# #1 APPENDIX

# BIRTH-PEAK-DEATH (BPD) Theorem
# to estimate SENILITY, even if only approximately.

This result is practically important for ESTIMATING the SENILITY instant (s)
given the birth (b), peak (p) and death (d) instants,
i.e. THE THREE MOST IMPORTANT INSTANTS in one's lifetime.

So, this theorem applies only when the death d is known also, and not just b and p only.
In other words, this result applies to DEAD civilizations or DEAD living organisms,
and NOT to alive civilizations or organisms that still have to die.

This approximated but important result was discovered by Claudio Maccone
only on April 4, 2015, at about 4 pm..

```
(%i1) kill(all);
```
(%o0) *done*

```
(%i1) assume(p>b,s>p,d>s);
```
(%o1) $[p>b, s>p, d>s]$

```
(%i2) History_mu:mu=log(s-b)+(2*s^2+(-3*d-b)*s+d^2+b*d)/((d-b)*s-b*d+b^2);
```
(%o2) $$\mu=\log(s-b)+\frac{2\,s^2+(-3\,d-b)\,s+d^2+b\,d}{(d-b)\,s-b\,d+b^2}$$

Factorized expression of mu, discovered by the author only on December 20, 2014, at 20:11.

```
(%i3) Maxima_factorized_History_mu:factor(History_mu-log(s-b))+log(s-b);
```
(%o3) $$\mu=\log(s-b)+\frac{(s-d)\,(2\,s-d-b)}{(d-b)\,(s-b)}$$

```
(%i4) Final_factorized_History_mu:mu=log(s-b)+((d-s)*(-2*s+d+b))/((d-b)*(s-b));
```
(%o4) $$\mu=\log(s-b)+\frac{(-2\,s+d+b)\,(d-s)}{(d-b)\,(s-b)}$$

```
(%i5) radcan(Final_factorized_History_mu-Maxima_factorized_History_mu);
```
(%o5) $0=0$

(%i6) History_sigma:sigma=(abs(d-s))/(sqrt(d-b)*sqrt(s-b));

(%o6) $$\sigma = \frac{d-s}{\sqrt{d-b}\,\sqrt{s-b}}$$

(%i7) History_Formulae:[Final_factorized_History_mu,History_sigma];

(%o7) $$[\mu = \log(s-b) + \frac{(-2\,s+d+b)\,(d-s)}{(d-b)\,(s-b)}, \sigma = \frac{d-s}{\sqrt{d-b}\,\sqrt{s-b}}]$$

(%i8) def_Peak_Height:Peak_Height=(%e^(sigma^2/2-mu))/(sqrt(2*%pi)*sigma);

(%o8) $$Peak\_Height = \frac{\%e^{\frac{\sigma^2}{2}-\mu}}{\sqrt{2}\,\sqrt{\pi}\,\sigma}$$

# Death_Formula

Let us find the Death Formula for b-lognormals,
appearing already in the 2012 book "Mathematical SETI",
page 163, equation (6.30).

(%i9) def_p:p=b+%e^(mu-sigma^2);

(%o9) $$p = \%e^{\mu-\sigma^2} + b$$

(%i10) towards_Death_Formula:def_p,History_Formulae;

(%o10) $$p = (s-b)\,\%e^{\frac{(-2\,s+d+b)\,(d-s)}{(d-b)\,(s-b)} - \frac{(d-s)^2}{(d-b)\,(s-b)}} + b$$

(%i11) linear_eq_in_d:radcan(towards_Death_Formula-b);

(%o11) $$p-b = (s-b)\,\%e^{\frac{s-d}{d-b}}$$

(%i12) Death_Formula:first(solve(linear_eq_in_d,d));

(%o12) $$d = \frac{s + b\,\log\left(\frac{b}{b-s} - \frac{p}{b-s}\right)}{\log\left(\frac{b}{b-s} - \frac{p}{b-s}\right) + 1}$$

Let us now APPROXIMATE TO FOURTH ORDER in (s-p) the Death Formula.

