Appendix A Proofs of Theorems

A.1 Theorems and Proofs from Section 3:

Theorem 15

Let $D = (S, L, C^{\mathbf{t}})$ be an ADF^+ and, for every $s \in S$, we define $C_s^{max} = \{R \in C_s^{\mathbf{t}} \mid \textit{there is no } R' \in C_s^{\mathbf{t}} \textit{ such that } R \subset R'\}$. Then, for every $s \in S$,

$$\varphi_s \equiv \bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s)-R} \neg b.$$

Proof

According to Equation (1), $\varphi_s \equiv \varphi_1 = \bigvee_{R \in C_s^{\mathbf{t}}} \left(\bigwedge_{a \in R} a \wedge \bigwedge_{b \in par(s)-R} \neg b\right)$. Let $\varphi_2 = \bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s)-R} \neg b$. We will show $\varphi_1 \equiv \varphi_2$, i.e., for any 2-valued interpretation v, $v(\varphi_1) = v(\varphi_2)$:

- If $v(\varphi_1) = \mathbf{t}$, then there exists $R \in C_s^{\mathbf{t}}$ such that for all $a \in R$, $v(a) = \mathbf{t}$ and for all $b \in par(s) - R$, $v(b) = \mathbf{f}$. As there exists $R' \in C_s^{max}$ such that $R \subseteq R'$, we obtain for all $b \in par(s) - R'$, $v(b) = \mathbf{f}$. Thus, $v(\varphi_2) = \mathbf{t}$.
- If $v(\varphi_1) = \mathbf{f}$, then for each $R \in C_s^{\mathbf{t}}$ there exists $a \in R$ such that $v(a) = \mathbf{f}$ or there exists $b \in par(s) - R$ such that $v(b) = \mathbf{t}$. In particular, for each $R \in C_s^{max}$ there exists $a \in R$ such that $v(a) = \mathbf{f}$ or there exists $b \in par(s) - R$ such that $v(b) = \mathbf{t}$, and[1] there exists $b \in par(s) - R'$ such that $v(b) = \mathbf{t}$, in which $R' = R - \{a \in R \mid v(a) = \mathbf{f}\}$. But then for each $R \in C_s^{max}$ there exists $b \in par(s) - R$ such that $v(b) = \mathbf{t}$. Thus, $v(\varphi_2) = \mathbf{f}$.

□

Theorem 16

Let $D = (S, L, C^{\mathbf{t}})$ be an ADF^+. A link $(r, s) \in L$ is redundant iff $r \in R$ for every $R \in C_s^{max}$.

Proof

($\Rightarrow$)

If $(r, s) \in L$ is a redundant link, then, in particular, it is a supporting link, i.e., for every $R \subseteq par(s)$, we have if $R \in C_s^{\mathbf{t}}$, then $(R \cup \{r\}) \in C_s^{\mathbf{t}}$.

By absurd, suppose there exists $R \in C_s^{max}$ such that $r \notin R$. This means $R \in C_s^{\mathbf{t}}$. But then we obtain $(R \cup \{r\}) \in C_s^{\mathbf{t}}$. It is an absurd as $R \in C_s^{max}$.

($\Leftarrow$)

Assume for any $R \in C_s^{max}$, we have $r \in R$. By absurd, suppose $(r, s) \in L$ is not redundant. Then there exists $R' \subseteq par(s)$ such that $C_s(R') = \mathbf{t}$ and $C_s(R' \cup \{r\}) = \mathbf{f}$.

As $r \in R$ for any $R \in C_s^{max}$, there exists $R'' \in C_s^{max}$ such that $R' \cup \{r\} \subseteq R''$ and $C_s(R'') = \mathbf{t}$. But then, as any link in L is attacking, we obtain $C_s(R' \cup \{r\}) = \mathbf{t}$. An absurd. □

[1] As D is an ADF^+, for each $R \in C_s^{max}$, for each $R' \subseteq R$, we have $R' \in C_s^{\mathbf{t}}$.

Corollary 17
Let $D = (S, L, C^{\mathbf{t}})$ be an ADF^{+}. For each $s \in S$, if φ_s is $\bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s) - R} \neg b$ and $L' = \{(r, s) \mid \neg r \ \textit{appears in} \ \varphi_s\}$, then L' has no redundant link.

