Appendix A Proofs of results

Proof of Proposition 2. Just note that, by construction, the evaluation of every 0-term w.r.t. $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ is the same to its evaluation w.r.t. $\hat{\mathcal{I}}$. Hence, for every 0-terms $\tau_1, \dots, \tau_n$ we have:
$\mathcal{I}, w \models_{\mathcal{S}} p(\tau_1, \dots, \tau_n)$
iff $p(\sigma^w(\tau_1), \dots, \sigma^w(\tau_n)) \in I^w$
iff $p(\hat{\sigma}^w(\tau_1), \dots, \hat{\sigma}^w(\tau_n)) \in I^w$
iff $\hat{\mathcal{I}}, w \models_{\mathcal{F}} p(\tau_1, \dots, \tau_n)$. Similarly, for any pair of 0-terms τ_1, τ_2, we have:
$\mathcal{I}, w \models_{\mathcal{S}} \tau_1 = \tau_2$
iff $\sigma(\tau_1) = \sigma(\tau_2)$
iff $\hat{\sigma}(\tau_1) = \hat{\sigma}(\tau_2)$
iff $\hat{\mathcal{I}}, w \models_{\mathcal{F}} \tau_1 = \tau_2$.
Then, the proof follows by induction noting that the rules of $\models_{\mathcal{S}}$ and $\models_{\mathcal{F}}$ are the same when considered the different signatures. □

Proof of Proposition 3. Just note that, by construction, the evaluation of every term w.r.t. $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ is the same to the evaluation of $\kappa(\tau)$ w.r.t. $\tilde{\mathcal{I}}$. Hence, for any terms $\tau_1, \dots, \tau_n$ we have:
$\mathcal{I}, w \models_{\mathcal{S}} p(\tau_1, \dots, \tau_n)$
iff $p(\sigma(\tau_1), \dots, \sigma(\tau_n)) \in I^w$
iff $p(\tilde{\sigma}(\kappa(\tau_1)), \dots, \tilde{\sigma}(\kappa(\tau_n))) \in I^w$
iff $\hat{\mathcal{I}}, w \models_{\mathcal{F}} p(\kappa(\tau_1), \dots, \kappa(\tau_n))$
iff $\hat{\mathcal{I}}, w \models_{\mathcal{F}} \kappa(p(\tau_1, \dots, \tau_n))$.
Then, the proof follows by induction noting that the rules of $\models_{\mathcal{S}}$ and $\models_{\mathcal{F}}$ are the same when considered the different signatures. □

Proof of Proposition 4. By definition, $\mathit{Coh}(\mathcal{I})$ is a coherent interpretation and, thus, we get: $\mathit{Coh}(\mathcal{I}) \models \varphi$ iff $\mathit{Coh}(\mathcal{I}), h \models_{\mathcal{S}} \varphi$. Furthermore, by definition, $\mathcal{I}$ and $\mathit{Coh}(\mathcal{I})$ agree on the evaluation of every 0-term and, since φ is a 0-formula, it follows that $\mathit{Coh}(\mathcal{I}), h \models_{\mathcal{S}} \varphi$ iff $\mathcal{J}, h \models_{\mathcal{S}} \varphi$ for any interpretation $\mathcal{J}$ such that $\mathcal{J} = \hat{\mathcal{I}}$. Hence, the statement follows directly from Proposition 2 □

Proof of Proposition 5. Let $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ be a coherent interpretation. Then, we have that $\mathcal{I}, w \models \varphi$ iff $\mathcal{I}, w \models_{\mathcal{S}} \varphi$ and it is obvious that $\mathcal{I}, w \models_{\mathcal{S}} \varphi$ implies $\mathcal{I}, t \models_{\mathcal{S}} \varphi$ when $w = t$. The proof that $\mathcal{I}, h \models_{\mathcal{S}} \varphi$ implies $\mathcal{I}, t \models_{\mathcal{S}} \varphi$ easily follows by structural induction. Note that, in case that φ is an atom $p(\tau_1, \dots, \tau_n)$, then $\mathcal{I}, h \models_{\mathcal{S}} \varphi$ implies $p(\tau_1, \dots, \tau_n) \in I^h \subseteq I^t$ which, in its turn, implies $\mathcal{I}, t \models_{\mathcal{S}} \varphi$. In case that φ is of the form $\tau_1 = \tau_2$, we have $\mathcal{I}, h \models_{\mathcal{S}} \varphi$ iff $\sigma^h(\tau_1) = \sigma^h(\tau_2) \neq \boldsymbol{u}$ which, in its turn, implies $\sigma^t(\tau_1) = \sigma^t(\tau_2) \neq \boldsymbol{u}$ and $\mathcal{I}, t \models_{\mathcal{S}} \varphi$. The rest of the cases are as usual in SQHT$^=$.

