# Appendix A. The MSP Scenario tree

The root node of the scenario tree is denoted by $n\_{0}$. Let $N$ be the set of all nodes in the tree, and $N\_{t}$ be the set of nodes at time $t$. For our retirement planning problem, time $0$ is the first stage and retirement time $T$ is the terminal stage. Thus, $N\_{0}=\{n\_{0}\}$ contains the root node only, $N\_{T}$ is the set of leaf nodes, and $N=\bigcup\_{t\in \left[0,T\right]}^{}N\_{t}$. The unconditional probability that a node, $n$, occurs is $pr\_{n}$ and, clearly $\sum\_{n\in N\_{t}}^{}pr\_{n}=1$. A node $n\ne n\_{0}$ will branch off from a parent node, denoted by $n^{-}$. A node $n\notin N\_{T}$ will give rise to a set of children nodes, denoted by $n^{+}$.

In the operations research literature, scenario trees are generated using three main methods: scenario reduction, state aggregation and moment matching (see Geyer et al., 2010). We choose the moment matching method (Høyland and Wallace, 2001; Klaassen, 2002) for generating scenario trees of accumulated equity returns and three Nelson-Siegel model parameters. The first-period sub-tree has outcomes corresponding to each child node in the set $n\_{0}^{+}$. The outcomes for the first period sub-tree are obtained by matching the first four moments of the distributions of state variables. For the second-period sub-trees, the conditional outcomes are obtained by matching the first four moments of the conditional distributions on outcomes of the first-period sub-tree. This procedure is executed sequentially for the third, fourth sub-trees and so on until the final-period sub-trees. By doing so, we ensure that all conditional distribution properties are fully matched throughout the multi-period scenario tree.

The scenario tree that we construct in our multi-stage stochastic programming problem has six stages. The time interval between the stage is $Δt$, so the stages occur at time $0$, $Δt$, $2Δt$, …, and $T=5Δt$. At each node $n$, we store the state variables $\left[R\_{n},r\_{n},β\_{1,n},β\_{2,n},β\_{3,n}\right]$ employing the same notation as before except that we index by node $n$ rather than by time. Thus, if node $n$ occurs at time $t$, $R\_{n}$ denotes the equity log-return over a $Δt$-long time interval ending at time $t$ (Equation (2)); $r\_{n}$ denotes equity log-return over a month ending at time $t$ (Equation (8)); and $β\_{1,n}$, $β\_{2,n}$ and $β\_{3,n}$ denote the Nelson-Siegel model parameters at time $t$ (Equation (7)). At the root node $n\_{0}$, the initial state values are set to equal the unconditional expected means in Table 3. In the scenario tree, every non-terminal node branches off to six children nodes. Six outcomes are the minimum required to perfectly match the first four moments of the five state variables.

Validating arbitrage opportunities among the financial assets (cash, bond and equity funds) is dealt with by using the two methods of Klaassen (2002) for two arbitrage types *ex-post* and the method of Geyer et al. (2014) for no-arbitrage bounds *ex-ante*. The detailed step procedures can be found in Owadally et al. (2021).

Since there are six child nodes for every non-terminal node and there are six stages (five periods) there are therefore $6^{5}=7,776$ scenarios and $\sum\_{j=0}^{5}6^{j}=9,331$ nodes. To improve the stability of our results, we aggregate two independently-generated scenario trees, with identical root nodes, into one large scenario tree (see Høyland and Wallace, 2001). This means that the total number of scenarios is 15,552 and the total number of nodes is 18,661.[[1]](#footnote-1)

From the generated outcomes on each node, the asset prices given in Equations (2) to (5) can be rewritten in a nodal form. Recall that any node $n$ in the scenario tree (except for the root node $n\_{0}$) branches off from a parent node $n^{-}$ at the previous time stage. The asset price in Equation (2), for example, is transformed into the nodal form simply by replacing $t$ with $n$ and $t-Δt$ with $n^{-}$ as follows:

|  |  |  |
| --- | --- | --- |
|  | $S\_{i,n}=S\_{i,n^{-}}⋅exp\left(R\_{i,n}\right)$ for $n\in N∖n\_{0}$ and $i\in \{C,B,E\}$, |  |

where $S\_{i,n\_{0}}=1$. Other pricing formulas are transformed in a similar way.