Proof
The result is straightforward: from Theorem 16, we know $(r, s) \in L$ is a redundant link iff for any $R \in C_s^{max}$, we have $r \in R$ iff $\neg r$ does not appear in $\bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s) - R} \neg b$ iff $(r, s) \notin L'$. $\square$

Theorem 19
Let $D = (S, L, C^{\mathbf{t}})$ be an ADF^{+}, $s \in S$; $r \in par(s)$ and $C_s^{\mathbf{t}}(r) = \big\{R \in C_s^{\mathbf{t}} \mid r \in R\big\}$. A link $(r, s) \in L$ is redundant iff $|C_s^{\mathbf{t}}(r)| = \dfrac{|C_s^{\mathbf{t}}|}{2}$.

Proof
The proof follows from the definition of ADF^{+}, a property of Power Sets and the Principle of Inclusion and Exclusion (PIE).

In D, for every $s \in S$ and $M \subseteq par(s)$, if $C_s(M) = \mathbf{t}$, then $C_s(M') = \mathbf{t}$ for every $M' \subseteq M$ (Definition 14). Then $C_s^{\mathbf{t}} = \{S \subseteq R \mid R \in C_s^{max}\} = \bigcup\big\{\wp(R) \mid R \in C_s^{max}\big\}$, where $C_s^{max} = \big\{R \in C_s^{\mathbf{t}} \mid \textit{there is no } R' \in C_s^{\mathbf{t}} \textit{ such that } R \subset R'\big\}$ and $\wp(R)$ denotes the power set of R.

Given a set S, we have $|\wp(S)| = 2^{|S|}$ and that, for each $r \in S$, r is an element of $\frac{2^{|S|}}{2}$ subsets of S, i.e., of precisely half the subsets of S. Then if $r \in S \cap T$, we have that r is an element of $\frac{2^{|S|}}{2}$ subsets of S, $\frac{2^{|T|}}{2}$ subsets of T and $\frac{2^{|S \cap T|}}{2}$ subsets of $S \cap T$. PIE ensures that $|\wp(S) \cup \wp(T)| = |\wp(S)| + |\wp(T)| - |\wp(S) \cap \wp(T)|$, which, because $\wp(S \cap T) = \wp(S) \cap \wp(T)$, leads to $|\wp(S) \cup \wp(T)| = |\wp(S)| + |\wp(T)| - |\wp(S \cap T)|$. That is, if $r \in S \cap T$, then $|\wp(S) \cup \wp(T)| = 2^{|S|} + 2^{|T|} - 2^{|S \cap T|}$ and r is an element of $\frac{2^{|S|}}{2} + \frac{2^{|T|}}{2} - \frac{2^{|S \cap T|}}{2} = \frac{|\wp(S) \cup \wp(T)|}{2}$ sets in $\wp(S) \cup \wp(T)$. By extension of PIE, if $r \in \bigcap\{S_1, \ldots, S_n\}$, then r is an element of $\frac{|\bigcup\{\wp(S_1), \ldots, \wp(S_n)\}|}{2}$ sets in $\bigcup\big\{\wp(S_1), \ldots, \wp(S_n)\big\}$.

Let (r, s) be a redundant link, then, for all $R \in C_s^{max}$, we have $r \in R$ (Theorem 16), i.e., $r \in \bigcap C_s^{max}$. Then r is an element of $\frac{|\bigcup\{\wp(R) \ \mid \ R \in C_s^{max} \ and \ r \in R\}|}{2} = \frac{|C_s^{\mathbf{t}}|}{2}$ sets in $\bigcup\big\{\wp(R) \ \mid \ R \in C_s^{max} \ and \ r \in R\big\} = C_s^{\mathbf{t}}$, i.e., $|C_s^{\mathbf{t}}(r)| = \frac{|C_s^{\mathbf{t}}|}{2}$. $\square$

Corollary 20
Let $D = (S, L, C^{\mathbf{t}})$ be an ADF^{+}. Deciding if a link $(r, s) \in L$ is redundant can be solved in sub-quadratic time on $|C_s^{\mathbf{t}}|$.

