Online appendix for the paper

Characterization of Logic Program Revision as an Extension of Propositional Revision

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Proposition 1
There is a one-to-one correspondence between the KM revision operators and the set of all faithful assignments.

Proof
Let $\circ_1, \circ_2$ be two KM revision operators. From Theorem 1 one can build two faithful assignments associating respectively with every formula ϕ the total preorders $\leq^1_\phi$ (for the first faithful assignment) and $\leq^2_\phi$ (for the second one), such that for all formulae ϕ, ψ, $mod(\phi \circ_1 \psi) = \min(mod(\psi), \leq^1_\phi)$ and $mod(\phi \circ_2 \psi) = \min(mod(\psi), \leq^2_\phi)$. Assume now that $\circ_1 \neq \circ_2$. This means that there exist two propositional formulae ϕ, ψ such that $\phi \circ_1 \psi \not\equiv \phi \circ_2 \psi$, so $mod(\phi \circ_1 \psi) \neq mod(\phi \circ_2 \psi)$, thus $\min(mod(\psi), \leq^1_\phi) \neq \min(mod(\psi), \leq^2_\phi)$. Hence, $\leq^1_\phi \neq \leq^2_\phi$, so the two faithful assignments associated respectively with $\circ_1$ and $\circ_2$ are different. Conversely, assume that the two faithful assignments associated respectively with $\circ_1$ and $\circ_2$ are different. Then, there exists a formula ϕ such that $\leq^1_\phi \neq \leq^2_\phi$. This means that there exists two interpretations I, J such that $I \leq^1_\phi J$ and $J <^2_\phi I$. Let ψ be any formula such that $mod(\psi) = \{I, J\}$. We have $I \in \min(mod(\psi), \leq^1_\phi)$ and $I \notin \min(mod(\psi), \leq^2_\phi)$. Hence, $mod(\phi \circ_1 \psi) \neq mod(\phi \circ_2 \psi)$, or equivalently, $\phi \circ_1 \psi \not\equiv \phi \circ_2 \psi$. This means that $\circ_1 \neq \circ_2$. $\square$

Proposition 2
$\star_D$ is a GLP revision operator.

Proof
Let P, Q be two logic programs. The fact that $P \star_D Q$ returns a GLP when P, Q are both GLPs is obvious from the definition. Postulates (RA1 - RA4) are directly satisfied from the definition. (RA5 - RA6) Let P, Q, R be three GLPs. If $(P \star_D Q) + R$ is not consistent then (RA5) is trivially satisfied, so assume that $(P \star_D Q) + R$ is consistent. We have to show that $(P \star_D Q) + R \equiv_s P \star_D (Q + R)$. We fall now into two cases. Assume first that $P + Q$ is consistent. By definition, $(P \star_D Q) + R = P + Q + R$. Yet since $(P \star_D Q) + R$ is consistent, so is $P + Q + R$, thus we get by definition $P \star_D (Q + R) = P + Q + R$. Therefore, $(P \star_D Q) + R \equiv_s P \star_D (Q + R)$. Now, assume that $P + Q$ is not consistent. By definition, $(P \star_D Q) + R = Q + R$. Since $P + Q$ is not consistent, we also have $P + Q + R$ not consistent. So by definition $P \star_D (Q + R) = Q + R$. Hence, $(P \star_D Q) + R \equiv_s P \star_D (Q + R)$. $\square$

Proposition 3
An LP operator $\star$ is a GLP revision operator if and only if there exists a pair (Φ, Ψ), where Φ is an LP faithful assignment associating with every GLP P a total preorder $\leq_P$, Ψ is a well-defined assignment associating with every GLP P and every interpretation Y a set of interpretations $P(Y)$, and such that for all GLPs P, Q,

$$SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}\cdot$$

Proof
(Only if part) In this proof, for every well-defined set of SE interpretations S, $\mathsf{lp}(S)$ denotes any GLP P such that $SE(P) = S$. To alleviate notations, when S is of the form $\{(Y, Y) \mid Y \in E\}$ for some set of interpretations E, we write $\mathsf{lp}(E)$ instead of $\mathsf{lp}(S)$. For instance, $\mathsf{lp}(\{(Y, Y), (Y', Y'), (Y^{(2)}, Y^{(2)})\})$ will simply be denoted by $\mathsf{lp}(\{Y, Y', Y^{(2)}\})$. The proof exploits on several occasions the following remarks:

Remark 2
If $\star$ is an LP revision operator satisfying the postulates (RA5) and (RA6), then for all GLPs P, Q, R such that $(P \star Q) + R$ is consistent, we have $(P \star Q) + R \equiv_s P \star (Q + R)$.