# Appendix B

*Table B.1. Average Stochastic Optimal Strategies (%) and Total Secured Retirement Income (£1,000 p.a.)
with Various Risk Aversion (*$γ$*), Time Preference (*$ρ$*), and Bequest Motive (*$κ$*) Parameters*

|  |  |  |  |
| --- | --- | --- | --- |
| Age | $$γ= 1.0, ρ=0.02, κ=2.0$$ |  | $$γ= 1.0, ρ=0.02, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 75.60 | 24.40 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |  | 0.00 | 75.79 | 24.21 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 9.94 | 55.88 | 31.80 | 2.38 |  | 2.13 | 0.00 | 0.00 | 25.38 |  | 9.89 | 57.83 | 31.85 | 0.42 |  | 0.35 | 0.00 | 0.00 | 0.02 |
| 50 | 12.46 | 44.17 | 40.44 | 2.93 |  | 2.61 | 0.00 | 0.00 | 18.30 |  | 12.78 | 45.70 | 41.02 | 0.50 |  | 0.35 | 0.00 | 0.00 | 0.01 |
| 55 | 14.00 | 48.58 | 33.60 | 3.81 |  | 3.26 | 0.00 | 0.00 | 23.32 |  | 14.59 | 50.79 | 34.16 | 0.46 |  | 0.36 | 0.00 | 0.00 | 0.03 |
| 60 | 10.80 | 33.74 | 32.79 | 22.68 |  | 17.94 | 0.00 | 0.01 | 67.02 |  | 16.21 | 43.89 | 35.41 | 4.49 |  | 3.52 | 0.00 | 0.02 | 23.18 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |   | 84.48 | 42.07 | 74.31 | 155.72 |   | 0.00 | 0.00 | 0.00 | 100.00 |   | 84.19 | 41.23 | 73.54 | 158.47 |
| Age | $$γ= 3.0, ρ=0.02, κ=2.0$$ |  | $$γ= 3.0, ρ=0.02, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 82.06 | 17.94 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |  | 15.27 | 67.74 | 16.99 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 10.37 | 53.41 | 24.42 | 11.80 |  | 7.58 | 0.00 | 0.00 | 48.03 |  | 14.38 | 60.76 | 24.86 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 16.94 | 34.45 | 24.94 | 23.67 |  | 16.19 | 0.00 | 15.55 | 55.67 |  | 20.57 | 52.95 | 26.48 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 55 | 11.69 | 33.46 | 19.67 | 35.18 |  | 24.81 | 0.00 | 24.43 | 59.25 |  | 18.66 | 58.91 | 22.43 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 60 | 8.28 | 22.24 | 16.04 | 53.44 |  | 40.33 | 2.21 | 40.45 | 76.35 |  | 23.11 | 48.50 | 20.73 | 7.66 |  | 5.69 | 0.00 | 1.54 | 18.06 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 78.95 | 50.89 | 71.35 | 130.96 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 78.81 | 47.15 | 71.44 | 134.22 |
| Age | $$γ= 5.0, ρ=0.02, κ=2.0$$ |  | $$γ= 5.0, ρ=0.02, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 83.09 | 12.93 | 3.98 |  | 1.69 | 1.69 | 1.69 | 1.69 |  | 58.82 | 32.34 | 8.84 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 10.97 | 48.81 | 16.85 | 23.37 |  | 13.06 | 1.69 | 10.08 | 41.10 |  | 34.97 | 47.45 | 17.57 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 15.56 | 28.55 | 15.79 | 40.10 |  | 24.70 | 2.06 | 24.10 | 48.65 |  | 27.50 | 54.75 | 17.75 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 55 | 9.29 | 