**Aims, Claims, and the Bargaining Model of War**

**Supplementary Information**

(Intended for on-line publication only)

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This appendix consists of four parts. The first three present the formal proofs of equilibria described in the text. Section 1 presents a proposition establishing the unique equilibrium in the game with only unlimited aims types. Section 2 considers the full game in which challengers can have limited or unlimited aims. Propositions 2-5 describe four variants of the equilibrium for this game, depicted in Figure 1. Section 3 considers a two-period versions of the game and develops several results discussed in the text. The fourth and final section of this appendix presents details of the estimated relationship between relative power and claim size depicted in Figure 3.

**1. Solution to Game with Unlimited Aims**

Proposition 1. The following strategies and beliefs constitute a perfect Bayesian equilibrium to the game with only unlimited types, with off-the-equilibrium path beliefs that satisfy the D1 criterion:

1. The challenger demands  if  and 0 otherwise.
2. The target accepts  with probability  . The target rejects all other demands.
3. The challenger fights if and only if .
4.  and  for all .

 Proof. We first establish that there exists a class of perfect Bayesian equilibria in which the low cost type demands some , the high cost type demands *x*\* with probability *s* and 0 with probability , and the target accepts *x*\* with probability *t* and rejects all other offers. Given *s*, we know from Bayes’ rule that

  .

From equation (2) in the text, the target is indifferent between accepting and rejecting *x*\* if . This implies that the target will be indifferent when

 .

For the high cost type to be indifferent between demanding 0 and *x*\*, the target must accept with probability *t* such that , which implies

 .

Finally, it is easy to show that, given *t*, the low cost type always wants to demand *x*\*.

We now impose the D1 refinement on OEP beliefs. To implement this refinement, we consider, for each deviation from the equilibrium path, which type of challenger has the most to gain (or the least to lose) relative to their payoff on the equilibrium path. In particular, for any OEP demand , assume that the target responds by accepting that demand with probability . Then, for each type, we calculate the values of  such that the challenger would want to deviate to this alternative demand. If the set of responses that causes one type to deviate is a true subset of the set of responses that causes the other type to deviate, then we rule out the former type as the source of the deviation. In other words, we assume that any deviation must come from the type that would deviate under the broader set of conditions.

Formally, consider a deviation to and let  denote the probability that the target accepts this demand off the equilibrium path. On the equilibrium path, the high cost challener expects a payoff of zero. In the event of a deviation to , that type expects a payoff of . Therefore, the high cost type will deviate if

  .

The low cost type expects a payoff of  on the equilibrium path, and a payoff of  from deviating. Therefore, the low cost type will deviate if

 .

Comparing these expressions, it is easy to show that  for all . Thus, the set of responses that will induce the high cost type to deviate to a higher demand () is a subset of the responses that will induce the low cost type to so deviate (). In plain English, this means that the resolved type is willing to deviate under a larger set of conditions than the unresolved type. As a result, the D1 restriction implies that any deviation to a demand  is assumed to be coming from a resolved type, or . Given those beliefs, as long as , the target would prefer to accept that demand, making it profitable for both types of challenger to deviate. As a result, both types have an incentive to deviate upwards from any equilibrium in which , and all such equilibria break down. The only equilibrium that does not create an incentive to deviate upward is the one in which , because any higher demand is rejected regardless of the target’s beliefs.

Similar logic rules out deviations to demands that are lower than *x*\*. Comparing and shows that  for all , meaning that it is the high cost type who has the greatest incentive to deviate downward. Thus, the D1 refinement implies , and no lower demand will be accepted. Under those conditions, the high cost challenger prefers to make no demand, thereby avoiding the audience costs.

Thus, the D1 refinement on OEP beliefs eliminates all of the perfect Bayesian equilibria identified above except for the one in which . Plugging  into and yields  and the expression for *t* posited in S1.2. 󠇃

**2. Solution to Game with Limited and Unlimited Aims**

 We now consider the full version of the game, in which the challenger can have limited or unlimited aims. Challenger types can be written as pairs, with  denoting a generic challenger type. Propositions 2-5 describe the equilibria marked in Table 1 as 1-4, respectively.

Proposition 2. The following strategies and beliefs constitute an equilibrium to this game when :

1. The challenger demands  if its type is  and  otherwise.
2. The target always accepts  and accepts  with probability *t*, defined below. The target rejects all other demands.
3. The challenger fights if and only if .
4. Beliefs on the equilibrium path are determined by Bayes’ rule. Off the equilibrium path, the target believes that  for any.