Remark 3
For all sets of interpretations E, F, $\mathsf{lp}(E) + \mathsf{lp}(F) \equiv_s \mathsf{lp}(E \cap F)$.

Remark 4
Let $\star$ be an LP revision operator satisfying the postulates (RA1) and (RA3). Then for any GLP P and any non-empty set of interpretations E, $mod(P \star \mathsf{lp}(E)) \neq \emptyset$ and $mod(P \star \mathsf{lp}(E)) \subseteq E$.

Let $\star$ be a GLP revision operator. For every GLP P, define the relation $\leq_P$ over interpretations such that $\forall Y, Y' \in \Omega$, $Y \leq_P Y'$ iff $Y \models P \star \mathsf{lp}(\{Y, Y'\})$. Moreover, for every GLP P, $\forall Y \in \Omega$, let $P(Y) = \{X \subseteq Y \mid (X, Y) \in SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))\}$. Let P be any GLP. We first show that $\leq_P$ is a total pre-order. Let $Y, Y', Y^{(2)} \in \Omega$.

(Totality of $\leq_P$): By Remark 4, $Y \models P \star \mathsf{lp}(\{Y, Y'\})$ or $Y' \models P \star \mathsf{lp}(\{Y, Y'\})$. Hence, $Y \leq_P Y'$ or $Y' \leq_P Y$.
(Reflexivity of $\leq_P$): By Remark 4, $Y \models P \star \mathsf{lp}(\{Y\})$, so $Y \leq_P Y$.
(Transitivity of $\leq_P$): Assume towards a contradiction that $Y \leq_P Y'$, $Y' \leq_P Y^{(2)}$ and $Y \not\leq_P Y^{(2)}$. We consider two cases:
Case 1: $(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y^{(2)}\})$ is consistent. Then we have

$$\begin{aligned}&(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y^{(2)}\})\\ &\quad \equiv_s P \star (\mathsf{lp}(\{Y, Y', Y^{(2)}\}) + \mathsf{lp}(\{Y, Y^{(2)}\})) \quad \text{(by Remark 2)}\\ &\quad \equiv_s P \star \mathsf{lp}(\{Y, Y^{(2)}\}) \quad \text{(by Remark 3)}\cdot\end{aligned}$$

Since $Y \not\leq_P Y^{(2)}$, by definition of $\leq_P$ we get that $Y \not\models P \star \mathsf{lp}(\{Y, Y^{(2)}\})$, hence $Y \not\models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$. By Remark 4, there are two remaining cases:

(i) $Y' \models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$. In this case, $(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y'\})$ is consistent, so

$$\begin{aligned}&(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y'\})\\ &\quad \equiv_s P \star (\mathsf{lp}(\{Y, Y', Y^{(2)}\}) + \mathsf{lp}(\{Y, Y'\})) \quad \text{(by Remark 2)}\\ &\quad \equiv_s P \star \mathsf{lp}(\{Y, Y'\}) \quad \text{(by Remark 3)}\cdot\end{aligned}$$

Since $Y \leq_P Y'$, by definition of $\leq_P$ we get that $Y \models P \star \mathsf{lp}(\{Y, Y'\})$, hence $Y \models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$. which contradicts the previous conclusion that $Y \not\models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$.

(ii) $Y' \not\models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$. Since we also have that $Y \not\models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$, by Remark 4 we must have that $Y^{(2)} \models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$ In this case, $(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y', Y^{(2)}\})$ is consistent, so

$$\begin{aligned}&(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y', Y^{(2)}\})\\ &\quad \equiv_s P \star (\mathsf{lp}(\{Y, Y', Y^{(2)}\}) + \mathsf{lp}(\{Y', Y^{(2)}\})) \quad \text{(by Remark 2)}\\ &\quad \equiv_s P \star \mathsf{lp}(\{Y', Y^{(2)}\}) \quad \text{(by Remark 3)}\cdot\end{aligned}$$

Since $Y' \leq_P Y^{(2)}$, by definition of $\leq_P$ we get that $Y' \models P \star \mathsf{lp}(\{Y', Y^{(2)}\})$, hence $Y' \models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$, which is a contradiction.

Case 2: $(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y^{(2)}\})$ is not consistent. Then by Remark 4, $Y' \models P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})$. Then $(P \star \mathsf{lp}(\{Y, Y', Y^{(2)}\})) + \mathsf{lp}(\{Y, Y'\})$ is consistent, and by using Remark 2 and 3 and following similar reasonings as in (i), we get that $Y' \models P \star \mathsf{lp}(\{Y, Y'\})$ and $Y \not\models P \star \mathsf{lp}(\{Y, Y'\})$. By definition of $\leq_P$ this contradicts $Y \leq_P Y'$ and concludes the proof that $\leq_P$ is a total preorder.