Let us show that $\mathcal{I}, w \models \neg\varphi$ iff $\mathcal{I}, t \not\models \varphi$. Note that, since $\mathcal{I}$ is coherent, we have:
$\mathcal{I}, w \models \neg\varphi$
iff $\mathcal{I}, w \models_{\mathcal{S}} \neg\varphi$
iff $\tilde{\mathcal{I}}, w \models_{\mathcal{F}} \kappa(\neg\varphi)$ (Proposition 3)
iff $\tilde{\mathcal{I}}, w \models_{\mathcal{F}} \neg\kappa(\varphi)$ (by definition)
iff $\tilde{\mathcal{I}}, t \not\models_{\mathcal{F}} \kappa(\varphi)$ (Proposition 1).

Furthermore, since $\mathcal{I}$ is coherent, we have:
$\mathcal{I}, t \not\models \varphi$
iff $\mathcal{I}, t \not\models_{\mathcal{S}} \varphi$
iff $\tilde{\mathcal{I}}, t \not\models_{\mathcal{F}} \kappa(\varphi)$ (Proposition 3).
Consequently, $\mathcal{I}, w \models \neg\varphi$ iff $\mathcal{I}, t \not\models \varphi$ holds. □

Proof of Proposition 6. Assume first that φ is a $\mathrm{SQHT}^{=}_{\mathcal{F}}$ tautology and suppose, for the sake of contradiction, that φ is not a $\mathrm{SQHT}^{=}_{\mathcal{S}}$ tautology. Let $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ be an interpretation such that $\mathcal{I} \not\models_{\mathcal{S}} \varphi$. Then, from Proposition 3, it follows that $\mathcal{I} \not\models_{\mathcal{F}} \kappa(\varphi)$ which is a contradiction. Hence, $\kappa(\varphi)$ must be a $\mathrm{SQHT}^{=}_{\mathcal{S}}$ tautology.

Assume now that φ is a 0-formula. Then $\kappa(\varphi) = \varphi$ and, as shown above, the only if direction holds. Hence, assume that φ is a $\mathrm{SQHT}^{=}_{\mathcal{S}}$ tautology and suppose, for the sake of contradiction, that φ is not a $\mathrm{SQHT}^{=}_{\mathcal{F}}$ tautology. Let $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ be an $\mathrm{SQHT}^{=}_{\mathcal{F}}$-interpretation such that $\mathcal{I} \not\models_{\mathcal{F}} \varphi$. From Proposition 4, this implies that $\mathit{Coh}(\mathcal{I}) \not\models_{\mathcal{S}} \varphi$ which is a contradiction with the fact that φ is a $\mathrm{SQHT}^{=}_{\mathcal{S}}$ tautology. Consequently, φ must be a $\mathrm{SQHT}^{=}_{\mathcal{S}}$ tautology. □

Lemma 1. *Any pair of* $\mathrm{SQHT}^{=}$*-interpretations* $\mathcal{I}_1$ *and* $\mathcal{I}_2$ *satisfy:*

i) $\mathcal{I}_1 \preceq \mathcal{I}_2$ *iff* $\mathit{Coh}(\mathcal{I}_1) \preceq \mathit{Coh}(\mathcal{I}_2)$*,*
ii) $\mathcal{I}_1 = \mathcal{I}_2$ *iff* $\mathit{Coh}(\mathcal{I}_1) = \mathit{Coh}(\mathcal{I}_2)$*, and*
iii) $\mathcal{I}_1 \prec \mathcal{I}_2$ *iff* $\mathit{Coh}(\mathcal{I}_1) \prec \mathit{Coh}(\mathcal{I}_2)$*.* □

Proof
First note that i) implies ii) and these two together imply iii). Hence, let us show that i) holds.