(%i13) eq_for_approx_FOURTH_ORDER_s_from_b_p_and_d:taylor(Death_Formula,s,p,4);

(%o13)/T/ $d+\ldots=p+2\,(s-p)-\frac{3\,(s-p)^2}{2\,b-2\,p}+\frac{5\,(s-p)^3}{6\,b^2-12\,p\,b+6\,p^2}-\frac{(s-p)^4}{2\,b^3-6\,p\,b^2+6\,p^2\,b-2\,p^3}+\ldots$

(%i14) Lodovico_Ferrari_formula_to_solve_this_quartic_eq:second(solve(eq_for_approx_FOURTH_OR

(%o14) $s = sqrt( -( -449\, p^{3} + 1347\, b\, p^{2} - 1347\, b^{2}\, p + 449\, b^{3}) / ( 18\, sqrt( ( 36\, ( ( (p-b)^{9/2}\, sqrt( -$
$2053\, p^{3} - (9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858$
$b^{2}\, d - 24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}$
$(-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 2/3) +$
$(-47\, p^{2} + 94\, b\, p - 47\, b^{2})\, ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} - (9393\, b - 15552\, d)\, p^{2} -$
$(2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d - 24740\, b^{3}) ) / 27 + ( 4\, p^{6}$
$+ 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}\, (-550\, p^{3} - 470\, d\, p^{2}) + b^{4}$
$(530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 1/3) + 52\, p^{4} + (-96\, d - 112\, b)\, p^{3} +$
$(288\, b\, d + 24\, b^{2})\, p^{2} + (80\, b^{3} - 288\, b^{2}\, d)\, p + 96\, b^{3}\, d - 44\, b^{4}) / ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} -$
$(9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d -$
$24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}$
$(-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 1/3) ) ) - ( ($
$(p-b)^{9/2}\, sqrt( -2053\, p^{3} - (9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} -$
$38889\, b\, d^{2} + 51858\, b^{2}\, d - 24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}$
$(295\, p^{4} + 470\, d\, p^{3}) + b^{3}\, (-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) /$
$9\, ) \wedge( 1/3) + ( -13\, p^{4} + b\, (28\, p^{3} - 72\, d\, p^{2}) + 24\, d\, p^{3} + b^{2}\, (72\, d\, p - 6\, p^{2}) + b^{3}\, (-20\, p - 24\, d) + 11\, b^{4})$
$/ ( 9\, ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} - (9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824$
$d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d - 24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}$
$(295\, p^{4} + 470\, d\, p^{3}) + b^{3}\, (-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) /$
$9\, ) \wedge( 1/3) ) - \frac{23\, p^{2} - 88\, b\, p + 29\, b^{2}}{18} + \frac{-4\, p^{2} + b\, p - 3\, b^{2}}{3}) / 2 - sqrt( ( 36\, ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} -$
$(9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d -$
$24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}$
$(-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 2/3) +$
$(-47\, p^{2} + 94\, b\, p - 47\, b^{2})\, ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} - (9393\, b - 15552\, d)\, p^{2} -$
$(2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d - 24740\, b^{3}) ) / 27 + ( 4\, p^{6}$
$+ 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}\, (-550\, p^{3} - 470\, d\, p^{2}) + b^{4}$
$(530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 1/3) + 52\, p^{4} + (-96\, d - 112\, b)\, p^{3} +$
$(288\, b\, d + 24\, b^{2})\, p^{2} + (80\, b^{3} - 288\, b^{2}\, d)\, p + 96\, b^{3}\, d - 44\, b^{4}) / ( ( (p-b)^{9/2}\, sqrt( -2053\, p^{3} -$
$(9393\, b - 15552\, d)\, p^{2} - (2583\, d^{2} + 25938\, b\, d - 22362\, b^{2})\, p + 13824\, d^{3} - 38889\, b\, d^{2} + 51858\, b^{2}\, d -$
$24740\, b^{3}) ) / 27 + ( 4\, p^{6} + 47\, d\, p^{5} + b\, (-71\, p^{5} - 235\, d\, p^{4}) + b^{2}\, (295\, p^{4} + 470\, d\, p^{3}) + b^{3}$
$(-550\, p^{3} - 470\, d\, p^{2}) + b^{4}\, (530\, p^{2} + 235\, d\, p) + b^{5}\, (-259\, p - 47\, d) + 51\, b^{6}) / 9\, ) \wedge( 1/3) ) / 12 +$
$\frac{7\, p + 5\, b}{12}$

Let us now APPROXIMATE TO THIRD ORDER in (s-p) the Death Formula.

