**Economic weights of maternal and direct traits of pigs calculated by applying gene flow methods**

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**Supplementary Material S1**

*Transmission matrix P for the three-way crossing system applying artificial insemination in all three breeds and crossbreeds*

For clarity, we have used A, B, and C to represent breeds Czech Large White (CLW), Czech Landrace (CL), and Pietrain (PN), respectively; AB represents the crossbred dams and crossbred progeny CLW × CL; and ABC represents terminal crosses (CLW × CL) × PN. Capital S indicates sires, capital D indicates dams, and capital SL indicates slaughter animals. The transmission matrix ***P*** had a 12 × 12 block structure. In the following scheme, only blocks with non-zero elements are presented:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Parents  Progeny | SA | DA | SB | DB | SC | DC | DAB | SLA | SLB | SLC | SLAB | SLABC |
| SA | **P11** | **P12** |  |  |  |  |  |  |  |  |  |  |
| DA | **P21** | **P22** |  |  |  |  |  |  |  |  |  |  |
| SB |  |  | **P33** | **P34** |  |  |  |  |  |  |  |  |
| DB |  |  | **P43** | **P44** |  |  |  |  |  |  |  |  |
| SC |  |  |  |  | **P55** | **P56** |  |  |  |  |  |  |
| DC |  |  |  |  | **P65** | **P66** |  |  |  |  |  |  |
| DAB |  | **P72** | **P73** |  |  |  | **P77** |  |  |  |  |  |
| SLA | **P81** | **P82** |  |  |  |  |  |  |  |  |  |  |
| SLB |  |  | **P93** | **P94** |  |  |  |  |  |  |  |  |
| SLC |  |  |  |  | **P10,5** | **P10,6** |  |  |  |  |  |  |
| SLAB |  | **P11,2** | **P11,3** |  |  |  |  |  |  |  |  |  |
| SLABC |  |  |  |  | **P12,5** |  | **P12,7** |  |  |  |  |  |

The submatrices **P11, P22, P33, P44, P55,** and **P66**are of structure **S1**; the submatrices **P12,** **P21, P34, P43, P56 ,P65, P72**, and **P73**are of structure **S2**; the submatrix **P77** is of structure **S3**; and the submatrices **P81, P82, P93, P94, P10,5, P10,6, P11,2, P11,3, P12,5**, and **P12,7** are of structure **S4**. As an example, the values of non-zero elements in the submatrices of breed A will be given here.

The submatrix for the transmission of genes of A sires to the next generation of A sires, **P11**, contained in our calculation the following values:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0.000 | 0.116 | 0.128 | 0.128 | 0.128 |
|  | 1 | 0 | 0 | 0 | 0 |
| **P11** = | 0 | 1 | 0 | 0 | 0 |
|  | 0 | 0 | 1 | 0 | 0 |
|  | 0 | 0 | 0 | 1 | 0 |

There were five age classes for A sires (boars): male replacement before starting reproduction and breeding boars that were assumed to produce sperm for 2 years (for four sow reproductive cycles). The values in columns 2–5 are one-half of the proportion of boars in each age class. The same elements as in the first row of this submatrix were also in the first rows of submatrices **P21**and **P81**, i.e., in the submatrices for A sires to A dams and A sires to A slaughter animals.

The submatrix for the transmission of genes of A dams to the next generation of A dams, **P22**, was as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0.000 | 0.000 | 0.091 | 0.074 | … | 0.023 |
|  | 1 | 0 | 0 | 0 | … | 0 |
| **P22** = | 0 | 1 | 0 | 0 | … | 0 |
|  | 0 | 0 | 1 | 0 | … | 0 |
|  | ... | ... | ... | ... | . . . | ... |
|  | 0 | 0 | 0 | 1 | … | 0 |
|  | 0 | 0 | 0 | 0 | 1 | 0 |

There were 12 age classes for A dams: two classes of female replacement before starting reproduction and 10 classes of breeding females that were assumed to have at most 10 parities (10 reproductive cycles). The values in columns 3–12 are one-half of the proportion of sows in each age class used for purebreeding, calculated from the age structure of sow herds of breed A (Krupa *et al*. 2017) and from the proportion of crosses in each parity. The same elements as in the first row of this submatrix were also in the first row of submatrices **P12**and **P82**, i.e., in the submatrices of A dams to A sires and A dams to A slaughter animals.