Now, let Q be any GLP. We have to show that $SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$. Let us denote by S the latter set and first show the first inclusion $SE(P \star Q) \subseteq_s \mathsf{S}$. Let $(X, Y) \in SE(P \star Q)$ and let us show that (i) $(X, Y) \in SE(Q)$, (ii) $\forall Y' \models Q, Y \leq_P Y'$ and that (iii) $X \in P(Y)$. (i) is direct from (RA1). For (ii), let $Y' \models Q$. Since $\star$ returns a GLP, $SE(P \star Q)$ is well-defined. That is, since $(X, Y) \in SE(P \star Q)$, we have $Y \models P \star Q$. Therefore, $(P \star Q) + \mathsf{lp}(\{Y, Y'\})$ is consistent. So by Remark 2 and 3, $Y \models P \star \mathsf{lp}(\{Y, Y'\})$. Hence, $Y \leq_P Y'$. For (iii), since $(X, Y) \in SE(P \star Q)$, $(P \star Q) + \mathsf{lp}(\{(X, Y), (Y, Y)\})$ is consistent, so we have $(X, Y) \in SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))$ by Remark 2 and 3; hence, $X \in P(Y)$. Let us now show the other inclusion $\mathsf{S} \subseteq_s SE(P \star Q)$. Assume $(X, Y) \in \mathsf{S}$. Then $\forall Y' \models Q$, $Y \leq_P Y'$ and $X \in P(Y)$. First, from the definition of $P(Y)$ we have $Y \in P(Y)$, so also $(Y, Y) \in \mathsf{S}$. Since $\mathsf{S} \neq \emptyset$, Q is consistent, thus by Remark 4 there exists $Y_* \models Q$, $Y_* \models P \star Q$. Let $R_\# = \mathsf{lp}(\{(X, Y), (Y, Y), (Y_*, Y_*)\})$. Note that $R_\# \subseteq_s Q$ and that $(P \star Q) + R_\#$ is consistent since Y_* is a model of both $P \star Q$ and $R_\#$. Then by Remark 2 we get that $(P \star Q) + R_\# \equiv_s P \star (Q + R_\#) \equiv_s P \star R_\#$. Since we have to show that $(X, Y) \in SE(P \star Q)$, it comes down to show that $(X, Y) \in SE(P \star R_\#)$. Assume towards a contradiction that $(X, Y) \notin SE(P \star R_\#)$. By Remark 4 and since $Y_* \models P \star R_\#$, we have two cases: (i) $Y \not\models P \star R_\#$. Since $(P \star R_\#) + \mathsf{lp}(\{(Y, Y), (Y_*, Y_*)\})$ is consistent, by Remark 2 and 3 we get that $Y \not\models P \star \mathsf{lp}(\{(Y, Y), (Y_*, Y_*)\})$. This contradicts $Y \leq_P Y_*$. (ii) $Y \models P \star R_\#$. Since $(P \star R_\#) + \mathsf{lp}(\{(X, Y), (Y, Y)\})$ is consistent, by Remark 2 and 3 we get that $(X, Y) \notin SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))$. This contradicts $X \in P(Y)$.

It remains to verify that all conditions (1 - 3) of the faithful assignment and conditions (a - e) of the well-defined assignment are satisfied:

(1) Assume $Y \models P$ and $Y' \models P$. By (RA2), $P \star \mathsf{lp}(\{Y, Y'\}) \equiv_s P + \mathsf{lp}(\{Y, Y'\})$. So $Y \models P \star \mathsf{lp}(\{Y, Y'\})$ and $Y' \models P \star \mathsf{lp}(\{Y, Y'\})$, hence $Y \simeq_P Y'$;
(2) Assume $Y \models P$ and $Y' \not\models P$. By (RA2), $P \star \mathsf{lp}(\{Y, Y'\}) \equiv_s P + \mathsf{lp}(\{Y, Y'\})$. So $Y \models P \star \mathsf{lp}(\{Y, Y'\})$ and $Y' \not\models P \star \mathsf{lp}(\{Y, Y'\})$, hence $Y <_P Y'$;
(3) Obvious from (RA4);
(a) By definition of $P(Y)$ and by (RA1) and (RA3), we must have $Y \models P \star \mathsf{lp}(\{(X, Y), (Y, Y)\})$, i.e., $Y \models P(Y)$;
(b) If $X \in P(Y)$ then $X \subseteq Y$ by definition of $P(Y)$;
(c) Assume $(X, Y) \in SE(P)$. Then $Y \models P$. By (RA2), $P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}) \equiv_s P + \mathsf{lp}(\{(X, Y), (Y, Y)\}) \equiv_s \mathsf{lp}(\{(X, Y), (Y, Y)\})$, so $(X, Y) \in SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))$. Therefore, $X \in P(Y)$.
(d) Assume $(X, Y) \notin SE(P)$ and $Y \models P$. By (RA2), $P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}) \equiv_s P + \mathsf{lp}(\{(X, Y), (Y, Y)\}) \equiv_s \mathsf{lp}(\{Y\})$, so $(X, Y) \notin \mathsf{lp}(\{(X, Y), (Y, Y)\})$. Therefore, $X \notin P(Y)$.
(e) Obvious from (RA4).

