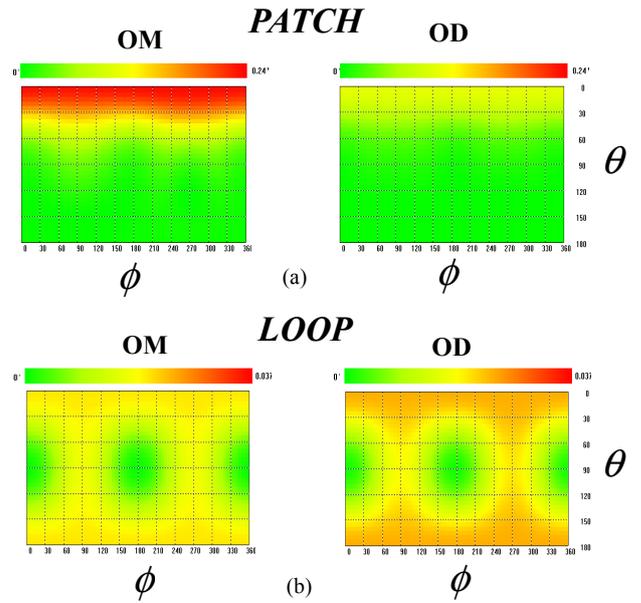


Fig. 5. Far-field power patterns on the  $\theta\phi$ -plane for the patch (a) and loop (b) antennas, on OM and OD substrates.

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