Proof
Because $|C_s^{\mathbf{t}}(r)| = \frac{|C_s^{\mathbf{t}}|}{2}$, where $C_s^{\mathbf{t}}(r) = \big\{R \in C_s^{\mathbf{t}} \mid r \in R\big\}$, to find if (r, s) is a redundant link, it suffices to check for each $R \in C_s^{\mathbf{t}}$, if $r \in R$. For each $R \in C_s^{\mathbf{t}}$, checking if $r \in R$ can be done by checking, for each $s \in R$, if $s = r$. Clearly, each $R \in C_s^{\mathbf{t}}$ has at most $k = max\,\{|R| \mid R \in \bigcup C_s^{max}\}$ elements. Because $C_s^{max} \subset C_s^{\mathbf{t}}$ and $C_s^{\mathbf{t}}$ is subset-complete, we have $|C_s^{\mathbf{t}}| \geq 2^k$. Then k is $O(ln|C_s^{\mathbf{t}}|)$, which means that deciding if a link $(r, s) \in L$ is redundant is $O(|C_s^{\mathbf{t}}|.ln(|C_s^{\mathbf{t}}|))$. $\square$

Theorem 21
Let $D = (S, L, C^\varphi)$ be an ADF^+, v be a 3-valued interpretation over S, and for each $s \in S$, φ_s is the formula $\bigvee\limits_{R \in C_s^{max}} \bigwedge\limits_{b \in par(s)-R} \neg b$ depicted in Theorem 15. It holds for every $s \in S$, $\Gamma_D(v)(s) = v(\varphi_s)$.

Proof
For each $s \in S$, let φ_s be

$$\bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s)-R} \neg b$$

It is enough to prove for each $s \in S$, $v(\varphi_s) = \bigsqcap\{w(\varphi_s) \mid w \in [v]_2\}$, where $[v]_2 = \{w \mid w \textit{ is two-valued and } v \leq_i w\}$. We have three possibilities:

- $v(\varphi_s) = \mathbf{t}$ iff there exists $R \in C_s^{max}$ such that for each $b \in par(s) - R$, $v(b) = \mathbf{f}$ iff there exists $R \in C_s^{max}$ such that for each $b \in par(s) - R$, for each $w \in [v]_2$, $w(b) = \mathbf{f}$ iff for each $w \in [v]_2$, $w(\varphi_s) = \mathbf{t}$ iff $\bigsqcap\{w(\varphi_s) \mid w \in [v]_2\} = \mathbf{t}$.
- $v(\varphi_s) = \mathbf{f}$ iff for each $R \in C_s^{max}$, there exists $b \in par(s) - R$ such that $v(b) = \mathbf{t}$ iff for each $w \in [v]_2$, for each $R \in C_s^{max}$, there exists $b \in par(s) - R$ such that $w(b) = \mathbf{t}$ iff for every $w \in [v]_2$, $w(\varphi_s) = \mathbf{f}$ iff $\bigsqcap\{w(\varphi_s) \mid w \in [v]_2\} = \mathbf{f}$.
- $v(\varphi_s) = \mathbf{u}$, then for each $R \in C_s^{max}$, there exists $b \in par(s) - R$ such that $v(b) \in \{\mathbf{t}, \mathbf{u}\}$ and there exists $R \in C_s^{max}$ such that for each $b \in par(s) - R$, it holds $v(b) \in \{\mathbf{f}, \mathbf{u}\}$. Hence,
  - there exists $w \in [v]_2$ such that for each $R \in C_s^{max}$, there exists $b \in par(s) - R$ such that $w(b) = \mathbf{t}$. This means there exists $w \in [v]_2$ such that $w(\varphi_s) = \mathbf{f}$;
  - there exists $w' \in [v]_2$, there exists $R \in C_s^{max}$ such that for each $b \in par(s) - R$, it holds $w'(b) = \mathbf{f}$. This means there exists $w \in [v]_2$ such that $w(\varphi_s) = \mathbf{t}$.

  But then we have $\bigsqcap\{w(\varphi_s) \mid w \in [v]_2\} = \mathbf{u}$.

□

Theorem 22
Let $D = (S, L, C^\varphi)$ be an ADF^+. Then v is a stable model of D iff v is a 2-valued complete model of D.

Proof
(⇒) Let v be a stable model of D. It is trivial v is a complete model of D as every stable model is a complete model.
(⇐)
Let v be a 2-valued complete model of D. We will show v is a stable model of D, i.e., v is a grounded model of $D^v = (E_v, L^v, C^v)$, in which $E_v = \{s \in S \mid v(s) = \mathbf{t}\}$, $L^v = L \cap (E_v \times E_v)$ and for every $s \in E_v$, we set $\varphi_s^v = \varphi_s[b/\mathbf{f} : v(b) = \mathbf{f}]$.