(If part) We consider a faithful assignment that associates with every GLP P a total preorder $\leq_P$ and a well-defined assignment that associates with every GLP P and every interpretation Y a set $P(Y) \subseteq \Omega$. For all GLPs P, Q, let $\mathsf{S}(P, Q)$ be the set of SE interpretations defined as $\mathsf{S}(P, Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$. Let P, Q be two GLPs and let us show that $\mathsf{S}(P, Q)$ is well-defined. Let $(X, Y) \in \mathsf{S}(P, Q)$. By condition (a) of the well-defined assignment and since $X \subseteq Y$, we have $Y \in P(Y)$, so $(Y, Y) \in \mathsf{S}(P, Q)$. Hence, $\mathsf{S}(P, Q)$ is well-defined. Then let us define an operator $\star$ associating two GLPs P, Q with a new GLP $P \star Q$ such that for all GLPs P, Q, $SE(P \star Q) = \mathsf{S}(P, Q)$.

It remains to show that postulates (RA1 - RA6) are satisfied. Let P, Q bet two GLPs.

(RA1) By definition, $SE(P \star Q) \subseteq SE(Q)$.

(RA2) Assume that $P+Q$ is consistent. We have to show that $SE(P\star Q) = SE(P+Q)$. We first show the inclusion $SE(P \star Q) \subseteq SE(P + Q)$. Let $(X, Y) \in SE(P \star Q)$. Towards a contradicton, assume that $(X, Y) \notin SE(P + Q)$. By definition of $\star$ we have $(X, Y) \in SE(Q)$, thus $(X, Y) \notin SE(P)$. We fall into two cases:
(i) $(Y, Y) \in SE(P)$. Then from condition (d), we have $X \notin P(Y)$. This contradicts $(X, Y) \in SE(P \star Q)$;
(ii) $(Y, Y) \notin SE(P)$. Then from condition (2), $\forall Y' \models P$, $Y' <_P Y$. In particular, $\forall Y' \models P + Q$, $Y' <_P Y$. This contradicts $(X, Y) \in SE(P \star Q)$.
We now show the other inclusion $SE(P + Q) \subseteq SE(P \star Q)$. Let $(X, Y) \in SE(P + Q)$. So $(X, Y) \in SE(Q)$. From conditions (1) and (2), $\forall Y' \in \Omega$, $Y <_P Y'$. Moreover from condition (c), since $(X, Y) \in SE(P)$ we get that $X \in P(Y)$. Therefore, $(X, Y) \in SE(P \star Q)$.

(RA3) Suppose that Q is consistent, i.e., $SE(Q) \neq \emptyset$. As Ω is a finite set of interpretations, we have no infinite descending chain of inequalities w.r.t. $\leq_P$. Moreover, $\leq_P$ is a total relation. Hence, there is an interpretation $Y_* \models Q$ such that $\forall Y' \models Q$, $Y_* \leq_P Y'$. Lastly by condition (a), $Y_* \in P_{Y_*}$. Hence, $Y_* \models P \star Q$, i.e., $P \star Q$ is consistent.

(RA4) Obvious by definition of $\star$ and from conditions (3) and (e).

(RA5) Let $(X, Y) \in SE((P \star Q) + R)$. So by definition of $\star$, $\forall Y' \models Q$, $Y \leq_P Y'$ and $X \in P(Y)$. In particular, $\forall Y' \models Q + R$, $Y \leq_P Y'$ and $X \in P(Y)$. So $(X, Y) \in SE(P \star (Q + R))$.

(RA6) Assume that $(P \star Q) + R$ is consistent. Let $Y_* \models (P \star Q) + R$. Let $(X, Y) \in SE(P \star (Q + R))$. Assume towards a contradiction that $(X, Y) \notin SE((P \star Q) + R)$. Since $(X, Y) \in SE(R)$, we have $(X, Y) \notin SE(P \star Q)$. But $(X, Y) \in SE(Q)$, this means that $Y_* <_P Y$ or $X \notin P(Y)$. Yet $Y_* \models Q + R$, so $(X, Y) \notin SE(P \star (Q + R))$. This leads to a contradiction.