Let $\mathcal{I}_1 = \langle \sigma_1^h, \sigma_1^t, I_1^h, I_1^t \rangle$ and $\mathcal{I} = \langle \sigma_2^h, \sigma_2^t, I_2^h, I_2^t \rangle$ such that $\mathcal{I}_1 \preceq \mathcal{I}_2$. Then, $\sigma_1^w \preceq \sigma_2^w$ and $I_1^w \subseteq I_2^w$ with $w \in \{h, t\}$. By definition, we have that $\mathit{Coh}(\mathcal{I}_1) = \langle \sigma_{\mathcal{I}_1}, \sigma_{\mathcal{I}_1^t}, I_1^h, I_1^t \rangle$ and $\mathit{Coh}(\mathcal{I}_2) = \langle \sigma_{\mathcal{I}_2}, \sigma_{\mathcal{I}_2^t}, I_2^h, I_2^t \rangle$ and, to show $\mathit{Coh}(\mathcal{I}_1) \preceq \mathit{Coh}(\mathcal{J}_2)$, it is enough to prove $\sigma_{\mathcal{J}_1} \preceq \sigma_{\mathcal{J}_2}$ for $\mathcal{J} \in \{\mathcal{I}, \mathcal{I}^t\}$. Note that, for every term $\tau \in \mathit{Terms}^0(\mathcal{C} \cup \mathcal{F})$, we have that

$$\begin{aligned} \sigma_{\mathcal{I}}(\tau) &= \sigma_1^h(\tau) \preceq \sigma_2^h(\tau) = \sigma_{\mathcal{I}}(\tau) \\ \sigma_{\mathcal{I}^t}(\tau) &= \sigma_1^t(\tau) \preceq \sigma_2^t(\tau) = \sigma_{\mathcal{I}^t}(\tau) \end{aligned}$$

and, for every intensional set $\tau = \{\vec{\tau}(\vec{x}) : \varphi(\vec{x})\}$ we have that

$$\begin{aligned} \sigma_{\mathcal{I}}(\tau) &\preceq \sigma_{\mathcal{I}}(\tau) \\ \sigma_{\mathcal{I}^t}(\tau) &\preceq \sigma_{\mathcal{I}^t}(\tau) \end{aligned}$$

follows from $I_1^w \subseteq I_2^w$. The rest of the proof follows by structural induction and the fact that functions preserve their interpretation through subterms. That is, $\tau = f(\tau_1, \ldots, \tau_n)$ and $\sigma_{\mathcal{J}_1}(\tau_i) \preceq \sigma_{\mathcal{J}_2}(\tau_i)$. By definition, if $\sigma_{\mathcal{J}_1}(\tau_i) = \boldsymbol{u}$ for some $1 \leq i \leq n$, then $\sigma_{\mathcal{J}_1}(\tau) = \boldsymbol{u} \preceq \sigma_{\mathcal{J}_2}(\tau)$. Otherwise, $\sigma_{\mathcal{J}_1}(\tau_i) = \sigma_{\mathcal{J}_2}(\tau_i)$ for all $1 \leq i \leq n$ and, thus

$$\sigma_{\mathcal{J}_1}(\tau) = \sigma_{\mathcal{J}_1}(f(\sigma_{\mathcal{J}_1}(\tau_1), \ldots, \sigma_{\mathcal{J}_1}(\tau_n))) = \sigma_{\mathcal{J}_1}(f(\sigma_{\mathcal{J}_2}(\tau_1), \ldots, \sigma_{\mathcal{J}_2}(\tau_n))) \preceq \sigma_{\mathcal{J}_2}(\tau)$$

and, by induction hypothesis, we get

$$\sigma_{\mathcal{J}_1}(f(\sigma_{\mathcal{J}_2}(\tau_1), \ldots, \sigma_{\mathcal{J}_2}(\tau_n))) \preceq \sigma_{\mathcal{J}_2}(f(\sigma_{\mathcal{J}_2}(\tau_1), \ldots, \sigma_{\mathcal{J}_2}(\tau_n))) = \sigma_{\mathcal{J}_2}(\tau)$$

Hence, $\sigma_{\mathcal{J}_1}(\tau) \preceq \sigma_{\mathcal{J}_2}(\tau)$ $\square$