As v is a complete model of D, if $v(s) = \mathbf{t}$, then $v(\varphi_s) = v(\bigvee_{R \in C_s^{max}} \bigwedge_{b \in par(s)-R} \neg b) = \mathbf{t}$. This means there exists $R \in C_s^{max}$ such that for each $b \in par(s) - R$, $v(b) = \mathbf{f}$. Thus, for each $s \in E_v$, $\varphi_s^v \equiv \mathbf{t}$. As consequence, E_v is the grounded extension of D^v, i.e., v is a stable model of D. □

A.2 Theorems and Proofs from Section 4:

Proposition 29
Let P be an *NLP*. The corresponding $\Xi(P)$ is an ADF^+.

Proof
Let $\Xi(P) = (A, L, C^{\mathbf{t}})$ be the *ADF* corresponding to the *NLP* P over a set of atoms A. By absurd, suppose $\Xi(P)$ is not an ADF^+. This means there exists a link $(b, a) \in L$ for which some $R \subseteq par(a)$ we have $C_a(R) = \mathbf{f}$ and $C_a(R \cup \{b\}) = \mathbf{t}$ (Definition 13). As $C_a(R \cup \{b\}) = \mathbf{t}$, from Definition 28, we obtain there exists $B \in Sup_P(a)$ such that $R \cup \{b\} \subseteq \{c \in par(a) \mid \neg c \notin B\}$. Then we can say there exists $B \in Sup_P(a)$ such that $R \subseteq \{c \in par(a) \mid \neg c \notin B\}$. But then $C_a(R) = \mathbf{t}$. An absurd! $\square$

Proposition 30
Let P be an *NLP* and $\Xi(P) = (A, L, C^{\mathbf{t}})$ the corresponding ADF^+. The acceptance condition φ_a for each $a \in A$ is given by

$$\varphi_a \equiv \bigvee_{B \in \mathtt{Sup}_P(a)} \left(\bigwedge_{\neg b \in B} \neg b \right).$$

In particular, if $\mathtt{Sup}_P(a) = \{\emptyset\}$, then $\varphi_a \equiv \mathbf{t}$ and if $\mathtt{Sup}_P(a) = \emptyset$, then $\varphi_a \equiv \mathbf{f}$.

Proof
As $\Xi(P)$ is an ADF^+, we obtain from Theorem 15 that for every $a \in A$,

$$\varphi_a \equiv \bigvee_{R \in C_a^{max}} \left(\bigwedge_{b \in par(a) - R} \neg b \right),$$

where $C_a^{max} = \left\{R \in C_a^{\mathbf{t}} \mid \textit{there is no } R' \in C_a^{\mathbf{t}} \textit{ such that } R \subset R'\right\}$. From Definition 28, we know $C_a^{max} = \{R \subseteq \{b \in par(a) \mid \neg b \notin B\} \mid B \in \mathtt{Sup}_P(a)$ *and there is no* $R' \in C_a^{\mathbf{t}}$ *such that* $R \subset R'\} = \{\{b \in par(a) \mid \neg b \notin B\} \mid B \in min\,\{\mathtt{Sup}_P(a)\}\}$, in which $min\,\{\mathtt{Sup}_P(a)\}$ returns the minimal sets (w.r.t. set inclusion) of $\mathtt{Sup}_P(a)$. Thus for every $a \in A$,

$$\varphi_a \equiv \bigvee_{R \in C_a^{max}} \left(\bigwedge_{b \in par(a) - R} \neg b \right) \equiv \bigvee_{B \in min\{\mathtt{Sup}_P(a)\}} \left(\bigwedge_{\neg b \in B} \neg b \right),$$

But then, we obtain

$$\varphi_a \equiv \bigvee_{B \in min\{\mathtt{Sup}_P(a)\}} \left(\bigwedge_{\neg b \in B} \neg b \right) \equiv \bigvee_{B \in \mathtt{Sup}_P(a)} \left(\bigwedge_{\neg b \in B} \neg b \right).$$

$\square$

Theorem 32
Let P be an *NLP* and $\Xi(P)$ be the corresponding ADF^+. v is a partial stable model of P iff v is a complete model of $\Xi(P)$.