□

Proposition 4
An LP revision operator is a GLP revision operator if and only if it is a propositional-based GLP revision operator.

Proof
(Only If part) Let $\star$ be a GLP revision operator. We have to show that there exists a KM revision operator $\circ$ and a mapping f from Ω to 2^Ω such that $\forall Y \in \Omega$, $Y \in f(Y)$ and if $X \in f(Y)$ then $X \subseteq Y$, and such that for all GLPs P, Q, $SE(P \star Q) = SE(P \star^{\circ,f} Q)$. Yet from Proposition 3 there exists a GLP parted assignment (Φ, Ψ), where Φ associates with every GLP P a total preorder $\leq_P$ and Ψ associates with every GLP P and every interpretation Y a set of interpretations $P(Y)$, such that for all GLPs P, Q, $SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$. Then, let $\circ$ be the KM revision operator associated with the faithful assignment (cf. Definition 3) that associates with every propositional formula ϕ the total preorder $\leq_\phi = \leq_P$, where P is any GLP

such that $\phi \equiv \alpha_P^2$ (from Remark 1, such an assignment is, indeed, faithful and unique). Then from Theorem 1, for every $Y \in \Omega$, $Y \in \min(mod(Q), \leq_P)$ if and only if $Y \models \alpha_P^2 \circ \alpha_Q^2$. Then define f as the mapping from Ω to 2^Ω such that $\forall Y \in \Omega$, $f(Y) = P(Y)$. From conditions (a) and (b) of the well-defined assignment (cf. Definition 15), f is such that $\forall Y \in \Omega$, $Y \in f(Y)$ and if $X \in f(Y)$ then $X \subseteq Y$. Now, given two GLPs P, Q, if $P + Q$ is consistent, we directly get $SE(P \star Q) = SE(P \star^{\circ,f} Q)$ from Definition 15 and postulate (RA2). So assume that $P + Q$ is inconsistent. Given an SE interpretation (X, Y), we have $(X, Y) \in SE(P \star Q)$ if and only if $(X, Y) \in SE(Q)$, $Y \in \min(mod(Q), \leq_P)$ and $X \in P(Y)$, if and only if $(X, Y) \in SE(Q)$, $Y \models \alpha_P^2 \circ \alpha_Q^2$ and $X \in f(Y)$, if and only if $(X, Y) \in SE(P \star^{\circ,f} Q)$. That is to say, $SE(P \star Q) = SE(P \star^{\circ,f} Q)$.

(If part) Let $\star^{\circ,f}$ be a propositional-based GLP revision operator. We have to show that there exists a GLP revision operator $\star$ such that $SE(P \star^{\circ,f} Q) = SE(P \star Q)$. Since $\circ$ is a KM revision operator, from Theorem 1 there is a faithful assignment associating with every propositional formula ϕ a total preorder $\leq_\phi$. Then using Remark 1, let Φ be the LP faithful assignment associating with every GLP P the total preorder $\leq_P = \leq_\phi$, where ϕ is any propositional formula such that $\alpha_P^2 \equiv \phi$. From Theorem 1, for all GLPs P, Q and for every $Y \in \Omega$, $Y \models \alpha_P^2 \circ \alpha_Q^2$ if and only if $Y \in \min(mod(Q), \leq_P)$. Now, let Ψ be the mapping associating with every GLP P and every interpretation Y the set of interpretations $P(Y) = \{X \in \Omega \mid (X, Y) \in SE(P)\} \cup \{X \in f(Y) \mid Y \not\models P\}$. By definition, Ψ satisfies conditions (a) - (e) of a well-defined assignment (cf. Definition 13). Then, let us consider the GLP revision operator $\star$ associated with the GLP parted assignment (Φ, Ψ). We need to check that for all GLPs P, Q, $SE(P \star Q) = SE(P \star^{\circ,f} Q)$. Given two GLPs P, Q, if $P + Q$ is consistent, we directly get $SE(P \star Q) = SE(P \star^{\circ,f} Q)$ from Definition 15 and postulate (RA2). So assume that $P + Q$ is inconsistent. We first prove that $SE(P \star Q) \subseteq SE(P \star^{\circ,f} Q)$. Let $(X, Y) \in SE(P \star Q)$. We have $(X, Y) \in SE(Q)$, $Y \in \min(mod(Q), \leq_P)$ and $X \in P(Y)$. Thus $(X, Y) \in SE(Q)$, $Y \models \alpha_P^2 \circ \alpha_Q^2$ and $X \in P(Y)$. We need to show that $X \in f(Y)$. Yet since $P + Q$ is inconsistent, we have $(X, Y) \notin SE(P)$; and since $(X, Y) \in SE(Q)$, we also have $(Y, Y) \in SE(Q)$, so $(Y, Y) \notin SE(P)$, thus $Y \not\models P$. By definition of $P(Y)$, this means that $X \in f(Y)$. Since $(X, Y) \in SE(Q)$, $Y \models \alpha_P^2 \circ \alpha_Q^2$ and $X \in f(Y)$, we have $(X, Y) \in SE(P \star^{\circ,f} Q)$. Therefore, $SE(P \star Q) \subseteq SE(P \star^{\circ,f} Q)$. We prove now that $SE(P \star^{\circ,f} Q) \subseteq SE(P \star Q)$. Let $(X, Y) \in SE(P \star Q)$. We have $(X, Y) \in SE(Q)$, $Y \in \alpha_P^2 \circ \alpha_Q^2$ and $X \in f(Y)$. Thus $(X, Y) \in SE(Q)$, $Y \in \min(mod(Q), \leq_P)$ and $X \in f(Y)$. We need to show that $X \in P(Y)$. Yet since $P + Q$ is inconsistent and since we have $(X, Y) \in SE(Q)$, we also have $(Y, Y) \in SE(Q)$, so $(Y, Y) \notin SE(P)$, thus $Y \not\models P$. So by definition of $P(Y)$, we get that $X \in P(Y)$. Since $(X, Y) \in SE(Q)$, $Y \in \min(mod(Q), \leq_P)$ and $X \in P(Y)$, we have $(X, Y) \in SE(P \star Q)$. Therefore, $SE(P \star^{\circ,f} Q) \subseteq SE(P \star Q)$. Hence, $SE(P \star^{\circ,f} Q) = SE(P \star Q)$. □