Proof of Proposition 7. Assume first that I is a stable model of Γ w.r.t. Definition 10. Then, there is some total coherent interpretation $\mathcal{I} = \langle \sigma, I \rangle$ such that $\mathcal{I} \models \Gamma$ and that satisfies $\mathcal{I}' \not\models \Gamma$ for all $\mathcal{I}'$ with $\mathcal{I}' \prec \mathcal{I}$. From $\mathcal{I} \models \Gamma$, it follows that $\hat{\mathcal{I}} \models_{\mathcal{F}} \varphi$ (Proposition 2). Suppose, for the sake of contradiction, that I is not a stable model according to Definition 4. Then, $\hat{\mathcal{I}} \models_{\mathcal{F}} \varphi$ implies that there is some interpretation $\mathcal{I}' \prec \hat{\mathcal{I}}$ such that $\mathcal{I}' \models_{\mathcal{F}} \Gamma$. From Proposition 4, this implies that $\mathit{Coh}(\mathcal{I}') \models \Gamma$. Furthermore, from Lemma 1, it follows that $\mathcal{I}' \prec \hat{\mathcal{I}}$ implies $\mathit{Coh}(\mathcal{I}') \prec \mathit{Coh}(\hat{\mathcal{I}}) = \mathcal{I}$ which is a contradiction.

The other way around. Assume now that I is a stable model of Γ w.r.t. Definition 4. Then, there is some interpretation $\mathcal{I} = \langle \sigma, I \rangle$ such that $\mathcal{I} \models_{\mathcal{F}} \Gamma$ and that $\mathcal{I}' \not\models_{\mathcal{F}} \Gamma$ for all $\mathcal{I}'$ with $\mathcal{I}' \prec \mathcal{I}$. From Proposition 4, this implies that $\mathit{Coh}(\mathcal{I}) \models \Gamma$. Suppose now that I is not a stable model according to Definition 10. Then, there is some coherent interpretation $\mathcal{I}' = \langle \sigma^h, \sigma^t, I^h, I \rangle \prec \mathit{Coh}(\mathcal{I})$ such that $\mathcal{I}' \models \Gamma$. From Proposition 2, this implies that $\hat{\mathcal{I}}' \models_{\mathcal{F}} \Gamma$ and that $\hat{\mathcal{I}}' \prec \mathcal{I}$ which is a contradiction. $\square$

Proposition 9. *Given a ground GZ-formula φ and a total coherent interpretation of the form $\mathcal{I} = \langle \sigma, T \rangle$, we have: $\mathcal{I} \models \varphi$ iff $T \models_{cl} \varphi$.* $\square$

Proof of Proposition 9. The proof follows by induction assuming φ is an i-formula and that the statement holds for every subformula of φ and for every $(i-1)$-formula. Note that iii) is the unique non-trivial case.

Let $A = (f\{\vec{x}\!:\!\varphi(\vec{x})\} \trianglelefteq n)$ be a set atom. Then, we have that
$T \models_{cl} A$
iff $\hat{f}\big(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \big|\ T \models_{cl} \varphi(\vec{c})\ \}\big) = k$ and $k \trianglelefteq n$ (Definition 12)
iff $\hat{f}\big(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \big|\ \mathcal{I} \models \varphi(\vec{c})\ \}\big) = k$ and $k \trianglelefteq n$ (induction hypothesis).
On the other hand, we also have that
$\mathcal{I} \models A$
iff $\trianglelefteq(\sigma(f\{\vec{x} : \varphi(\vec{x})\}), \sigma(n)) \in I^h$ (Definition 7)
iff $\sigma(f\{\vec{x} : \varphi(\vec{x})\}) \trianglelefteq \sigma(n)$
iff $\hat{f}(\sigma(\{\vec{x} : \varphi(\vec{x})\})) \trianglelefteq \sigma(n)$ (Definition 11)
iff $\hat{f}(\{\ \vec{x}[\vec{x}/\vec{c}] \ \big|\ \mathcal{I} \models \varphi(\vec{c})$ with $\vec{c} \in \mathcal{D}^{|\vec{x}|}\ \}) \trianglelefteq \sigma(n)$ (Definition 8)
iff $\hat{f}(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \big|\ \mathcal{I} \models \varphi(\vec{c})\ \}) \trianglelefteq \sigma(n)$
iff $\hat{f}(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \big|\ \mathcal{I} \models \varphi(\vec{c})\ \}) \trianglelefteq n$ (term evaluation)
Then, the result follows directly by defining k as the result of evaluating the expression $\hat{f}(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \big|\ \mathcal{I} \models \varphi(\vec{c})\ \})$. $\square$

Proposition 10. *Given a ground GZ-formula φ and some coherent interpretation $\mathcal{I}$, we have:*
i) $\mathcal{I},t \models \varphi$ iff $T \models_{cl} \varphi$, and
ii) $\mathcal{I} \models \varphi$ iff $H \models_{cl} \varphi^T$. $\square$

Proof of Proposition 10. First, note that i) follows directly from Proposition 9. So, let us prove ii).