(%i15) eq_for_approx_THIRD_ORDER_s_from_b_p_and_d:taylor(Death_Formula,s,p,3);

(%o15)/T/ $d+\ldots=p+2\,(s-p)-\frac{3\,(s-p)^2}{2\,b-2\,p}+\frac{5\,(s-p)^3}{6\,b^2-12\,p\,b+6\,p^2}+\ldots$

(%i16) Cardan_formula_to_solve_this_cubic_eq:solve(eq_for_approx_THIRD_ORDER_s_from_b_p_an

So, we found the third order approximated s from b, p and d :

(%i17) approx_THIRD_ORDER_s_from_b_p_and_d:last(Cardan_formula_to_solve_this_cubic_eq);

(%o17) $s=(\frac{(p-b)^2\sqrt{59\,p^2+(-72\,d-46\,b)\,p+225\,d^2-378\,b\,d+212\,b^2}}{25}-\frac{12\,p^3+b\,(39\,p^2+150\,d\,p)-75\,d\,p^2+b^2\,(-114\,p-75\,d)+63\,b^3}{125})\hat{}(1/3)-(11\,p^2-22\,b\,p+11\,b^2)/(25\,(\frac{(p-b)^2\sqrt{59\,p^2+(-72\,d-46\,b)\,p+225\,d^2-378\,b\,d+212\,b^2}}{25}-\frac{12\,p^3+b\,(39\,p^2+150\,d\,p)-75\,d\,p^2+b^2\,(-114\,p-75\,d)+63\,b^3}{125})\hat{}(1/3))+\frac{6\,p+9\,b}{15}$

Let us now find the second order approximated s from p and d.

(%i18) eq_for_approx_SECOND_ORDER_eq_for_s_from_b_p_and_d:taylor(Death_Formula,s,p,2);

(%o18)/T/ $d+\ldots=p+2\,(s-p)-\frac{3\,(s-p)^2}{2\,b-2\,p}+\ldots$

(%i19) solving_this_quadratic_for_s:solve(eq_for_approx_SECOND_ORDER_eq_for_s_from_b_p_and_

(%i20) approx_SECOND_ORDER_s_from_b_p_and_d:last(solving_this_quadratic_for_s);

(%o20) $s=\frac{\sqrt{2}\,\sqrt{-p^2+(3\,d-b)\,p-3\,b\,d+2\,b^2}+p+2\,b}{3}$

Finally, let usfind the first order approximated s from p and d.

(%i21) eq_for_approx_FIRST_ORDER_s_from_b_p_and_d:taylor(Death_Formula,s,p,1);
(%o21)/T/ $d+\ldots=p+2\,(s-p)+\ldots$

(%i22) approx_FIRST_ORDER_s_from_b_p_and_d:first(solve(eq_for_approx_FIRST_ORDER_s_from_

(%o22) $s = \frac{p + d}{2}$

Thus, the first-order approximated s from p and d simply falls HALF WAY BETWEEN p AND d,
a rather obvious result, that, however, encourages us to use NOT the first-order approximation to s,
but rather the second order one (quadratic), if not even the third-order one (cubic) based on the lengthy
Cardan formulae.

A practical application to the case of Egypt: s=-689.