Proposition 5

For all propositional-based GLP revision operators $\star^{\circ_1,f_1}, \star^{\circ_2,f_2}$, we have $\star^{\circ_1,f_1} = \star^{\circ_2,f_2}$ if and only if $\circ_1 = \circ_2$ and $f_1 = f_2$.

Proof
Let $\star^{\circ_1,f_1}, \star^{\circ_2,f_2}$ be two propositional-based GLP revision operators.
(If part) Obvious by Definition 15.
(Only If part) Let us prove the contraposite, i.e., assume that $\circ_1 \neq \circ_2$ or $f_1 \neq f_2$ and let us show that $\star^{\circ_1,f_1} \neq \star^{\circ_2,f_2}$. First, assume that $\circ_1 \neq \circ_2$. This means that there exist two propositional formulae ϕ, ψ such that $\phi \circ_1 \psi \not\equiv \phi \circ_2 \psi$. Then, let P, Q be two GLPs defined such that $\alpha_P^2 \equiv \phi$ and $\alpha_Q^2 \equiv \psi$. We have $mod(\alpha_P^2 \circ_1 \alpha_Q^2) \neq mod(\alpha_P^2 \circ_2 \alpha_Q^2)$. By Definition 15 since $\star^{\circ_1,f_1}, \star^{\circ_2,f_2}$ are both propositional-based GLP revision operators, $\circ_1$ and $\circ_2$ are both KM revision operators. This means that $\circ_1$ and $\circ_2$ satisfy the postulate (R2) (see Definition 2), but since $mod(\alpha_P^2 \circ_1 \alpha_Q^2) \neq mod(\alpha_P^2 \circ_2 \alpha_Q^2)$, this also means that $\alpha_P^2 \wedge \alpha_Q^2$ is inconsistent, i.e., $P + Q$ is inconsistent. Hence, from Definition 15 we can see that for every propositional-based LP revision operator $\star^{\circ,f}$, we have $mod(P \star^{\circ,f} Q) = mod(\alpha_P^2 \circ \alpha_Q^2)$, This means that $mod(P \star^{\circ_1,f_1} Q) \neq mod(P \star^{\circ_2,f_2} Q)$, thus $SE(P \star^{\circ_1,f_1} Q) \neq SE(P \star^{\circ_2,f_2} Q)$. Therefore, $\star^{\circ_1,f_1} \neq \star^{\circ_2,f_2}$.
Now, assume that $f_1 \neq f_2$. So there exists an interpretation Y such that $f_1(Y) \neq f_2(Y)$. We fall into at least one of the two following cases: (i) there exists $X \in f_1(Y)$ such that $X \notin f_2(Y)$, or (ii) there exists $X \in f_2(Y)$ such that $X \notin f_1(Y)$. Assume that we fall into the first case (i) (the second case (ii) leads to the same result by symmetry). Now, let P, Q be two GLPs defined such that $Y \not\models P$ and $SE(Q) = \{(X, Y), (Y, Y)\}$. $P + Q$ is inconsistent. Then by Definition 15 we get that $SE(P \star^{\circ_1,f_1} Q) = \{(X, Y), (Y, Y)\}$ and $SE(P \star^{\circ_2,f_2} Q) = \{(Y, Y)\}$, thus $SE(P \star^{\circ_1,f_1} Q) \neq SE(P \star^{\circ_2,f_2} Q)$. Therefore, $\star^{\circ_1,f_1} \neq \star^{\circ_2,f_2}$. □

Proposition 6
For every $(\Phi, \Psi) \in GLP_{part}$ and every $\Gamma \in GLP_{faith}$, $((\Phi, \Psi), \Gamma) \in \sigma_{part \to faith}$ if and only if for all GLPs P, Q, $\min(SE(Q), \leq_P^*) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$.