Assume that $\mathcal{I}$ is of the form $\mathcal{I} = \langle \sigma^h, \sigma^t, H, T \rangle$. If φ is an ground GZ-atom a, then $\mathcal{I} \models \varphi$

iff $a \in H \subseteq T$ iff $H \models_{cl} \varphi^T$. Otherwise, we proceed by induction assuming that φ is an i-formula and that the statement holds for all subformulas of φ and all $(i-1)$-formulas.

If φ is a set atom of the form $A = f\{\vec{x}:\psi(\vec{x})\} \trianglelefteq n$. Then, $\mathcal{I} \models A$ implies $\mathcal{I}, t \models A$ (Proposition 5) which, in its turn, implies $T \models_{cl} A$ (Proposition 9) and, thus, we get $A^T = \big(\bigwedge \mathtt{Gr}^+_T(\psi(\vec{x})) \big)^T$. Furthermore, $\mathcal{I} \models A$ also implies

$$\sigma^h(f\{\vec{x}:\psi(\vec{x})\}) \neq \boldsymbol{u} \qquad\qquad \sigma^h(n) = n \neq \boldsymbol{u}$$

By definition of term evaluation the former implies

$$\sigma^h(\{\vec{x}:\psi(\vec{x})\}) \neq \boldsymbol{u}$$

and, by the definition of coherent interpretation, this implies

$$\begin{aligned}\sigma^h(\{\vec{x}:\psi(\vec{x})\}) &= \sigma^t(\{\vec{x}:\psi(\vec{x})\}) \\ &= \{\ \sigma^h(\vec{\tau}[\vec{x}/\vec{c}]) \mid \mathcal{I}, h \models_{\mathcal{S}} \psi(\vec{c}) \ \text{ for some } \vec{c} \in \mathcal{D}^{|\vec{x}|}\ \} \\ &= \{\ \sigma^t(\vec{\tau}[\vec{x}/\vec{c}]) \mid \mathcal{I}, t \models_{\mathcal{S}} \psi(\vec{c}) \ \text{ for some } \vec{c} \in \mathcal{D}^{|\vec{x}|}\ \}\end{aligned}$$

and, thus, $\mathcal{I}, h \models \psi(\vec{c})$ iff $\mathcal{I}, t \models \psi(\vec{c})$ iff $T \models_{cl} \psi(\vec{c})$ (Proposition 9) for all $\vec{c} \in \mathcal{D}^{|\vec{x}|}$. This implies

$$\mathcal{I} \models \bigwedge \mathtt{Gr}^+_T(\psi(\vec{x})) = \bigwedge \{\ \psi(\vec{c}) \in \mathtt{Gr}(T) \mid \text{ and } T \models_{cl} \psi(\vec{c})\ \}$$

Since this is a $(i-1)$-formula, by induction hypothesis, we get

$$H \models_{cl} \big(\bigwedge \mathtt{Gr}^+_T(\psi(\vec{x})) \big)^T = A^T$$

Assume now that $H \models_{cl} A^T$, then $T \models_{cl} A$ and we get

$$\hat{f}\big(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \mid T \models_{cl} \psi(\vec{c})\ \}\big) \trianglelefteq n$$

which implies $\hat{f}\big(\{\ \vec{c} \in \mathcal{D}^{|\vec{x}|} \mid \mathcal{I}, t \models_{\mathcal{S}} \psi(\vec{c})\ \}\big) \trianglelefteq n$. From this and Definition 8, we get $\hat{f}\big(\sigma^t(\{\vec{x} : \psi(\vec{x})\})\big) \trianglelefteq n$ and, in its turn, from this and Definition 11 we get