Proof
Let P be an *NLP* and $\Xi(P) = (A, L, C^{\mathbf{t}})$ be the corresponding ADF^+. Let v be a 3-valued interpretation. We will prove v is a partial stable model of P iff v is a complete model of $\Xi(P)$, i.e., $\Omega_P(v) = v$ iff for each $a \in A$, $v(a) = v(\varphi_a)$.

We will prove by induction on j that for each $a \in A$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{j}(a) = \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{j}(r)$, $v(x) = \mathbf{t}$.

Base Case: We know ${\Psi^{\uparrow}_{\frac{P}{v}}}^{1}(a) = \mathbf{t}$ iff $a \in \frac{P}{v}$ iff there is a rule $a \leftarrow \mathtt{not}\ b_1, \ldots, \mathtt{not}\ b_n \in P$ $(n \geq 0)$ such that for each b_i, $(1 \leq i \leq n)$, $v(b_i) = \mathbf{f}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that ${\mathtt{Sup}^{\uparrow}_P}^{1}(r) = \{\neg b_1, \ldots, \neg b_n\}$ and for each $\neg b_i \in {\mathtt{Sup}^{\uparrow}_P}^{1}(r)$, $v(\neg b_i) = \mathbf{t}$.

Inductive Hypothesis: Assume for each $a' \in A$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n}(a') = \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a')$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n}(r)$, $v(x) = \mathbf{t}$.

Inductive Step: We will prove ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n+1}(a) = \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n+1}(r)$, $v(x) = \mathbf{t}$:
We know ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n+1}(a) = \mathbf{t}$ iff there exists $a \leftarrow a_1, \ldots, a_m \in \frac{P}{v}$ such that for each a_i, $1 \leq i \leq m$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n}(a_i) = \mathbf{t}$ iff there exists $a \leftarrow a_1, \ldots, a_m, \mathtt{not}\ b_1, \ldots, \mathtt{not}\ b_n \in P$ such that for each a_i, $1 \leq i \leq m$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n}(a_i) = \mathbf{t}$, and for each b_j, $1 \leq j \leq n$, $v(b_j) = \mathbf{f}$ iff according to the Inductive Hypothesis, there exists $a \leftarrow a_1, \ldots, a_m, \mathtt{not}\ b_1, \ldots, \mathtt{not}\ b_n \in P$ such that for each a_i, $1 \leq i \leq m$, there exists $\mathtt{Sup}_P(r_i) \in \mathtt{Sup}_P(a_i)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n}(r_i)$, $v(x) = \mathbf{t}$, and for each b_j, $1 \leq j \leq n$, $v(b_j) = \mathbf{f}$ iff there exists $a \leftarrow a_1, \ldots, a_m, \mathtt{not}\ b_1, \ldots, \mathtt{not}\ b_n \in P$ and there are statements r, r_i, $(1 \leq i \leq m)$ in P with $\mathtt{Conc}_P(r) = a$ and $\mathtt{Conc}_P(r_i) = a_i$ such that for each r_i, for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n}(r_i)$, $v(x) = \mathbf{t}$, and for each b_j, $1 \leq j \leq n$, $v(\neg b_j) = \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n+1}(r)$, $v(x) = \mathbf{t}$.

The above result guarantees for a 3-valued interpretation v of P, $\Omega_P(v)(a) = \mathbf{t}$ iff there exists $B = \mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in B$, $v(x) = \mathbf{t}$, i.e.,

$$\Omega_P(v)(a) = \mathbf{t} \ \textit{iff}\ v\left(\bigvee_{B \in \mathtt{Sup}_P(a)}\left(\bigwedge_{\neg b \in B} \neg b\right)\right) = \mathbf{t} \ \textit{iff}\ v(\varphi_a) = \mathbf{t}. \tag{A1}$$

Similarly now we will prove by induction on j that for each $a \in A$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{j}(a) \neq \mathbf{f}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{j}(r)$, $v(x) \neq \mathbf{f}$.

