Can we detect deep axisymmetric toroidal magnetic fields in stars?

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INTRODUCTION

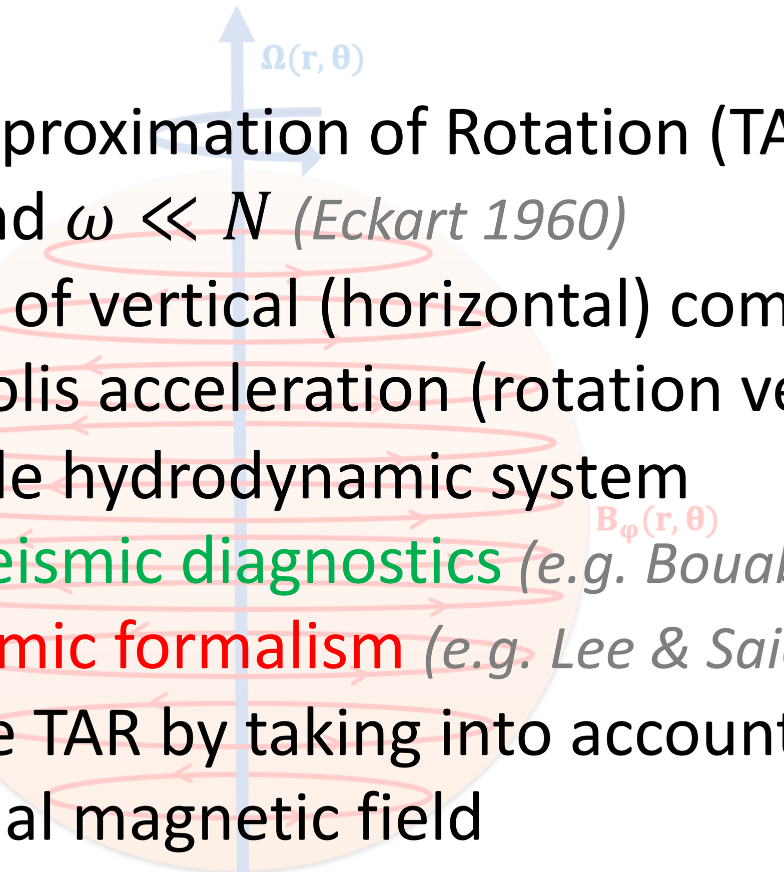
- **One of the major unsolved question of stellar astrophysics** : strong extraction of angular momentum (AM) in stars' radiative zones (RZ)
- **Possible answer** : magnetic field with its various topologies
- **Interesting topology** : strong axisymmetric toroidal fields *(e.g. Heger+2005, Fuller+2019)*
  - ✓ Tayler MHD instability *(Tayler 1973)*
  - ✓ Dynamo in RZ *(Spruit 2002)*
  - ➢ Magnetic torque ⟶ Efficient transport of AM

Is it possible to detect signatures of these deep toroidal magnetic fields?

- **Only way to answer : Asteroseismology**

➢ Study stellar pulsations in stably stratified, rotating, and strongly magnetised zones : Magneto-Gravito-Inertial Modes (MGIM)

MAGNETIC TAR



- Traditional Approximation of Rotation (TAR)
  - $2\Omega \ll N$ and $\omega \ll N$ *(Eckart 1960)*
  - Negligence of vertical (horizontal) component of the Coriolis acceleration (rotation vector)
    - ➢ Separable hydrodynamic system
  - ✓ Powerful seismic diagnostics *(e.g. Bouabid+2013)*
  - ❖ Hydrodynamic formalism *(e.g. Lee & Saio 1997)*