(%i23) Egypt:[b=-3100, s=-689, d=-30];

(%o23) $[b = -3100, s = -689, d = -30]$

(%i24) Final_factorized_History_mu,Egypt;

(%o24) $\mu = \log(2411) - \frac{577284}{3700885}$

(%i25) mu_Egypt:ev(%,numer);

(%o25) $\mu = 7.631811485498069$

(%i26) History_sigma,Egypt;

(%o26) $\sigma = \frac{659}{\sqrt{2411}\,\sqrt{3070}}$

(%i27) sigma_Egypt:ev(%,numer);

(%o27) $\sigma = 0.24222425100085$

(%i28) def_p_Egypt:p=b+%e^(mu-sigma^2);

(%o28) $p = \%e^{\mu - \sigma^2} + b$

(%i29) def_p_Egypt:ev(%,Egypt,mu_Egypt,sigma_Egypt);

(%o29) $p = -1154.763392556425$

(%i30) b_p_and_d_Egypt:[b=-3100,def_p_Egypt,d=-30];

(%o30) $[b = -3100, p = -1154.763392556425, d = -30]$

Let us compute the APPROXIMATED s in this case of Egypt, for which s=-689.

(%i31) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,b_p_and_d_

(%o31) $s = -688.8249920336343$

(%i32) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,b_p_and_d_Egypt;
(%o32) $s = -687.7068130243333$

(%i33) approx_SECOND_ORDER_s_from_b_p_and_d,b_p_and_d_Egypt;
(%o33) $s = \frac{3759.213175069583\sqrt{2} - 7354.763392556425}{3}$

(%i34) numeric_SECOND_ORDER_s:ev(%,numer);
(%o34) $s = -679.4777121737989$

(%i35) numeric_FIRST_ORDER_s:approx_FIRST_ORDER_s_from_b_p_and_d,b_p_and_d_Egypt;
(%o35) $s = -592.3816962782126$

Napoleon. Written April 4, 2015, at 11:57 pm.

(%i36) Napoleon:[b=1769, p=1812, d=1821];
(%o36) $[b = 1769, p = 1812, d = 1821]$

(%i37) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Napoleon$

(%i38) ev(%,numer);
(%o38) $s = 1816.178127561793$

(%i39) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Napoleon$

(%i40) ev(%,numer);
(%o40) $s = 1816.178955342549$

(%i41) History_Formulae,Napoleon,numeric_FOURTH_ORDER_s$

(%i42) Napoleon_blognormal:[first(Napoleon),ev(%,numer)];
(%o42) $[b = 1769, [\mu = 3.770679430224423, \sigma = 0.097351734804091]]$

(%i43) ev(def_Peak_Height,Napoleon_blognormal,numer);
(%o43) $Peak\_Height = 0.094850381900173$

Napoleon at St. Helena realizes he will never be allowed to go back to Europe.

Ancient Greece, from the first Olympic Games (-776) to Pericle's peak (-438, Parthenon completed) up to the death of Cleopatra (-30, the last Ellenistic kingdom, Egypt, becomes a Roman province).

(%i44) Greece:[b=-776,p=-438,d=-30];

(%o44) $[b=-776, p=-438, d=-30]$

(%i45) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Greece$

(%i46) ev(%,numer);

(%o46) $s=-293.8384888474267$

(%i47) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Greece$

(%i48) ev(%,numer);

(%o48) $s=-292.3453956762008$

-281: Seleucus I Nicator wins the battle of Corupedion against Lysimachus but shortly after is killed by Ptolemy Ceraunus, and so any residual hope to re-unify Alexander's empire is lost. That was the Ellenistic SENILITY.

(%i49) History_Formulae,Greece,numeric_FOURTH_ORDER_s$

(%i50) Greece_blognormal:[first(Greece),ev(%,numer)];

(%o50) $[b=-776, [\mu=6.018136748571353, \sigma=0.4399187445982]]$

(%i51) ev(def_Peak_Height,Greece_blognormal,numer);

(%o51) $Peak\_Height=0.0024317459289002$

Ancient Rome, from Rome's foundation (-753) to Trajan's peak (117, largest extent of the Roman Empire, even Susa captured) up to the fall of the Western Roman Empire (476).

(%i52) Rome:[b=-753,p=117,d=476];

(%o52) $[b=-753, p=117, d=476]$

(%i53) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Rome$

(%i54) ev(%,numer);

(%o54) $s=273.1566057502638$

(%i55) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Rome$

(%i56) ev(%,numer);

(%o56) $s=273.3289719109718$

273: Costruction of the Aurelian's Walls around imperial Rome (271-275). This means that the town was no longer regarded as secure against the invasions of the Barbarians, and this was ROME'S SENILITY.