 Proof. Some elements of this equilibrium are identical to that in the basic game. When the challenger demands *m*, the target is certain that it is facing the  type and is indifferent between refusing and accepting the demand. Given this, the target mixes in response to this demand, with a mix probability that makes the  at least indifferent between demanding and receiving  and demanding mimicking the low cost challenger with unlimited aims. Thus,

 .

If the challenger makes the limited demand, Bayes’ rule implies that

 .

Rejecting the demand will lead to war only if the challenger has low costs and limited aims, so

 ,

so the target will accept the demand if

 .

Thus, for the target to accept , it must be the case that

 , or

 ,

which is also equivalent to the condition on  in the statement of the proposition.

Finally, we have to ensure that no type has an incentive to deviate to a different strategy. In equilibrium, both challenger types with limited aims receive their most preferred outcome and thus have no incentive to deviate. The strategy of the target in response to *x*\* ensures that the unresolved challenger with unlimited aims cannot benefit by deviating to that demand. For the  type, mimicking the others leads to a payoff of  for certain, so the equilibrium demand is preferable as long as

  , or

  .

The fact that  ensures that there are values of *t* that meet both and simultaneously.

Finally, we consider deviations to strategies not played on the equilibrium path. First, recall that types with limited aims are already, in equilibrium, getting their most preferred outcome; hence, there is no way that deviation could be profitable. By the intuitive criterion (Cho and Kreps 1987), OEP beliefs have to assign zero probability to the types with limited aims. For the types with unlimited aims, the logic of universal divinity invoked above still applies. In particular, deviating to a demand of  can potentially benefit the  more than the  type. As a result, this restriction leads to the assumption about OEP beliefs in S2.4 above. Given those beliefs, any OEP demand is rejected, and all types are better off playing their equilibrium strategy. 󠇃

Proposition 3. The following strategies and beliefs constitute an equilibrium to this game when :

1. The challenger demands  if its type is , demands  if its type is  or , and mixes between  and 0 if its type is . The mix probability, *s*, is defined below.
2. The target accepts  with probability  and  with probability *t*, both defined below. The target rejects all other demands.
3. The challenger fights if and only if .
4. Beliefs on the equilibrium path are determined by Bayes’ rule. Off the equilibrium path, the target believes that  for any.

 Proof. In order to make the  type indifferent between the two demands, it must be the case that

 .

Notice that this acceptance probability makes the  type strictly prefer to demand  (for which it gets a payoff of 1 if accepted).

Given a mix probability of *s*, Bayes’ rule implies.

 .

In order for the target to be indifferent between accepting and rejecting , the condition in must hold as an equality. This implies

  .

The condition in the statement of the proposition ensure that .

To ensure that the  type does not deviate to , the target has to accept this demand with a probability that at least makes this type indifferent between deviation and its equilibrium payoff, which is zero in expectation. This implies

 .

Notice that  implies . Finally, to ensure that the  type will not deviate down to , it must be the case that

 

which is ensured by . 󠇃

Proposition 4. The following strategies and beliefs constitute an equilibrium to this game when :

1. The challenger demands  if its type is , demands  if its type is , mixes between  and 0 if its type is , and makes no demand if its type is . The mix probability, *s*, is defined below.
2. The target accepts  with probability  and  with probability *t*, both defined below. The target rejects all other demands.
3. The challenger fights if and only if .
4. Beliefs on the equilibrium path are determined by Bayes’ rule. Off the equilibrium path, the target believes that  for any.

Proof. In order to make the  type indifferent between the two demands, it must be the case that

 .

Notice that this acceptance probability makes the  type strictly prefer to make no demand rather than deviate to .

Given a mix probability of *s*, Bayes’ rule implies.

 .

In order for the target to be indifferent between accepting and rejecting , the condition in must hold as an equality. This implies

  .

The conditions in the statement of the proposition ensure that .

To ensure that the  type does not deviate to , the target has to accept this demand with a probability that at least makes that type indifferent between deviation and its equilibrium payoff, zero. This implies

 .

Notice that  implies . Finally, because its maximal demand is accepted with higher probability than the limited demand, the  type has no incentive to deviate to . 󠇃

Proposition 5. The following strategies and beliefs constitute an equilibrium to this game when :

1. The challenger demands  if its type is , demands  if its type is , and makes no demand if its type is  or . The mix probability, *s*, is defined below.
2. The target accepts  with probability  and  with probability *t*, both defined below. The target rejects all other demands.
3. The challenger fights if and only if .
4. Beliefs on the equilibrium path are determined by Bayes’ rule. Off the equilibrium path, the target believes that  for any.