Base Case: We know ${\Psi^{\uparrow}_{\frac{P}{v}}}^{1}(a) \neq \mathbf{f}$ iff either $a \in \frac{P}{v}$ or $a \leftarrow \mathbf{u} \in \frac{P}{v}$ iff there exists a rule $a \leftarrow \mathtt{not}\ b_1, \ldots, \mathtt{not}\ b_n \in P$ $(n \geq 0)$ such that for each b_i, $(1 \leq i \leq n)$, $v(b_i) \neq \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that ${\mathtt{Sup}^{\uparrow}_P}^{1}(r) = \{\neg b_1, \ldots, \neg b_n\}$ and for each b_i, $(1 \leq i \leq n)$, $v(b_i) \neq \mathbf{t}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $\neg b_i \in {\mathtt{Sup}^{\uparrow}_P}^{1}(r)$, $v(\neg b_i) \neq \mathbf{f}$.

Inductive Hypothesis: Assume for each $a' \in A$, ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n}(a') \neq \mathbf{f}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a')$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n}(r)$, $v(x) \neq \mathbf{f}$.

Inductive Step: We will prove ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n+1}(a) \neq \mathbf{f}$ iff there exists $\mathtt{Sup}_P(r) \in \mathtt{Sup}_P(a)$ such that for each $x \in {\mathtt{Sup}^{\uparrow}_P}^{n+1}(r)$, $v(x) \neq \mathbf{f}$:
We know ${\Psi^{\uparrow}_{\frac{P}{v}}}^{n+1}(a) \neq \mathbf{f}$ iff there exists $a \leftarrow a_1, \ldots, a_m \in \frac{P}{v}$ such that for each a_i, $1 \leq$

$i \leq m$, $\Psi^{\uparrow n}_{\frac{P}{v}}(a_i) \neq \mathbf{f}$ iff there exists $a \leftarrow a_1, \ldots, a_m, \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P$ such that for each a_i, $1 \leq i \leq m$, $\Psi^{\uparrow n}_{\frac{P}{v}}(a_i) \neq \mathbf{f}$, and for each b_j, $1 \leq j \leq n$, $v(b_j) \neq \mathbf{t}$ iff according to the Inductive Hypothesis, there exists $a \leftarrow a_1, \ldots, a_m, \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P$ such that for each a_i, $1 \leq i \leq m$, there exists $\texttt{Sup}_P(r_i) \in \texttt{Sup}_P(a_i)$ such that for each $x \in \texttt{Sup}^{\uparrow n}_P(r_i)$, $v(x) \neq \mathbf{f}$, and for each b_j, $1 \leq j \leq n$, $v(b_j) \neq \mathbf{t}$ iff there exists $a \leftarrow a_1, \ldots, a_m, \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P$ and there are statements r, r_i, $(1 \leq i \leq m)$ in P with $\texttt{Conc}_P(r) = a$ and $\texttt{Conc}_P(r_i) = a_i$ such that for each r_i, for each $x \in \texttt{Sup}^{\uparrow n}_P(r_i)$, $v(x) \neq \mathbf{f}$, and for each b_j, $1 \leq j \leq n$, $v(\neg b_j) \neq \mathbf{f}$ iff there exists $\texttt{Sup}_P(r) \in \texttt{Sup}_P(a)$ such that for each $x \in \texttt{Sup}^{\uparrow n+1}_P(r)$, $v(x) \neq \mathbf{f}$.

The above result guarantees for a 3-valued interpretation v of P, $\Omega_P(v)(a) \neq \mathbf{f}$ iff there exists $B = \texttt{Sup}_P(r) \in \texttt{Sup}_P(a)$ such that for each $x \in B$, $v(x) \neq \mathbf{f}$, i.e.,

$$\Omega_P(v)(a) = \mathbf{f} \textit{ iff } v\left(\bigvee_{B \in \texttt{Sup}_P(a)} \left(\bigwedge_{\neg b \in B} \neg b\right)\right) = \mathbf{f} \textit{ iff } v(\varphi_a) = \mathbf{f}. \tag{A2}$$

From (A1) and (A2), we conclude v is a partial stable model of P iff for all $a \in A$, $v(a) = \Omega_P(v)(a) = v\left(\bigvee_{B \in \texttt{Sup}_P(a)} \left(\bigwedge_{\neg b \in B} \neg b\right)\right) = v(\varphi_a)$, i.e., v is a complete model of $\Xi(P)$. □

Theorem 33
Let P be an *NLP* and $\Xi(P) = (A, L, C^{\mathbf{t}})$ the corresponding ADF^+. We have

- v is a well-founded model of P iff v is a grounded model of $\Xi(P)$.
- v is a regular model of P iff v is a preferred model of $\Xi(P)$.
- v is a stable model of P iff v is a stable model of $\Xi(P)$.
- v is an L-stable model of P iff v is an L-stable model of $\Xi(P)$.