(%i57) History_Formulae,Rome,numeric_FOURTH_ORDER_s$

(%i58) Rome_blognormal:[first(Rome),ev(%,numer)];

(%o58) $[b=-753,[\mu=6.801153565071694,\sigma=0.1806251018937]]$

(%i59) ev(def_Peak_Height,Rome_blognormal,numer);

(%o59) $Peak\_Height=0.0024975432542786$

Renaissance Italy, from the death of Holy Roman Emperor Frederick II (1250) to the shutting down of the Accademia del Cimento in Florence, the last scientific istitution of Renaissance's Italy.
This was the Italian Renaissance's SENILITY.

(%i60) Renaissance_Italy:[b=1250,p=1497,d=1660];

(%o60) $[b=1250,p=1497,d=1660]$

(%i61) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Renaissance

(%i62) ev(%,numer);

(%o62) $s=1562.996690578324$

(%i63) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Renaissance_Italy$

(%i64) ev(%,numer);

(%o64) $s=1563.207783080236$

1563: The Council of Trent (1545-1563) ends, assuring absolute power for the Roman Catholic Church in Italy. This was the SENILITY oof Renaissanse Italy.

(%i65) History_Formulae,Renaissance_Italy,numeric_FOURTH_ORDER_s$

(%i66) Renaissance_Italy_blognormal:[first(Renaissance_Italy),ev(%,numer)];

(%o66) $[b=1250,[\mu=5.582923745446211,\sigma=0.27078509155415]]$

(%i67) ev(def_Peak_Height,Renaissance_Italy_blognormal,numer);

(%o67) $Peak\_Height=0.0057487651546898$

Portuguese Empire: it started in 1419 with the colonization of Madeira, the first portuguese step towards the circumnavigtion of Africa. It was strongly re-inforced by the trade of black slaves from Angola accross the Antlantic into Brasil, and Brasil became by far the most important Portuguese colony.
The SENILITY of the Portuguese colonial empire started in 1822, when Brasil declared it independence of Portugal.

```
(%i68) Portuguese_Empire:[b=1419,p=1716,d=1999];
```

(%o68) $[b=1419, p=1716, d=1999]$

```
(%i69) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Portuguese_
```

```
(%i70) ev(%,numer);
```

(%o70) $s=1822.169804735767$

```
(%i71) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Portuguese_Empire$
```

```
(%i72) ev(%,numer);
```

(%o72) $s=1822.88352210571$

1822: Brazil declared its independence from Portugal.
This was the Portuguese colonial Empire's SENILITY.

```
(%i73) History_Formulae,Portuguese_Empire,numeric_FOURTH_ORDER_s$
```

```
(%i74) Portuguese_Empire_blognormal:[first(Portuguese_Empire),ev(%,numer)];
```

(%o74) $[b=1419, [\mu=5.828198329751777, \sigma=0.36567766113674]]$

```
(%i75) ev(def_Peak_Height,Portuguese_Empire_blognormal,numer);
```

(%o75) $Peak\_Height=0.0034331614179115$

Spanish Empire: it started in 1402 with the colonization of the first Canary island of Lanzarote.
The discovery of America by Christopher Columbus (Cristoforo Colombo) boosted the Spanish Empire, that kept growing enormously in America (and the Philippines) until it reached its maximum extension in America in 1798 (peak), having conquered California, Texas and the Gulf of Mexico coastline up to Florida inclusive. Having lost her fleet at Trafalgar in 1805, Spain gradually started losing its American colonies that proclaimed independence, virtually complete after the battle of Ayacucho (1825).
The final death of the Spanish Empire arrived in 1898, with the loss of Cuba, Puerto Rico and the Philippine
The SENILITY of the Spanish colonial empire is found hereafter as 1843, and actually in 1836 the Spanish government went so far as to renounce sovereignty over all of continental America.

```
(%i76) Spanish_Empire:[b=1402,p=1798,d=1898];
```