$$\sigma^t(f\{\vec{x} : \psi(\vec{x})\}) \trianglelefteq \sigma^t(n)$$

Hence, we obtain that $\mathcal{I}, t \models A$. Furthermore, $H \models_{cl} A^T = \big(\bigwedge \mathtt{Gr}^+_T(\psi(\vec{x})) \big)^T$ implies that, for all $\vec{c} \in \mathcal{D}^{|\vec{x}|}$, $H \models_{cl} \psi(\vec{c})^T$ whenever $T \models_{cl} \psi(\vec{c})$. By induction hypothesis, this implies that $\mathcal{I} \models \psi(\vec{c})^T$ holds whenever $T \models_{cl} \psi(\vec{c})$. and, thus, we get that $\mathcal{I}, t \models_{\mathcal{S}} \psi(\vec{c})$ implies $\mathcal{I} \models_{\mathcal{S}} \psi(\vec{c})$ for all $\vec{c} \in \mathcal{D}^{|\vec{x}|}$. Hence,

$$\sigma^t(\{\vec{x}:\psi(\vec{x})\}) \quad = \quad \sigma^h(\{\vec{x}:\psi(\vec{x})\})$$

and, thus, that $\mathcal{I} \models A$.

The cases for connective $\wedge$, $\vee$ and $\to$ follow by structural induction as in Lemma 1 from (**?**): $\mathcal{I} \models \varphi_1 \wedge \varphi_2$ (resp. $\mathcal{I} \models \varphi_1 \wedge \varphi_2$)
iff $\mathcal{I} \models \varphi_1$ and (resp. or) $\mathcal{I} \models \varphi_2$
iff $H \models_{cl} \varphi_1^T$ and (resp. or) $H \models_{cl} \varphi_2^t$
iff $H \models_{cl} \varphi_1^T \wedge \varphi_2^T$ (resp. $H \models_{cl} \varphi_1^T \vee \varphi_2^T$)
iff $H \models_{cl} (\varphi_1 \wedge \varphi_2)^T$ (resp. $H \models_{cl} (\varphi_1 \vee \varphi_2)^T$).

Finally, $\mathcal{I} \models \varphi_1 \to \varphi_2$

iff both $\mathcal{I}, h \not\models \varphi_1$ or $\mathcal{I}, h \models \varphi_2$ and $\mathcal{I}, t \not\models \varphi_1$ or $\mathcal{I}, t \models \varphi_2$
iff both $\mathcal{I} \not\models \varphi_1$ or $\mathcal{I} \models_{cl} \varphi_2$ and $T \not\models_{cl} \varphi_1$ or $T \models_{cl} \varphi_2$
iff both $H \not\models_{cl} \varphi_1^T$ or $H \models_{cl} \varphi_2^T$ and $T \not\models_{cl} \varphi_1$ or $T \models_{cl} \varphi_2$
iff both $H \models_{cl} \varphi_1^T \to \varphi_2^T$ and $T \models_{cl} \varphi_1 \to \varphi_2$
iff both $H \not\models_{cl} \varphi_1^T \to \varphi_2^T$ and $\varphi^T = \varphi_1^T \to \varphi_2^T$
iff $H \models_{cl} \varphi^T$ □

Lemma 2. *Let Γ be any GZ-theory and let $\mathcal{I}$ be any coherent interpretation and T be a set of atoms. Then,*
i) $\mathcal{I} \models \Gamma$ iff $\mathcal{I} \models \mathtt{Gr}(\Gamma)$,
ii) T is a stable model of Γ iff T is a stable model of $\mathtt{Gr}(\Gamma)$. □

Proof
By definition, we get: $\mathcal{I} \models \Gamma$
iff $\mathcal{I} \models \forall\vec{x}\varphi(\vec{x})$ for all $\varphi(\vec{x}) \in \Gamma$
iff $\mathcal{I} \models \varphi(\vec{c})$ for all $\varphi(\vec{x}) \in \Gamma$ and all $\vec{c} \in \mathcal{D}^{|\vec{x}|}$
iff $\mathcal{I} \models \varphi(\vec{c})$ for all $\varphi(\vec{x}) \in \mathtt{Gr}(\Gamma) = \{ \ \varphi(\vec{c}) \ \big| \ \forall\vec{x}\varphi(\vec{x}) \in \Gamma \text{ and } \vec{c} \in \mathcal{D}^{|\vec{x}|} \ \}$
iff $\mathcal{I} \models \mathtt{Gr}(\Gamma)$.