Proof
This proof is a straightforward consequence from Theorem 32:

- v is a well-founded model of P iff v is the $\leq_i$-least partial stable model of P iff (according to Theorem 32) v is the $\leq_i$-least complete model of $\Xi(P)$ iff v is the grounded model of $\Xi(P)$.
- v is a regular model of P iff v is a $\leq_i$-maximal partial stable model of P iff (according to Theorem 32) v is a $\leq_i$-maximal complete model of $\Xi(P)$ iff v is a preferred model of $\Xi(P)$.
- v is a stable model of P iff v is a partial stable model of P such that $\texttt{unk}(v) = \{s \in S \mid v(s) = \mathbf{u}\} = \emptyset$ iff (according to Theorem 32) v is a complete model of $\Xi(P)$ such that $\texttt{unk}(v) = \emptyset$ iff (based on Theorem 22) v is a stable model of $\Xi(P)$.
- v is an L-stable model of P iff v is a partial stable model of P with minimal $\texttt{unk}(v) = \{s \in S \mid v(s) = \mathbf{u}\}$ (w.r.t. set inclusion) among all partial stable models of P iff (according to Theorem 32) v a complete model of $\Xi(P)$ with minimal $\texttt{unk}(v)$ among all complete models of P iff v is an L-stable model of $\Xi(P)$.

□

A.3 Propositions and Proofs from Section 5:

Proposition 37
Let P be an *NLP*, where each rule is either a fact or its body has only default literals as in $a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n$. Let $\Xi(P)$ be the *ADF* obtained from P via Definition 28 and $\Xi_2(P)$ the *ADF* obtained from P via Definition 36. Then $\Xi(P) = \Xi_2(P)$.

Proof
Firstly, let P be an *NLP* defined over a set A of atoms, where each rule is like $a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n$. We know from Definitions 25 and 26 $\texttt{Sup}_P(a) = \{\{\neg b_1, \ldots, \neg b_n\} \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P\}$. Then, according to Definition 28, we obtain the *ADF* $\Xi(P) = (A, L, C^{\mathbf{t}})$, where

- $L = \{(c, a) \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P \textit{ and } c \in \{b_1, \ldots, b_n\}\}$;
- For $a \in A$, $C^{\mathbf{t}}_a = \{B' \subseteq \{b \in par(a) \mid \neg b \notin \{b_1, \ldots, b_n\} \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P\}\}$ $= \{B' \subseteq par(a) \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P \textit{ and } \{b_1, \ldots, b_n\} \cap B' = \emptyset\}$.

According to Definition 36, we obtain the *ADF* $\Xi_2(P) = (A, L_2, C^{\mathbf{t}}_2)$, where

- $L_2 = \{(c, a) \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P \textit{ and } c \in \{b_1, \ldots, b_n\}\} = L$;
- For each $a \in A$, $C^{\mathbf{t}}_{2a} = \{B' \in par(a) \mid a \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_n \in P, \{b_1, \ldots, b_n\} \cap B' = \emptyset\} = C^{\mathbf{t}}_a$.

Hence, $\Xi(P) = \Xi_2(P)$. □

Proposition 39
Let $SF = (A, R)$ be a *SETAF* and $DF^{SF} = (A, L, C)$ be the corresponding *ADF*. Then, DF^{SF} is an ADF^+.

Proof
In order to show $DF^{SF} = (A, L, C)$ is an ADF^+, we will guarantee any $(r, s) \in L$ is an attacking link, i.e., for every $B \subseteq par(s)$, if $C_s(B \cup \{r\}) = \mathbf{t}$, then $C_s(B) = \mathbf{t}$:

Suppose $C_s(B \cup \{r\}) = \mathbf{t}$. Then according to the translation from *SETAF* to *ADF*, there is no $(X_i, s) \in R$ such that $X_i \subseteq B \cup \{r\}$. Thus there is no $(X_i, s) \in R$ such that $X_i \subseteq B$. This implies $C_s(B) = \mathbf{t}$. □