In[∘]:= << PrSAT`

Let's just consider a pair {ℍ,𝔼}, and the following UR-prior **m**, such that the Likelihood Ratio in **m** for {ℍ,𝔼} is 8, LR_m[ℍ,𝔼] = 8. (We also represent **m*** algebraically).

In[∘]:=

ℍ	𝔼	m	m*
T	T	$\frac{1}{4}$	d
T	F	$\frac{1}{4}$	e
F	T	$\frac{1}{32}$	f
F	F	$\frac{15}{32}$	$1-(d+e+f)$

Out[∘]= $\{\{ℍ, 𝔼, \{\{𝔼 \to \{a_2, a_4\}, ℍ \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \{a_1 \to \frac{15}{32}, a_2 \to \frac{1}{32}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{4}\}\}, \{\{𝔼 \to \{a_2, a_4\}, ℍ \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \{a_1 \to \frac{3}{512}(59+5\sqrt{161}), a_2 \to \frac{1}{32}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{512}(191-15\sqrt{161})\}\}\}, \{T, T, \frac{1}{4}, d\}, \{T, F, \frac{1}{4}, e\}, \{F, T, \frac{1}{32}, f\}, \{F, F, \frac{15}{32}, 1-d-e-f\}\}$

First, let's verify that LR_m[ℍ,𝔼] = 8. (Note that PrSAT indexes truth table rows in descending order, with FF indexed to 4 and TT indexed to 1 in this case. I follow the more common usage in philosophy when indexing them in the text.)

In[∘]:= **m = PrSAT[{Pr[ℍ && 𝔼] == $\frac{1}{4}$, Pr[ℍ && ¬ 𝔼] == $\frac{1}{4}$, Pr[¬ ℍ && 𝔼] == $\frac{1}{32}$}]**

Out[∘]= $\{\{𝔼 \to \{a_2, a_4\}, ℍ \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \{a_1 \to \frac{15}{32}, a_2 \to \frac{1}{32}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{4}\}\}$

In[∘]:= **EvaluateProbability[$\frac{\text{Pr}[𝔼 \mid ℍ]}{\text{Pr}[𝔼 \mid \neg ℍ]}$ == 8, m]**

Out[∘]= True

Now, we can ask: what is the closest probability function **m*** to **m** such that LR_{m^*}[¬ℍ,𝔼] = 8? This is easily answered, as follows (here, it's algebraically easier to use **ManhattanDistance**):

In[∘]:= **sol = Minimize[{ManhattanDistance[{$\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{32}$}, {d, e, f}], $\frac{f}{\frac{(1-(d+e))\,d}{d+e}}$ == 8}, {d, e, f}] // FullSimplify**

Out[∘]= $\{\frac{3}{512}(-21+5\sqrt{161}), \{d \to \frac{1}{512}(191-15\sqrt{161}), e \to \frac{1}{4}, f \to \frac{1}{32}\}\}$

In[∘]:= **m* = PrSAT[{Pr[ℍ && 𝔼] == d //. sol⟦2⟧, Pr[ℍ && ¬ 𝔼] == e //. sol⟦2⟧, Pr[¬ ℍ && 𝔼] == f //. sol⟦2⟧}]**

Out[∘]= $\{\{𝔼 \to \{a_2, a_4\}, ℍ \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \{a_1 \to \frac{3}{512}(59+5\sqrt{161}), a_2 \to \frac{1}{32}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{512}(191-15\sqrt{161})\}\}$

Let’s verify that $LR_{m^*}[\neg \mathbb{H}, \mathbb{E}] = 8$.

In[∘]:=
```
EvaluateProbability[Pr[𝔼 | ¬ ℍ] / Pr[𝔼 | ℍ] == 8, m*]
```

Out[∘]=
```
True
```

As you can see below, the closest such distribution `m*` is ~0.25 away from `m` in Manhattan distance!

In[∘]:=
```
sol // N
```

Out[∘]=
```
{0.248689, {d → 0.0013112, e → 0.25, f → 0.03125}}
```

We can write a function `f` which takes the constraint $LR_{m'}[\mathbb{H}, \mathbb{E}]$ as an argument, and returns the distance of the closest `m'` to `m`, satisfying this constraint.

In[∘]:=
```
f[l_] := f[l] =
  Minimize[{ManhattanDistance[{1/4, 1/4, 1/32}, {d, e, f}], f / (((1-(d+e)) d)/(d+e)) == 1/l}, {d, e, f}][[1]]
```

Sanity check on the known value:

In[∘]:=
```
N[f[1/8]]
```

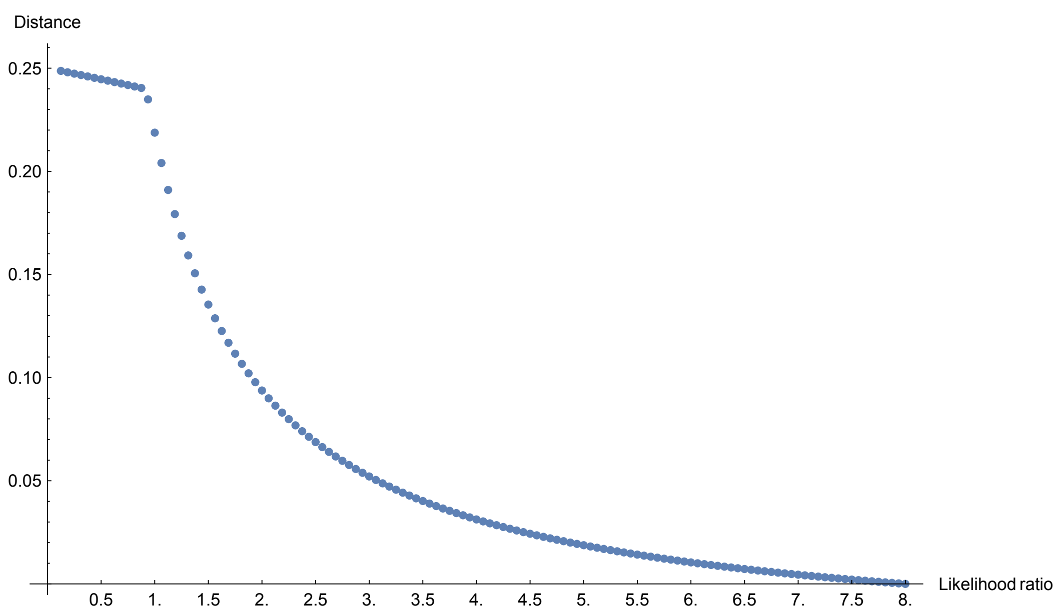
Out[∘]=
```
0.248689
```

Plot of `f` as l goes from 8 to $\frac{1}{8}$, by increments of $\frac{1}{16}$.

In[∘]:=
```
points = Table[{l, f[l]}, {l, 8, 1/8, -1/16}];
```

In[∘]:=
```
ListPlot[points, PlotRange → All, Ticks → {Table[l, {l, 0, 8, 0.5}], Automatic},
 AxesLabel → {Likelihood ratio , Distance}]
```

Out[∘]=



As suggested, `m'` *gets monotonically closer to* `m`, as `m'`’s likelihood ratio approaches 8 (which is `m`’s).

(There is a non-smoothness at $LR_{m'}[\mathbb{H},\mathbb{E}] = 1$, since that's the shift from dis-confirmation to confirmation.)