➢ Generalise the TAR by taking into account a general toroidal magnetic field

MAGNETIC LAPLACE TIDAL EQUATION

$$\mathcal{L}^{\mathrm{magn.}}_{\omega^{\mathrm{in}}\,m} = \omega^2 \partial_x \left[\frac{1}{\mathcal{A}}\,\frac{1-x^2}{D_{\mathrm{M}}}\,\partial_x\right] + m\omega^2\partial_x\left(\frac{\nu_{\mathrm{M}}x}{\mathcal{A}D_{\mathrm{M}}}\right) - m^2\,\frac{\omega^2}{\mathcal{A}D_{\mathrm{M}}\left(1-x^2\right)} + m^2\,\frac{\omega^2}{\mathcal{A}^2}\,\frac{x}{D_{\mathrm{M}}}\,\partial_x\omega_{\mathrm{A}}^2$$

$$\mathcal{L}^{\mathrm{magn.}}_{\omega^{\mathrm{in}}m}\left[w^{\mathrm{magn.}}_{\omega^{\mathrm{in}}km}(r,x)\right] = -\Lambda^{\mathrm{magn.}}_{\omega^{\mathrm{in}}km}(r)\,w^{\mathrm{magn.}}_{\omega^{\mathrm{in}}km}(r,x)$$

Eigenvalues — Hough functions

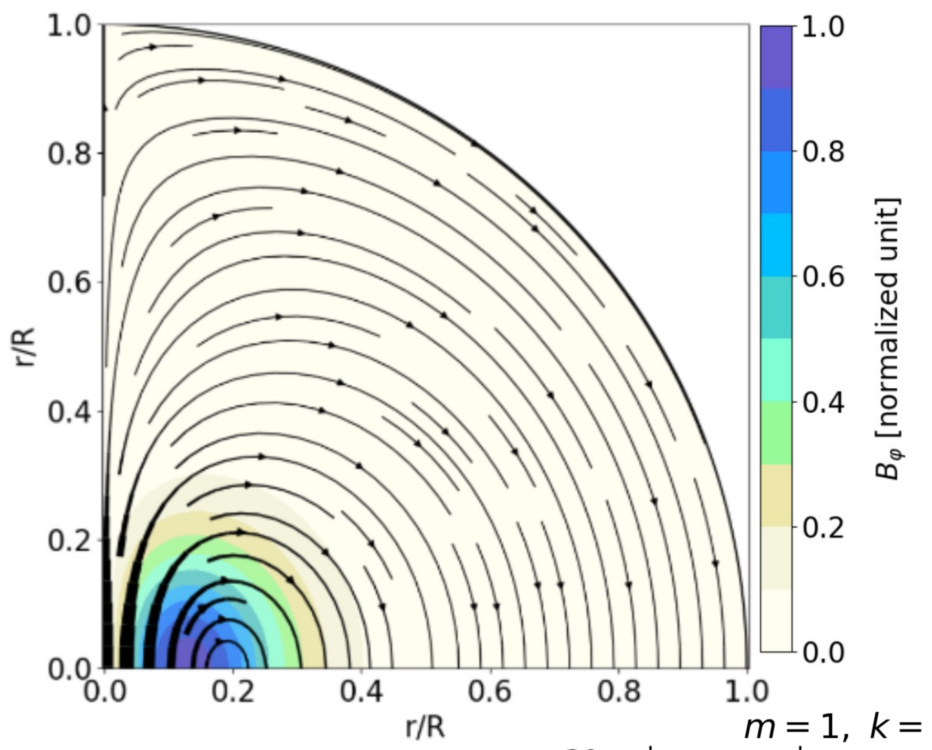
- **Dispersion relation + Radial quantification :**

$$\int_{r_1}^{r_2} \frac{N\sqrt{\Lambda^{\mathrm{magn.}}_{\omega^{\mathrm{in}}km}}}{r\omega}\,\mathrm{d}r = (n+1/2)\pi$$