Proof
In this proof, for every well-defined set of SE interpretations S, $\mathsf{lp}(S)$ denotes any GLP P such that $SE(P) = S$. Let $(\Phi, \Psi) \in GLP_{part}$ and $\Gamma \in GLP_{faith}$. We have to show that $((\Phi, \Psi), \Gamma) \in \sigma$, i.e., conditions (i) and (ii) involved in the definition of $\sigma_{part \to faith}$ are satisfied, if and only if for all GLP P, Q, we have $\min(SE(Q), \leq_P^*) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$. For simplicity reasons we abuse notations and respectively denote $S_{faith} = \min(SE(Q), \leq_P^*)$ and $S_{part} = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P(Y)\}$.

(If part) Assume that for all GLP P, Q, $S_{faith} = S_{part}$. We have to show that conditions (i) and (ii) involved in the definition of $\sigma_{part \to faith}$ are satisfied.

We first prove that (i) for every GLP P and all interpretations $Y, Y' \in \Omega$, $(Y, Y) \leq_P^* (Y', Y')$ if and only if $Y \leq_P Y'$. Let $Y, Y' \in \Omega$, assume that $(Y, Y) \leq_P^* (Y', Y')$ and assume toward a contradiction that $Y' <_P Y$. Let Q be the GLP $Q = \mathsf{lp}(\{Y, Y'\})$. Then $Y \notin \min(mod(Q), \leq_P)$, thus $(Y, Y) \notin S_{part}$. Hence, $(Y, Y) \notin S_{faith}$, which contradicts $(Y, Y) \leq_P^* (Y', Y')$. The other way around, assume that $Y \leq_P Y'$ and assume toward a contradiction that $(Y', Y') <^*_P (Y, Y)$. Let Q be the GLP $Q = \mathsf{lp}(\{Y, Y'\})$. Then $Y \notin S_{faith}$, thus $Y \notin S_{part}$, which means that $Y \notin \min(mod(Q), \leq_P)$ or $Y \notin P(Y)$. Yet the fact that $Y \notin \min(mod(Q), \leq_P)$ contradicts $Y \leq_P Y'$ and $Y \notin P(Y)$ contradicts condition (a) required by the well-defined assignment Ψ. This proves (i).

We now prove that (ii) for every GLP P, $(X, Y) \leq^*_P (Y, Y)$ and all interpretations $X, Y \in \Omega$ s.t. $X \subseteq Y$, $(X, Y) \leq^*_P (Y, Y)$ if and only if $X \in P(Y)$. Let $X, Y \in \Omega$, $X \subseteq Y$, assume that $(X, Y) \leq^*_P (Y, Y)$ and assume toward a contradiction that $X \notin P(Y)$. Then for the GLP Q defined as $Q = \mathsf{lp}(\{(X, Y), (Y, Y)\})$, we have $(X, Y) \notin S_{faith}$, so $(X, Y) \notin S_{part}$, which contradicts $(X, Y) \leq^*_P (Y, Y)$. The other way around, assume that $X \in P(Y)$ and assume toward a contradiction that $(Y, Y) <^*_P (X, Y)$. Let Q be the GLP defined as $Q = \mathsf{lp}(\{(X, Y), (Y, Y)\})$. On the one hand Q has the only model Y, so $\min(mod(Q), \leq_P) = \{Y\}$. On the other hand, we have $(X, Y) \notin S_{faith}$, so $(X, Y) \notin S_{part}$, which means that we should have $Y \notin \min(mod(Q), \leq_P)$ since we assumed that $X \in P(Y)$. This leads to a contradiction. This proves (ii).

(Only If part) Assume that conditions (i) and (ii) involved in the definition of $\sigma_{part \rightarrow faith}$ are satisfied. We have to show that for all GLP P, Q, we have $S_{faith} = S_{part}$. Let P, Q be two GLPs.

