# Supplementary File: Full Mathematical Framework for Phytoclimatic Mapping

Here, we list the full set of mathematical equations that are applicable to defining the phytoclimatic maps of exoplanets, given assumptions about the kinds of organisms that would be considered as plants in a terrestrial setting. Some of these equations are set out in the main paper.

## Aridity index Qn2

The aridity index can be defined as follows (Defaut, 1991). Inclusion of the term “tc-tf” assesses this seasonal range, with the largest variation indicating the greatest degree of continentality. This is only relevant for planets with seasonal variations in temperature and precipitation.

$$Qn\_{2}=10\sqrt{\frac{50(P+10P^{'})}{(T+30)(tc-tf)}}$$

(1)

Here, P is the average annual rainfall; P’ the cumulative rainfall of the driest three consecutive months, where dryness is inversely proportional to the ratio between annual rainfall and average temperature; T is the average temperature; *tc* is the temperature of the warmest month and *tf* the temperature of the coldest. For surfaces that have a very low, average annual temperature, *T* will less than 4.5°C (±0.5°C). Here, the value of the average temperature of the warmest month can be used.

## Holdridge System

The Holdridge Life-Zone Model (HLZM) identifies several hundred, mathematically-defined zones, which are delineated by temperature, *T*, Potential Evapotranspiration, *PET* or *E0*, and precipitation, *P*. Throughout this section, the term biotemperature, *Tb*, is the average temperature of the surface in the range 4.5oC±0.5°C to 30oC.

### Biotemperature and Potential Evapotranspiration Ratio

If we multiply the mean annual biotemperature by 58.93 we can derive PET (*E0*, Holdridge, 1966).

$$PET=T\_{b}58.93$$

(2)

Where Tb is the biotemperature, the physiological range for photosynthesis, as discussed above.

Actual transpiration may be related to temperature and vegetation height as follows:

$$\% Actual Transpiration (E\_{T})=\frac{h\_{s}}{2T\_{b}}$$

(3)

Where *hs* is the height of the vegetation on the surface and *Tb* is the biotemperature. In turn, the actual transpiration, *ET*, may be derived from the potential evapotranspiration, PET (*E0*), by

$$E\_{T}=\%E\_{T}.E\_{0}$$

(4)

By substitution of equation 4 into 3, the following equation is derived:

$$E\_{T}=\frac{T\_{b}58.93h\_{s}}{2T\_{b}}$$

(5)

Where, *ET* is the actual transpiration rate; *hs* is the maximum height (in metres) of vegetation and 58.93 is the empirical relationship between biotemperature and evapotranspiration (Holdridge, 1966). This may be reduced to

$$E\_{T}=29.47h\_{s}$$

(6)

### PET Ratio

The final measurement we need to constrain habitability at a regional level is the Potential Evapotranspiration Ratio, or PET ratio, for short. This is derived from climate models, simply the inverse of the Aridity Index measure, which derived, through alternative means to equation 1, and is shown below.

$$PET (E\_{0}) Ratio=\frac{58.93T\_{b}}{P}$$

(7)

Where Tb has the same meaning as before (biotemperature) and P is the annual precipitation derived from climate models.

### Aridity Index (Qn) from PET

Aridity can be defined by equation 8, or through PET by the following relationship:

$$Qn=\frac{P}{PET}$$

(8)

Where the Aridity Index (Qn), is the ratio of annual precipitation, P, to the potential evapotranspiration, PET.

## Maximum Vegetation Height and Planetary Mass

Height of dominant vegetation, which is constrained by:

$$h\_{S}=h\_{E}.\frac{g\_{E}}{g\_{S}}$$

(9)

Where *hS* is the maximum height of vegetation on an exoplanet of mass, *M* and radius *R*; *hE* is the maximum height of vegetation on Earth (Koch et al, 2004); *gE* and *gS*are the gravitational field strengths of the Earth and the exoplanet in question, respectively (Stevenson, 2019a).

## Evapotranspiration and planetary mass

Where *hp* is the height of the dominant vegetation and the factor 29.47 is half the empirical factor used in equation 1. See Holdridge, 1966, for details. Then substituting equation 5 into 8 gives us:

$$E\_{T}=h\_{E}.\frac{29.47g\_{E}}{g\_{S}}$$

(10)

As terrestrial vegetation appears limited to heights of 122-130 m (Stevenson, 2019a) this equation can be simplified to

$$E\_{T}=\frac{3831.1g\_{E}}{g\_{s}}$$

(11)

If we adopt the maximum value of 130 m for terrestrial vegetation. The value 3831.1 is a simple multiplication of the absolute maximum height of trees on Earth by the evapotranspiration factor 29.47.

## Wind velocity and vegetation height

Surface wind velocity scales with the frictional coefficient of the surface and with the pressure gradient between geographical areas. In principle, when vegetation doubles in height, the wind velocity decreases by 0.33-0.48 m/s.

$$δU=aln(\frac{z\_{0}}{z\_{0c}})$$

(12)

Where U is the change in velocity of surface winds to a height of 10 m; *a* is a constant, which has a value of 0.48-0.37ms-1 when the surface is roughened (the lower end pertaining to a generic increase in roughness; and the upper value pertaining to an increase in the roughness of grassland); z0c is a standard roughness value and z0 representing the new value caused in this instance by an increase in vegetative cover.

### Evapotranspiration – alternative methods based on temperature only

Evapotranspiration can be derived using the Hargreaves Equation. This equation does not include relative humidity, therefore, it is only directly applicable to arid regions where the relative humidity is negligible.

Hargreaves$ET=0.0023(T\_{mean}+17.8)\sqrt{(T\_{max}-T\_{min})}R\_{a}$

(13)

*T* is the temperature measured in degrees Celsius; both *Ra*, the extra-terrestrial radiation and ET are given units millimetres per unit time.

### Precipitation distribution with distance from water sources

The distribution of rainfall along storm tracks can be defined by the linear regression,

$$lnP=a+bx$$

(14)

Where P is the precipitation rate, *x,* is the distance from the water source and b is defined by equation 15:

$$b= \frac{-1}{l}$$

(15)

Here, *l*, is the precipitation e-folding length.