Proof. Note that the key difference between this condition and the previous ones is that , which means that the  type cannot demand a border at , and the most the target is willing to concede to a type with limited aims is . Let  denote the maximum limited demand. The target mixes in response to both non-zero demands because both leave the target indifferent between accepting and rejecting.

In order to make the  type at least indifferent between the making no demand and demanding , it must be the case that

 .

Notice that this acceptance probability makes the  type strictly prefer to make no demand rather than deviate to .

In order to make the  type at least indifferent between the making no demand and demanding , it must be the case that

 .

Notice that, if both acceptance probabilities are set to their maximum value,  implies .

As before, because its maximal demand is accepted with higher probability than the limited demand, the  type has no incentive to deviate to . Finally, because the  type is no longer getting its most preferred outcome in equilibrium, we have to ensure that it does not want to deviate to . The assumption that  is sufficient (and stronger than needed) to ensure this. 󠇃

**3. The Two-period Game**

Finally, we consider a two-period game in which the challenger can make a second demand in the event that its first demand is accepted. Thus, the order of moves is as follows:

1. Nature determines the aims and war costs of the challenger.
2. The challenger makes some demand .
3. The target accepts or reject the demand.
4. If the target rejects, the challenger decides either to fight or not. Either decision ends the game. In the event of war, the challenger wins with probability  where *p* is a non-decreasing function of the amount of the target’s territory the challenger possesses.
5. If the target accepts, the challenger can make a new demand  .
6. The target accepts or rejects the new demand.
7. If the target rejects, the challenger decides either to fight or not. In the event of war, the challenger wins with probability .

For simplicity, we focus on the case in which , which means that the target would be willing to concede the ideal border of the type with limited aims rather than fight a war against that type of challenger. We also assume that  , so that the power shift induced by moving the border to  does not make the high cost challenger resolved to fight in the second period.

It is helpful to establish a few results.

Result 1. In any equilibrium in which the resolved type with limited aims makes the demand , no other first period demand  is accepted.

Proof. Under this condition, a demand of  can come from one of three types: the low cost challenger with unlimited aims or a high cost challenger with either limited or unlimited aims. The target clearly prefers to reject the demand of the latter two types, since doing so leads to its best possible outcome, the status quo. Now consider the target’s decision making if faced with the  type. Rejecting the demands leads to war with the status quo capabilities, for a payoff of . If the target accepts the demand, then by the logic of Proposition 1, the type can push the target to indifference between war and even greater concessions in the second period, for an expected payoff of . Since , the target is no worse off rejecting the demand and fighting a war in the first period. Thus, for any posterior belief upon seeing a first-period demand , the target prefers to reject the demand. 󠇃

This result implies that, in any equilibrium in which the resolved type with limited aims makes the demand , the resolved type with unlimited aims can do no worse by mimicking that strategy. Moreover, if the limited demand is accepted with non-zero probability, this type is strictly better off mimicking that demand than making any alternative demand.

Result 2. In any equilibrium in which  is accepted in the first period, the second-period equilibrium consists of the type demanding  , the target accepting the demand with probability , and the  retaining the new status quo.

Proof. This result follows from the that only type with unlimited aims would make a second-period demand once the ideal border of the limited aims type is agreed to. Therefore, in the event that  is accepted, the second period proceeds exactly as the one-period model with only unlimited aims types, but the new status quo is at , the new probability of victory is , and the maximum demand that the target will accept is . The equilibrium described then follows from the logic of Proposition 1. 󠇃

We can now state the following proposition.

Proposition 6. In any equilibrium of the two-period game in which the resolved challenger with limited aims demands , the target will only accept that demand with non-zero probability if (1)  is relatively small, (2) *r* is relatively large, and (3)  is relatively small, with the relevant conditions described in the proof.

Proof. Define  as probability that a challenger of type  makes the demand. By assumption , and by result 1, . Now define  as the posterior probability that a challenger is type  conditional on seeing the demand. It follows from Bayes’ rule that



The expected utility to the target of rejecting  is

 .

The first two terms reflect the value of the status in the event that the challenger has high costs. The second two terms reflect the expected value of war against low costs types with limited and unlimited aims, respectively, given the first period probability of victory,  . The expected utility to the target of accepting  is

.

The first term reflects the fact that the three indicated types make no second-period demand, while the low cost type with unlimited aims, following result 2, will make a new demand that drives the target to indifference between war and concessions of .

For the target to accept the demand with non-zero probability, it must be the case that



which implies



If the inequality holds strictly, then the target is certain to accept, in which case all types has an incentive to make the demand. Thus,  for all , and the expression becomes

 .

