North-South Trade and Standards: What Can General Equilibrium Analysis Tell Us?

Appendix

Production function:

The output y of any good z in the continuum is a function of combining both effective labor l and the bad b via the following constant returns to scale Cobb-Douglas technology:

$$y(l,b;z) = \begin{cases} l^{1-\alpha(z)}b^{\alpha(z)} & \text{if } b \leq \lambda l \\ 0 & \text{if } b > \lambda l \end{cases}, \tag{A1}$$

where $\lambda > 0$, $\alpha(z)$ varies across goods, and $\alpha(z) \in [\bar{\alpha}, \hat{\alpha}]$, with $0 < \bar{\alpha} < \hat{\alpha} < 1$. Differentiating (A1) with respect to l and b, allows derivation of the marginal rate of substitution, $\frac{1-\alpha(z)}{\alpha(z)}\frac{b}{l}$.

Consumption function:

Assuming consumption goods z and the public bad b are separable in utility, the indirect utility function of a representative consumer is:

$$V = \int_0^1 f(z)\ln[x(z)]dz - \int_0^1 f(z)\ln[p(z)]dz + \ln i - \frac{\beta D^{\gamma}}{\gamma}, \tag{A2}$$

where $x(z)$ is consumption of z, $f(z)$ is the budget share for each good in the continuum, and the sum of budget shares is $\int_0^1 f(z)dz = 1$; $p(z)$ is the continuum of prices for the consumption goods z; $i = I/L$ is income per capita of a representative consumer, where I is national income and L is the total number of workers in the economy; D is aggregate production of the public bad; β measures the representative consumer's disutility associated with the public bad; and $\gamma \geq 1$, implying consumers' willingness to pay for a reduction in the level of the public bad is non-decreasing in its aggregate level.

$T(\tilde{z})$ *function*:

By minimizing total costs subject to the production function, the unit cost function for a good *z* in the continuum is:

$$a(w_e, c_b; h, z) = \Omega(z) c_b^{\alpha(z)} [w / A(h)]^{1-\alpha(z)}, \qquad \text{(A3)}$$

where $\Omega(z) \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ is a good-specific constant, and *w* is the wage rate for raw labor. For given wages and compliance costs, a good *z* in the continuum will be produced in the North if $a(w_e, c_b; h, z) \leq a^*(w_e^*, c_b^*; h^*, z)$, such that:

$$\tilde{\omega} \equiv \frac{w}{w^*} \leq \frac{A}{A^*}\left(\frac{c_b^*}{c_b}\right)^{\alpha(\tilde{z})/(1-\alpha(\tilde{z}))} \equiv T(\tilde{z}). \qquad \text{(A4)}$$

Given (A4) the optimal level of compliance costs c_b is derived by maximizing (A2) with respect to the public bad:

$$V_p dp / dD + V_i di / dD + V_D = 0, \qquad \text{(A5)}$$

and assuming $dp/dD = 0$, (A5) can be re-arranged as:

$$di / dD = -\left(V_D / V_i\right). \qquad \text{(A6)}$$

From differentiation of (A2):

$$-(V_D / V_i) = (\beta D^{\gamma-1}) / (1/i) = \beta D^{\gamma-1} i, \qquad \text{(A7)}$$

which given the definition of *i* can be re-written as:

$$-L(V_D / V_i) = \beta D^{\gamma-1} I = c_b, \qquad \text{(A8)}$$

Balanced trade requires that $I = \psi(\tilde{z})(I + I^*)$ and $I^* = \psi^*(\tilde{z})(I^* + I)$, where $\psi(\tilde{z}) \equiv \int_0^{\tilde{z}} f(z)dz$ and $1 - \psi(\tilde{z}) = \psi^*(\tilde{z}) \equiv \int_{\tilde{z}}^1 f(z)dz$ are the shares of world spending on Northern and Southern goods respectively. Solving for $I(I^*)$ and $D(D^*)$ in terms of $\tilde{z}$, an expression for relative compliance costs as a function of $\tilde{z}$ can be derived as:

$$\frac{c_b^*}{c_b} = \left(\frac{\psi^*(\tilde{z})}{\psi(\tilde{z})}\right)^{1/\gamma}\left(\frac{\phi^*(\tilde{z})}{\phi(\tilde{z})}\right)^{(\gamma-1)/\gamma} \equiv C(\tilde{z}), \tag{A9}$$

where $\phi(\tilde{z}) \equiv \int_0^{\tilde{z}} \alpha(z) f(z) dz$ $\left(\phi^*(\tilde{z}) \equiv \int_0^{\tilde{z}} \alpha(z) f(z) dz\right)$ are the portions of the shares of world spending on Northern (Southern) goods that contribute to Northern (Southern) compliance costs, $C(\tilde{z}) < 1$ if compliance costs are higher in the North than the South. Substituting (A9) into expression (A4) gives:

$$\tilde{\omega} \equiv \frac{w}{w^*} \leq \frac{A}{A^*}[C(\tilde{z})]^{\alpha(\tilde{z})/(1-\alpha(\tilde{z})} \equiv T(\tilde{z}), \tag{A10}$$

where $d \ln T(\tilde{z}) / d\tilde{z} < 0$, and $T(1) = 0$.

If the cost of an aid transfer is τ, then compliance costs in the North become $c_b = \beta D^{\gamma-1}(I-\tau)$, (A9) being re-written as:

$$\frac{c_b^*}{c_b} = \left(\frac{\psi^*(\tilde{z})}{\psi(\tilde{z})} + h(\tau, \tilde{z})\right)^{1/\gamma}\left(\frac{\phi^*(\tilde{z})}{\phi(\tilde{z})}\right)^{(\gamma-1)/\gamma} \equiv C(\tilde{z}, \tau), \tag{A11}$$

where $h(\tau, \tilde{z}) = D / \psi(\tilde{z})(I - \tau)$. After substitution, (A10) becomes:

$$\tilde{\omega} \equiv \frac{w}{w^*} \leq \frac{A}{A^*}[C(\tilde{z}, \tau)]^{\alpha(\tilde{z})/(1-\alpha(\tilde{z})} \equiv T(\tilde{z}, \tau). \tag{A12}$$