Furthermore, T is a stable model of Γ
iff there is some total coherent interpretation $\mathcal{I} = \langle \sigma, T \rangle$ which is an equilibrium model of Γ
iff there is some total coherent interpretation $\mathcal{I} = \langle \sigma, T \rangle$ which is an $\prec$-minimal model of Γ
iff there is some total coherent interpretation $\mathcal{I} = \langle \sigma, T \rangle$ which is an $\prec$-minimal model of $\mathtt{Gr}(\Gamma)$
iff T is a stable model of Γ. □

Proof of Theorem 1. First note that, from Definition 13 and Lemma 2, we have that T is a stable model of Γ iff T is a stable model of $\mathtt{Gr}(\Gamma)$ according to both Definitions. Hence, we assume without loss of generality that Γ is ground.

Let $\mathcal{I} = \langle \sigma, T \rangle$ be a total coherent interpretation. Then, from Proposition 10, we get that $\mathcal{I} \models \Gamma$ iff $T \models_{cl} \Gamma^T$. Let us show now that if T is the $\subseteq$-minimal model of Γ^T, then $\mathcal{I}$ is an equilibrium model of Γ. Suppose, for the sake of contradiction, that this does not hold. Then, there is a some coherent interpretation $\mathcal{I}' = \langle \sigma^h, \sigma^t, H, T \rangle$ such that $\mathcal{I}' \preceq \mathcal{I}$ and $\mathcal{I} \models \Gamma$, but $\mathcal{I}' \not\succeq \mathcal{I}$. Note that, from Proposition 10, $\mathcal{I}' \models \Gamma$ implies $H \models_{cl} \Gamma^T$ while, since $\mathcal{I}'$ is coherent, $\mathcal{I}' \preceq \mathcal{I}$ and $\mathcal{I}' \not\succeq \mathcal{I}$ imply $H \subset T$ (note that all evaluable functions are aggregates and, thus, σ^h and σ^t are fully determined by H and T, respectively) which is a contradiction with the assumption.

The other way around. Suppose, for the sake of contradiction, that $\mathcal{I}$ is an equilibrium model of Γ, but T is not the $\subseteq$-minimal model of Γ^T. Then there is some set $H \subset T$ that satisfies $H \models_{cl} \Gamma^T$ and, from Proposition 10, this implies $\mathcal{I}' = \langle \sigma^h, \sigma^t, H, T \rangle \models \Gamma$ and that $\mathcal{I}' \prec \mathcal{I}$ which contradicts the fact that $\mathcal{I}$ is a stable model of Γ. □

Proof of Proposition 8. Let $\mathcal{I} = \langle \sigma^h, \sigma^t, I^h, I^t \rangle$ be some coherent interpretation. If φ is an atom, by definition, we have that $\mathcal{I} \models_{\mathcal{S}} \exists x, x = \tau_i \wedge p(\tau_1, \ldots, x, \ldots, \tau_n)$
iff $\mathcal{I} \models_{\mathcal{S}} c = \tau_i \wedge p(\tau_1, \ldots, c, \ldots, \tau_n)$ for some $c \in \mathit{Terms}^0(\mathcal{C})$
iff $\mathcal{I} \models_{\mathcal{S}} c = \tau_i$ and $\mathcal{I} \models_{\mathcal{S}} p(\tau_1, \ldots, c, \ldots, \tau_n)$ for some $c \in \mathit{Terms}^0(\mathcal{C})$
iff $\sigma^h(c) = \sigma^h(\tau_i) \neq \boldsymbol{u}$ and $p(\sigma^h(\tau_1), \ldots, \sigma^h(c), \ldots, \sigma^h(\tau_n)) \in I^h$ for some constant $c \in \mathit{Terms}^0(\mathcal{C})$
iff $\sigma^h(c) = \sigma^h(\tau_i) \neq \boldsymbol{u}$ and $p(\sigma^h(\tau_1), \ldots, \sigma^h(\tau_i), \ldots, \sigma^h(\tau_n)) \in I^h$ for some constant $c \in \mathit{Terms}^0(\mathcal{C})$
iff $\sigma^h(\tau_i) \neq \boldsymbol{u}$ and $p(\sigma^h(\tau_1), \ldots, \sigma^h(\tau_i), \ldots, \sigma^h(\tau_n)) \in I^h$
iff $p(\sigma^h(\tau_1), \ldots, \sigma^h(\tau_i), \ldots, \sigma^h(\tau_n)) \in I^h$
iff $\mathcal{I} \models_{\mathcal{S}} p(\tau_1, \ldots, \tau_i, \ldots, \tau_n)$. $\square$