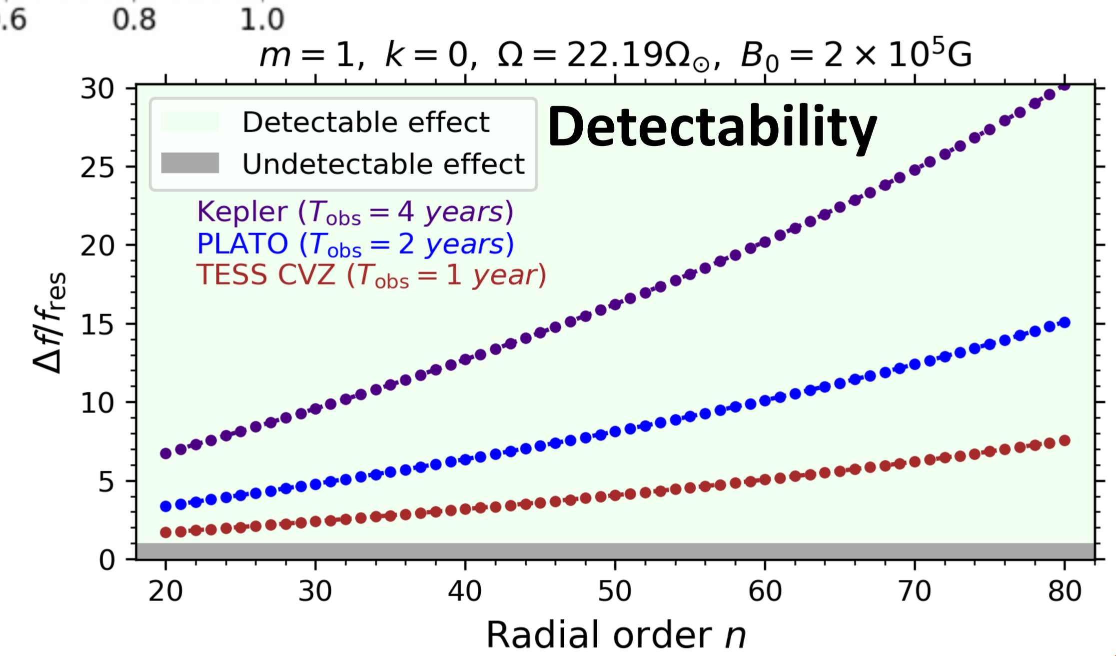
Frequency of the wave in the rotating frame

$$x = \cos\theta$$
$$\mathcal{A} = \omega^2 - m^2\omega_{\mathrm{A}}^2$$
$$\mathcal{B} = 2\left(\Omega\omega + m\omega_{\mathrm{A}}^2\right)$$
$$\nu_{\mathrm{M}} = \mathcal{B}/\mathcal{A}$$
$$D_{\mathrm{M}}(r\,,x) = 1 - \nu_{\mathrm{M}}^2x^2 - \left(1-x^2\right)\frac{x}{\mathcal{A}}\,\partial_x\omega_{\mathrm{A}}^2$$