26.40 | 12.87 | 51.44 |  | 34.10 | 10.79 | 34.60 | 53.32 |  | 21.57 | 63.51 | 14.92 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 60 | 6.96 | 17.66 | 10.57 | 64.81 |  | 46.47 | 21.08 | 46.17 | 77.22 |  | 26.14 | 49.62 | 13.84 | 10.40 |  | 7.15 | 0.00 | 7.53 | 15.77 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 74.20 | 53.14 | 68.60 | 114.30 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 72.19 | 46.17 | 67.83 | 111.82 |
| Age | $$γ= 8.0, ρ=0.02, κ=2.0$$ |  | $$γ= 8.0, ρ=0.02, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 4.28 | 73.39 | 6.59 | 15.74 |  | 6.69 | 6.69 | 6.69 | 6.69 |  | 77.67 | 16.84 | 5.49 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 11.75 | 48.45 | 8.41 | 31.40 |  | 16.01 | 6.69 | 17.07 | 27.19 |  | 66.39 | 23.89 | 9.72 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 13.40 | 28.03 | 9.95 | 48.62 |  | 27.95 | 14.13 | 29.17 | 42.29 |  | 54.13 | 35.67 | 10.20 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 55 | 8.62 | 24.15 | 8.51 | 58.73 |  | 36.78 | 21.07 | 36.79 | 52.94 |  | 29.04 | 60.93 | 10.01 | 0.02 |  | 0.01 | 0.00 | 0.00 | 0.00 |
| 60 | 6.80 | 15.66 | 7.25 | 70.29 |  | 47.52 | 30.21 | 46.98 | 67.37 |  | 31.49 | 48.69 | 9.17 | 10.65 |  | 6.42 | 0.00 | 6.40 | 14.28 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 69.33 | 53.94 | 65.98 | 94.95 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 62.10 | 41.93 | 59.30 | 91.41 |
| Age | $$γ= 1.0, ρ=0.04, κ=2.0$$ |  | $$γ= 1.0, ρ=0.04, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 75.61 | 24.39 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |  | 0.00 | 75.79 | 24.21 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 9.94 | 56.20 | 31.80 | 2.06 |  | 1.83 | 0.00 | 0.00 | 20.64 |  | 9.92 | 58.19 | 31.83 | 0.06 |  | 0.05 | 0.00 | 0.00 | 0.00 |
| 50 | 12.50 | 44.57 | 40.55 | 2.38 |  | 2.05 | 0.00 | 0.00 | 12.66 |  | 12.95 | 45.93 | 41.04 | 0.08 |  | 0.05 | 0.00 | 0.00 | 0.00 |
| 55 | 14.18 | 49.25 | 33.69 | 2.88 |  | 2.44 | 0.00 | 0.00 | 17.48 |  | 14.71 | 51.01 | 34.21 | 0.08 |  | 0.05 | 0.00 | 0.00 | 0.00 |
| 60 | 11.26 | 34.82 | 33.16 | 20.77 |  | 16.37 | 0.00 | 0.01 | 63.67 |  | 16.95 | 44.90 | 35.60 | 2.55 |  | 2.01 | 0.00 | 0.01 | 16.00 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 84.47 | 42.05 | 74.30 | 156.19 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 84.12 | 41.15 | 73.45 | 158.84 |