The submatrix for the transmission of genes from A dams to AB crossbred dams **P72** had the form:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0.0 | 0.0 | 0.106 | 0.086 | … | 0.018 |
|  | 0 | 0 | 0 | 0 | … | 0 |
| **P72** = | 0 | 0 | 0 | 0 | … | 0 |
|  | 0 | 0 | 0 | 0 | … | 0 |
|  | ... | ... | ... | ... | . . . | ... |
|  | 0 | 0 | 0 | 0 | … | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 |

The values in columns 3–12 are one-half of the proportion of A sows in each age class used for crossbred matings. The same elements as in the first row of this submatrix were also in the row of submatrix **P11,2**, i.e., in the submatrices of A dams to crossbred AB slaughter animals.

**Supplementary Material S2**

*Realization vectors for direct traits (****hd****) and for maternal traits (****hm****)*

Both realisation vectors have the same dimension as matrix ***P***, i.e., 70. In the realisation vector for direct traits, only the last five elements belonging to the age classes of slaughter animals had values different from zero. The first three non-zero values belonged to traits expressed in purebred progeny of breeds A, B, and C; therefore, the proportions of sows of breeds A, B, and C, respectively (variables *pdA*, *pdB*, and *pdC* from Table 1) in the three-way crossing system were inserted for these values (all calculated marginal economic values were expressed per sow, not per slaughtered animal). The last but one element of the vector was again proportion of sows of breed A (*pdA*) because the economic values of traits in crossbred progeny AB were also expressed per sow of breed A. The last element of vector **hd** was the proportion of crossbred AB sows (variable *pdAB* from Table 1):



In the realisation vector for maternal traits, only the elements belonging to age classes of dams of breeds A, B, and C and the crossbreed AB had non-zero values:



where the zeros represent all age classes of A, B, and C sires and the first two age classes of A, B, C, and AB dams. The values *pA3* to *pA12* (0.20–0.04) are proportions of breeding females of breed A in age classes 3–12 calculated as one-half of the proportions of sows at parity 1 to *RR*. The variables pB3 to pB12 (also 0.20–0.04), pC3 to pC10 (0.22–0.05), and pAB3 to pAB10 (0.19 to 0.04) had the same meaning for breeds B and C and the crossbreed AB.

The non-zero elements (*hedi*j) of the matrix **hed**, which was needed for the calculation of *EWdi*, were obtained by multiplying the non-zero elements *hdj* (the last five elements) of the vector **hd** by the appropriate marginal economic values (MEVs) for the direct traits *i* taking into account the ratios of the number of sows in the appropriate links of the crossing system:

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where the values of the second index *j* give the position of the calculated values in the vector hed that has the dimension *acm*. *MEV0Ai*, *MEV0Bi*, and *MEV0Ci* are MEVs of direct traits *i* expressed in purebred progeny of sows of breeds A, B, and C, respectively. *MEV1Ai* and *MEV1ABi* are the MEVs of direct trait *i* expressed in crossbred progeny of sows of breed A and of the crossbreed AB, respectively, and nA, nB, nC, and nAB are the number of sows of breeds A, B, and C and of the crossbreed AB, respectively, in the entire three-way crossing system.

Similarly, the non-zero elements (*hemi*j) of matrix **hem** were obtained by multiplying the non-zero elements *hmj*, of the vector **hm** (see above) by the appropriate MEVs of maternal traits expressed in sows with purebred and crossbred litters. Proportions belonging to sows of breed A were multiplied by the sum (*MEV0Ai + MEV1Ai*), proportions belonging to sows of breed B were multiplied by *MEV0Bi*, those belonging to breed C were multiplied by *MEV0Ci*, and proportions belonging to the crossbreed AB were multiplied by *MEV0ABi.* The MEVs *MEV0Ai*, *MEV0Bi*, *MEV0Ci*, and *MEV0ABi* are the economic values of maternal trait *i* expressed in litters of genotypes A, B, C, and AB, respectively, and *MEV1Ai* is the economic value of maternal trait *i* expressed in crossbred litters of the genotype AB.