(%o76) $[b=1402, p=1798, d=1898]$

```
(%i77) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,Spanish_Em
```

```
(%i78) ev(%,numer);
(%o78) s = 1843.761579677793
```

```
(%i79) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,Spanish_Empire$
```

```
(%i80) ev(%,numer);
(%o80) s = 1843.776414682098
```

1836: the Spanish government renounces sovereignty over all of continental America.
This is the Spanish colonial Empire's SENILITY.

```
(%i81) History_Formulae,Spanish_Empire,numeric_FOURTH_ORDER_s$
```

```
(%i82) Spanish_Empire_blognormal:[first(Spanish_Empire),ev(%,numer)];
(%o82) [b = 1402, [μ = 5.994844602256773, σ = 0.11587032442965]]
```

```
(%i83) ev(def_Peak_Height,Spanish_Empire_blognormal,numer);
(%o83) Peak_Height = 0.0086362522002876
```

French Empire: the history of the French colonial empire is rather unusual among the histories
of European colonial empires. In fact, there were basically TWO French colonial empires:
1) the FIRST French colonial empire in North America and India (approximately 1525-1763),and
2) the SECOND French colonial empire in Africa and "Indochine", (approximately 1624-1954).
We assume here that Giovanni da Verrazzano's first recorded visit to New York Bay (1525), in the service
of France, marks the beginning (b) of the French colonial empire.
We also assume that the peak of the French (second) colonial empire was reached in 1920, after the
French victory over Germany in World War 1 (1914-1918).
Finally, we assume 1962 as the date of the "death" of the French colonial empire, due to the loss of Algeria.
Our mathematical conclusion is that the SENILITY of the French colonial empire is found to have occurred
just when Nazi Germany defeated and occupied France entirely.

```
(%i84) French_Empire:[b=1525,p=1920,d=1962];
(%o84) [b = 1525, p = 1920, d = 1962]
```

```
(%i85) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,French_Emp
```

```
(%i86) ev(%,numer);
(%o86) s = 1940.202364250324
```

```
(%i87) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,French_Empire$
```

(%i88) ev(%,numer);
(%o88) $s = 1940.20298991006$

1940: Nazi Germany invades the whole of France: the Germans entered Paris on June 14, 1940.
This was the French colonial Empire's SENILITY.

(%i89) History_Formulae,French_Empire,numeric_FOURTH_ORDER_s$

(%i90) French_Empire_blognormal:[first(French_Empire),ev(%,numer)];
(%o90) $[b = 1525, [\mu = 5.981504503169573, \sigma = 0.051172750852446]]$

(%i91) ev(def_Peak_Height,French_Empire_blognormal,numer);
(%o91) $Peak\_Height = 0.019710858294224$

British Empire: the history of the British colonial empire pratically started in 1583,
just five years before the British fleet of Elisabeth I defeated the Spanish Armada (1588).

(%i92) British_Empire:[b=1583,p=1922,d=1974];
(%o92) $[b = 1583, p = 1922, d = 1974]$

(%i93) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,British_Emp

(%i94) ev(%,numer);
(%o94) $s = 1946.604324260161$

(%i95) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,British_Empire$

(%i96) ev(%,numer);
(%o96) $s = 1946.606432564159$

1947: Britain grants India its own independence. Since India was the most important British colony,
it independence marked the SENILITY of the British Empire.

(%i97) History_Formulae,British_Empire,numeric_FOURTH_ORDER_s$

(%i98) British_Empire_blognormal:[first(British_Empire),ev(%,numer)];
(%o98) $[b = 1583, [\mu = 5.831279668210668, \sigma = 0.072657277179497]]$

(%i99) ev(def_Peak_Height,British_Empire_blognormal,numer);
(%o99) $Peak\_Height = 0.016154171990071$

USA Empire: the history of the American colonial empire pratically started in 1898,
when the Americans took Cuba, Puerto Rico and the Philippines from Spain.
The peak of America were the set of six Moon Landings, 1969-1972, so we take p=1972.
The putative end of American supremacy to China we guess might occur around 2050...