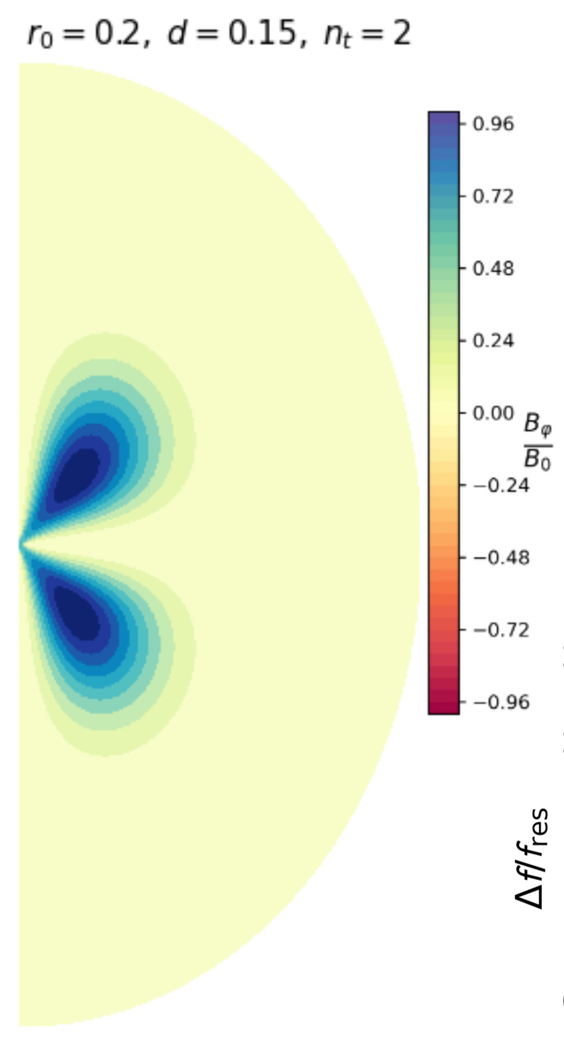
EQUATORIAL TOROIDAL FIELD



- Fossil origin
- High detectability
- High impact on GIM

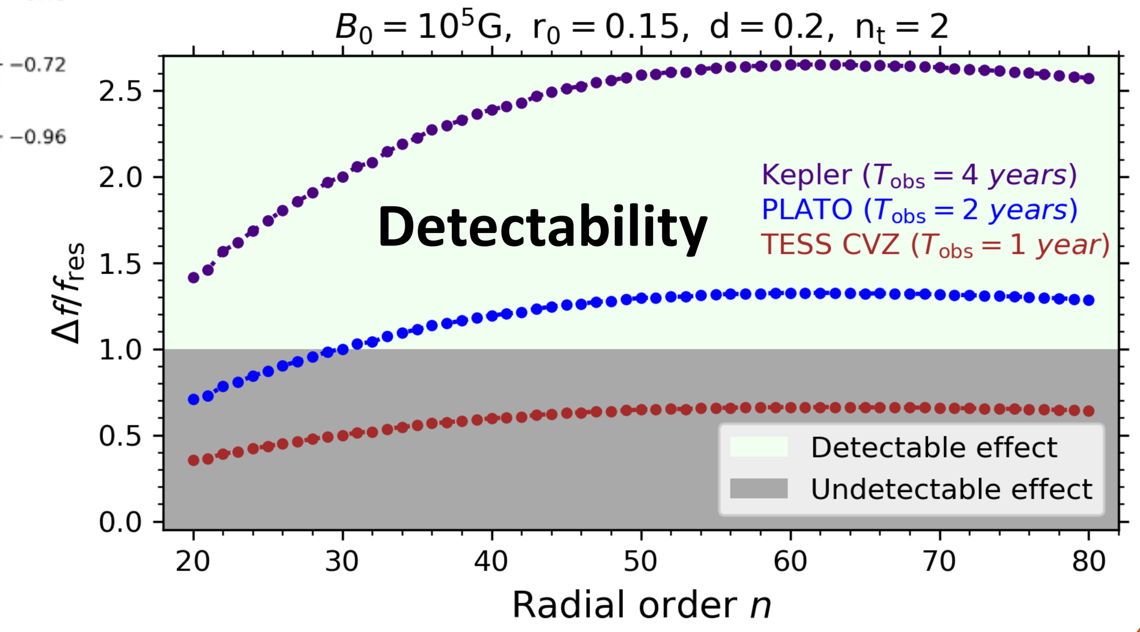


HEMISPHERIC TOROIDAL FIELD



$$B_\varphi(r,\theta) = B_0\exp\left[-\frac{(r-r_0)^2}{2d^2}\right]\sin^{n_t}(2\theta)$$

- Depend on the history of the differential rotation
- Low detectability
- Low impact on GIM



SUMMARY

- Derivation of a new generalisation of the TAR
- Better magneto-asteroseismic modelling
- Dependency of the detectability of magnetic fields on their configurations
- Possible detection of equatorial fields
- Signature similar to rotation
- Greater difficulty in detecting fields located far from the equator

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For more details, check out
Dhouib et al. 2022, A&A, 661, A133