Notice that this condition is more likely to be met if (1)  is small, (2) *r* is large, and/or (3)  is small, as stated in the proposition.

If, in equilibrium, the target is indifferent between accepting and rejecting the offer, then the target might mix, and one of the high cost types will play a mixed strategy in response. We saw above that any acceptance probability that incentivizes the  type to be indifferent between no demand and a demand of  causes the  type to strictly prefer no demand. Thus  implies . In that case, becomes

.

To ensure that , it must be the case that

 ,

which can only be met under the three conditions cited in the proposition.

Alternatively, if the equilibrium is such that the  is indifferent, then the  type strictly prefer to make the demand. Thus  implies . In that case, becomes

 

To ensure that , it must be the case that

 .

This expression is more likely to be true as  goes to zero (which ensures that the expression inside the braces is positive), *r* is large, and  is small, as conjectured. 󠇃

**4. Estimating the Relationship between Relative Power and Claim Size**

As discussed in the text, the sample consists of 138 homeland territory disputes that started after 1947. One complication is that some homeland disputes began as disputes over colonial or dependent territory before one or both states were independent. For example, Ethiopian demands on Sudan and Kenya began when the latter two were still under British rule and then become interstate conflict with their independence in 1956 and 1963, respectively. Although many such disputes were inherited unchanged upon independence, others were dropped or reduced in size. For example, Liberian claims against French holdings in Guinea and the Ivory Coast were both dropped upon their independence; Saudi claims on Britain for large swaths of what became the United Arab Emirates were reduced in size on the eve of the latter’s independence. These observations suggest that states have discretion over claim making, even when disputes are inherited from the colonial period. Therefore, in the main analysis (depicted in Figure 3), we count disputes that continued after the independence of both participants as new onsets for the purpose of this exercise. However, one might worry that these disputes weaken the link between the states’ relative power and the size of the claim because the claims were to some extent inherited from a period when the power considerations were different (i.e., a colonial power was present on the challenger or target side or both). To address this concern, we also run the analysis without those cases, which leaves a sample of 111 disputes.

To capture the possibility of heterogeneity in the relationship between the challenger’s share of capabilities and the share of the target’s territory it claims, we estimate a finite mixture model. In particular, we assume that each observation can belong to one of two classes, but class membership is unobserved, and the relationship between power and claim size differs by class. In particular, for each latent class  , let  denote the probability that an observation is in that class, and let  denote the model for the claim size (*y*) as a function of relative power (*x*). Although in principle the class probabilities can be written as linear combination of covariates, we assume that the  are the same for each observation. In particular, the probability of being in class 1 is given by  , where  is estimated along with the other parameters. This model was estimated using the fmm commands in Stata 15.1.

Table A1 presents the estimates based on the full sample (column 1) and the sample that excludes disputes that were inherited from the colonial period (column 2). In both samples, the estimates imply that there is no relationship between relative power and claim size in class 1 (i.e., ) and modest positive relationship in class 2. In the full sample, the estimate of  is statistically distinguishable from both zero and the estimate of . The estimated frequency of the two classes is shown. Comparing columns (1) and (2) suggests that the challenger’s share of dyadic capabilities has less effect on claim size when we drop disputes inherited from the colonial period. This means that the inclusion of these cases does not obscure the relationship between power and claim size, as might be feared. The predicted values from the model in column (1) are depicted in Figure 3.

**Table A1. The Estimated Relationship between Capabilities and Claim Size**

|  |  |  |
| --- | --- | --- |
|  | (1) | (2) |
| **Class 1** |  |  |
|  |  |  |
| Challenger’s Share of Capabilities | 0.00021 | 0.0014 |
|  | (0.00047) | (0.0012) |
| Constant | 0.00029 | 0.00 |
|  | (0.00047) | (0.00) |
|  |  |  |
|  |  |  |
| **Class 2** |  |  |
|  |  |  |
| Challenger’s Share of Capabilities | 0.29\* | 0.16 |
|  | (0.12) | (0.16) |
| Constant | 0.036 | 0.14 |
|  | (0.073) | (0.10) |
|  |  |  |
| **Class Probabilities** |  |  |
|  |  |  |
| **1 | 0.57 | 0.75 |
|  | (0.068) | (0.053) |
| **2 | 0.43 | 0.25 |
|  | (0.068) | (0.053) |
|  |  |  |
| Observations | 138 | 111 |

Note: Robust standard errors in parentheses. \*\* p<0.01, \* p<0.05