(%i100) USA_Empire:[b=1898,p=1972,d=2050];
(%o100) $[b=1898, p=1972, d=2050]$

(%i101) numeric_FOURTH_ORDER_s:Lodovico_Ferrari_formula_to_solve_this_quartic_eq,USA_Empir

(%i102) ev(%,numer);
(%o102) $s=2000.554521535127$

(%i103) numeric_THIRD_ORDER_s:approx_THIRD_ORDER_s_from_b_p_and_d,USA_Empire$

(%i104) ev(%,numer);
(%o104) $s=2000.786403465213$

2001, 9/11: With the terrorist attack to the Twin Towers and the Pentagon, America's SENILITY arrived.

(%i105) History_Formulae,USA_Empire,numeric_FOURTH_ORDER_s$

(%i106) USA_Empire_blognormal:[first(USA_Empire),ev(%,numer)];
(%o106) $[b=1898, [\mu=4.461934627734692, \sigma=0.39602935949885]]$

(%i107) ev(def_Peak_Height,USA_Empire_blognormal,numer);
(%o107) $Peak\_Height=0.012573213963496$

# ALTERNATIVE WAY

(%i108) linear_eq_in_d;
(%o108) $p-b=(s-b)\,\%e^{\frac{s-d}{d-b}}$

(%i109) FOURTH_ORDER_DIRECTLY_FORM_p:taylor(linear_eq_in_d,s,p,4);

(%o109)/T/ $(-b+p)+\ldots=\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,b+\frac{\left(2\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)}{b-d}-\frac{\left(3\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-2\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^2}{2\,b^2-4\,d\,b+2\,d^2}+\frac{\left(4\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-3\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^3}{6\,b^3-18\,d\,b^2+18\,d^2\,b-6\,d^3}-\frac{\left(5\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-4\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^4}{24\,b^4-96\,d\,b^3+144\,d^2\,b^2-96\,d^3\,b+24\,d^4}+\ldots$

(%i110) s_to_FOURTH_ORDER:second(solve(FOURTH_ORDER_DIRECTLY_FORM_p,s))$

(%i111) s_to_FOURTH_ORDER,b_p_and_d_Egypt;

(%o111) $s=-688.9942540986185$

(%i112) THIRD_ORDER_DIRECTLY_FORM_p:taylor(linear_eq_in_d,s,p,3);

(%o112)/T/ $(-b+p)+\ldots=\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,b+\frac{\left(2\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)}{b-d}-\frac{\left(3\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-2\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^2}{2\,b^2-4\,d\,b+2\,d^2}+\frac{\left(4\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-3\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^3}{6\,b^3-18\,d\,b^2+18\,d^2\,b-6\,d^3}+\ldots$

(%i113) s_to_THIRD_ORDER:last(solve(THIRD_ORDER_DIRECTLY_FORM_p,s))$

(%i114) s_to_THIRD_ORDER,b_p_and_d_Egypt;

(%o114) $s=-688.8429186197122$

(%i115) SECOND_ORDER_DIRECTLY_FORM_p:taylor(linear_eq_in_d,s,p,2);

(%o115)/T/ $(-b+p)+\ldots=\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,b+\frac{\left(2\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)}{b-d}-\frac{\left(3\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-2\,\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)^2}{2\,b^2-4\,d\,b+2\,d^2}+\ldots$

(%i116) s_to_SECOND_ORDER:first(solve(SECOND_ORDER_DIRECTLY_FORM_p,s))$

(%i117) s_to_SECOND_ORDER,b_p_and_d_Egypt;

(%o117) $s=-685.6488137412043$

(%i118) FIRST_ORDER_DIRECTLY_FORM_p:taylor(linear_eq_in_d,s,p,1);

(%o118)/T/ $(-b+p)+...=\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,b+\frac{\left(2\,\%e^{\frac{p-d}{d-b}}\,b-\%e^{\frac{p-d}{d-b}}\,p-\%e^{\frac{p-d}{d-b}}\,d\right)(s-p)}{b-d}+...$

(%i119) s_to_FIRST_ORDER:first(solve(FIRST_ORDER_DIRECTLY_FORM_p,s))$

(%i120) s_to_FIRST_ORDER,b_p_and_d_Egypt;

(%o120) $s=-627.8670498411429$