| Age | $$γ= 3.0, ρ=0.04, κ=2.0$$ |  | $$γ= 3.0, ρ=0.04, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 82.05 | 17.95 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |  | 21.15 | 62.33 | 16.52 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 10.41 | 54.21 | 24.42 | 10.95 |  | 7.05 | 0.00 | 0.00 | 44.81 |  | 15.73 | 59.63 | 24.64 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 17.09 | 35.50 | 25.12 | 22.29 |  | 15.23 | 0.00 | 14.44 | 52.03 |  | 20.84 | 52.62 | 26.54 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 55 | 12.00 | 34.42 | 19.88 | 33.70 |  | 23.74 | 0.00 | 22.72 | 55.27 |  | 18.77 | 58.74 | 22.49 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 60 | 8.58 | 22.83 | 16.26 | 52.33 |  | 39.49 | 2.20 | 39.80 | 74.65 |  | 23.84 | 49.49 | 20.83 | 5.84 |  | 4.32 | 0.00 | 0.01 | 14.77 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |   | 79.02 | 50.78 | 71.39 | 131.60 |   | 0.00 | 0.00 | 0.00 | 100.00 |   | 78.62 | 46.99 | 71.36 | 133.90 |
| Age | $$γ= 5.0, ρ=0.04, κ=2.0$$ |  | $$γ= 5.0, ρ=0.04, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 0.00 | 85.04 | 12.95 | 2.01 |  | 0.86 | 0.86 | 0.86 | 0.86 |  | 60.42 | 30.76 | 8.81 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 11.62 | 50.94 | 16.69 | 20.76 |  | 11.78 | 0.86 | 9.00 | 38.52 |  | 38.08 | 44.66 | 17.26 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 16.10 | 29.91 | 15.86 | 38.14 |  | 23.55 | 1.59 | 22.77 | 46.25 |  | 29.54 | 52.63 | 17.83 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 55 | 9.64 | 27.24 | 13.04 | 50.08 |  | 33.19 | 10.25 | 33.86 | 52.54 |  | 22.04 | 62.91 | 15.05 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 60 | 7.19 | 18.10 | 10.68 | 64.03 |  | 45.94 | 20.76 | 45.56 | 76.80 |  | 26.94 | 50.50 | 13.90 | 8.66 |  | 5.90 | 0.00 | 5.95 | 13.56 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 74.28 | 52.97 | 68.67 | 114.84 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 71.93 | 45.62 | 67.55 | 111.84 |
| Age | $$γ= 8.0, ρ=0.04, κ=2.0$$ |  | $$γ= 8.0, ρ=0.04, κ=10.0$$ |
| Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |  | Avg. Stochastic Optimal Strategy |  | Total Secured Retirement Income |
| Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |  | Cash | Bond | Equity | DA |  | Avg. | 5th | 50th | 95th |
| 40 | 5.07 | 77.29 | 6.04 | 11.60 |  | 4.93 | 4.93 | 4.93 | 4.93 |  | 77.93 | 16.56 | 5.51 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | 12.62 | 51.17 | 7.89 | 28.33 |  | 14.58 | 4.93 | 15.75 | 25.08 |  | 67.06 | 23.20 | 9.74 | 0.00 |  | 0.00 | 0.00 | 0.00 | 0.00 |
| 50 | 13.92 | 29.08 | 9.96 | 47.04 |  | 27.02 | 12.97 | 28.45 | 41.37 |  | 57.08 | 32.69 | 10.22 | 0.01 |  | 0.01 | 0.00 | 0.00 | 0.00 |
| 55 | 8.91 | 24.81 | 8.55 | 57.73 |  | 36.11 | 19.62 | 36.26 | 52.23 |  | 31.31 | 58.65 | 10.02 | 0.02 |  | 0.02 | 0.00 | 0.00 | 0.01 |
| 60 | 6.96 | 15.94 | 7.28 | 69.82 |  | 47.17 | 29.87 | 46.77 | 66.67 |  | 32.25 | 49.22 | 9.20 | 9.33 |  | 5.56 | 0.00 | 5.41 | 13.08 |
| 65 | 0.00 | 0.00 | 0.00 | 100.00 |  | 69.29 | 53.69 | 65.98 | 94.69 |  | 0.00 | 0.00 | 0.00 | 100.00 |  | 61.78 | 41.62 | 58.94 | 91.26 |

Note: Upfront and selling fees are 0.0% for the cash fund and 0.5% for the bond and equity funds. Expense loadings on annuities are 3.0%. Management fees are ignored.

Source: Author’s Calculation

Table B.2. Monte Carlo Simulation Results (£1,000 p.a.) with different risk aversion and fee structures

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ($γ$, Feea) | Strategy | Mean | StdDev | Mean/StdDev | 5th Pctl. | 50th Pctl. | 95th Pctl. | CEb |
| (1, I) | New Glide Path (DA)  | 55.2354 | 14.3625 | 3.8458 | 36.5790 | 52.8877 | 81.8111 | 53.5514 |
|  | New Glide Path (IA) | 54.0105 | 14.9657 | 3.6090 | 34.7661 | 51.4905 | 81.7533 | 52.1548 |
|  | Glide Path (Equity-to-Bond) | 56.6303 | 28.8644 | 1.9619 | 24.8626 | 49.9473 | 110.7636 | 50.8668 |
|  | Glide Path (Equity-to-Cash) | 50.9162 | 25.6795 | 1.9828 | 22.5801 | 45.0195 | 98.9467 | 45.8338 |
| (3, I) | New Glide Path (DA)  | 56.1819 | 9.9666 | 5.6370 | 42.2370 | 54.9670 | 74.2333 | 53.7750 |
|  | New Glide Path (IA) | 52.8627 | 10.5864 | 4.9934 | 38.3043 | 51.4654 | 72.1647 | 50.0278 |
|  | Glide Path (Equity-to-Bond) | 56.6303 | 28.8644 | 1.9619 | 24.8626 | 49.9473 | 110.7636 | 41.9321 |
|  | Glide Path (Equity-to-Cash) | 50.9162 | 25.6795 | 1.9828 | 22.5801 | 45.0195 | 98.9467 | 37.9330 |
| (5, I) | New Glide Path (DA)  | 57.1334 | 7.9538 | 7.1831 | 45.5161 | 56.3952 | 71.2707 | 54.5818 |
|  | New Glide Path (IA) | 52.0478 | 8.2361 | 6.3195 | 40.1720 | 51.2051 | 66.8108 | 49.0862 |
|  | Glide Path (Equity-to-Bond) | 56.6303 | 28.8644 | 1.9619 | 24.8626 | 49.9473 | 110.7636 | 35.4050 |
|  | Glide Path (Equity-to-Cash) | 50.9162 | 25.6795 | 1.9828 | 22.5801 | 45.0195 | 98.9467 | 32.1310 |
| (8, I) | New Glide Path (DA)  | 58.1401 | 6.0280 | 9.6450 | 49.2704 | 57.6070 | 68.8223 | 55.8784 |
|  | New Glide Path (IA) | 51.3565 | 6.8259 | 7.5238 | 41.1714 | 50.8061 | 63.4013 | 48.0088 |
|  | Glide Path (Equity-to-Bond) | 56.6303 | 28.8644 | 1.9619 | 24.8626 | 49.9473 | 110.7636 | 28.4282 |
|  | Glide Path (Equity-to-Cash) | 50.9162 | 25.6795 | 1.9828 | 22.5801 | 45.0195 | 98.9467 | 25.8880 |
| (3, II) | New Glide Path (DA)  | 55.8462 | 9.7047 | 5.7545 | 42.2138 | 54.6927 | 73.4043 | 53.5415 |
|  | New Glide Path (IA) | 52.2352 | 10.2227 | 5.1097 | 38.1173 | 50.9105 | 70.8437 | 49.5485 |
|  | Glide Path (Equity-to-Bond) | 55.9526 | 28.4682 | 1.9654 | 24.5905 | 49.3711 | 109.3376 | 41.4596 |
|  | Glide Path (Equity-to-Cash) | 50.5617 | 25.4659 | 1.9855 | 22.4375 | 44.7213 | 98.1971 | 37.6860 |
| (3, III) | New Glide Path (DA)  | 55.1213 | 9.7612 | 5.6470 | 41.4591 | 53.9340 | 72.7986 | 52.7673 |
|  | New Glide Path (IA) | 51.8558 | 10.3848 | 4.9934 | 37.5747 | 50.4851 | 70.7902 | 49.0749 |
|  | Glide Path (Equity-to-Bond) | 55.5516 | 28.3146 | 1.9619 | 24.3890 | 48.9959 | 108.6538 | 41.1334 |
|  | Glide Path (Equity-to-Cash) | 49.9463 | 25.1903 | 1.9828 | 22.1500 | 44.1620 | 97.0620 | 37.2105 |
| (3, IV) | New Glide Path (DA)  | 54.7926 | 9.5040 | 5.7652 | 41.4375 | 53.6653 | 71.9870 | 52.5389 |
|  | New Glide Path (IA) | 51.2402 | 10.0280 | 5.1097 | 37.3913 | 49.9408 | 69.4943 | 48.6047 |
|  | Glide Path (Equity-to-Bond) | 54.8869 | 27.9260 | 1.9654 | 24.1221 | 48.4307 | 107.2550 | 40.6699 |
|  | Glide Path (Equity-to-Cash) | 49.5986 | 24.9809 | 1.9855 | 22.0101 | 43.8695 | 96.3267 | 36.9682 |

a Fee structure labelling refers to Table 6.
b Certainty equivalent values to the expected utility of total secured retirement income at retirement is achieved by solving $u^{-1}\left(E\left[u\left(T,X\_{A,T}\right)\right]\right)=CE$; $ρ$ is ignored.

Note: For the upper panel, time preference, and bequest coefficients are $ρ=0.0$ and $κ=0.0$ respectively. Upfront and selling fees are 0.0% for the cash fund and 0.5% for the bond and equity funds. Expense loadings on annuities are 3.0%. Management fees are ignored. For the lower panel, constant risk aversion, time preference, and bequest coefficients are $γ=3.0$, $ρ=0.0$ and $κ=0.0$ respectively.

Source: Author’s calculations.

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1. Generating each scenario takes about 20 minutes with Matlab by using a parallel loop *parfor* on a HP desktop computer with Intel CPU i7-7700 3.60 Ghz and 32 Gbyte memory. [↑](#footnote-ref-1)