We first prove that $S_{faith} \subseteq S_{part}$. Let $(X, Y) \in S_{faith}$. This means that for every $(X', Y') \in SE(Q)$, $(X, Y) \leq^*_P (X', Y')$. In particular, $(X, Y) \leq^*_P (Y', Y')$. And condition (4) required by the GLP compliant faithful assignment Γ states that $(Y, Y) \leq^*_P (X, Y)$. Hence, $(Y, Y) \leq^*_P (Y', Y')$. So by condition (i) involved in the definition of $\sigma_{part \rightarrow faith}$, we get that $Y \leq_P Y'$ for every $Y' \in \Omega$. So we showed that $Y \in \min(mod(Q), \leq_P)$. Furthermore, since for all $(X', Y') \in SE(Q)$, $(X, Y) \leq^*_P (X', Y')$, we also have that $(X, Y) \leq^*_P (Y, Y)$, and condition (ii) involved in the definition of $\sigma_{part \rightarrow faith}$ implies that $X \in P(Y)$. Since $Y \in \min(mod(Q), \leq_P)$ and $X \in P(Y)$, we get that $(X, Y) \in S_{part}$.

We prove now that $S_{part} \subseteq S_{faith}$. Let $(X, Y) \in S_{part}$. Since $Y \in \min(mod(Q), \leq_P)$, condition (i) involved in the definition of $\sigma_{part \rightarrow faith}$ implies that $(Y, Y) \leq^*_P (Y', Y')$ for every $Y' \in \Omega$. Together with condition (4) required by the GLP compliant faithful assignment Γ, we get for all $X', Y' \in \Omega$ s.t. $X' \subseteq Y'$ that $(Y, Y) \leq^*_P (X', Y')$. And since $X \in P(Y)$, condition (ii) involved in the definition of $\sigma_{part \rightarrow faith}$ implies that $(X, Y) \leq^*_P (Y, Y)$. Therefore, for all $X', Y' \in \Omega$ s.t. $X' \subseteq Y'$, $(X, Y) \leq^*_P (X', Y')$. This is true in particular for every $(X', Y') \in SE(Q)$. This means that $(X, Y) \in S_{faith}$, and this concludes the proof. □

Proposition 7
Let $\circ$ be a KM revision operator. Then for all GLP revision operators $\star_1, \star_2 \in GLP(\circ)$, $\star_1 \preceq_\circ \star_2$ if and only if for all GLPs P, Q, we have $AS(P \star_1 Q) \subseteq AS(P \star_2 Q)$.

Proof
Let $\circ$ be a KM revision operator and $\star_1, \star_2 \in GLP(\circ)$.

(Only if part) Assume that $\star_1 \preceq_\circ \star_2$. By Definition 18, for every interpretation Y we have $f_2(Y) \subseteq f_1(Y)$. Let P, Q be two GLPs such that $P + Q$ is inconsistent (the case where $P + Q$ is consistent is trivial since by Definition 15, we would have $P \star_1 Q = P \star_2 Q = P + Q$) and let $Y \in AS(P \star_1 Q)$. We need to show that $Y \in AS(P \star_2 Q)$. We have $(Y, Y) \in SE(P \star_1 Q)$ and for every $X \subsetneq Y$, $(X, Y) \notin SE(P \star_1 Q)$. Since $\star_1$ is a propositional-based revision operator (cf. Proposition 4), from Definition 15 we get that $Y \models \alpha_P^2 \circ \alpha_Q^2$ (i) and for every $X \subsetneq Y$, $(X, Y) \notin SE(Q)$ or $X \notin f_1(Y)$, thus $(X, Y) \notin SE(Q)$ or $X \notin f_2(Y)$, therefore $(X, Y) \notin SE(P \star_2 Q)$ (ii). By (i) we get that $(Y, Y) \in SE(P \star_2 Q)$ and by (ii) we have for every $X \subsetneq Y$, $(X, Y) \notin SE(P \star_2 Q)$. Therefore, by Definition 15 we get that $Y \in AS(P \star_2 Q)$. Hence, $AS(P \star_1 Q) \subseteq AS(P \star_2 Q)$.

(If part) Assume that for all GLPs P, Q, $AS(P \star_1 Q) \subseteq AS(P \star_2 Q)$. Toward a contradiction, assume that $\star_1 \not\preceq_\circ \star_2$. This means that there exists an interpretation Y such that $f_2(Y) \not\subseteq f_1(Y)$, that is, there exists an interpretation $X \subsetneq Y$ such that $X \in f_2(Y)$ and $X \notin f_1(Y)$. Then, consider a GLP Q such that $SE(Q) = \{(X, Y), (Y, Y)\}$ and any GLP P such that $Y \not\models P$. Since Y is the only interpretation satisfying $Y \models Q$, from postulates (R1) and (R3) of a KM revision operator we have $Y \models \alpha_P^2 \circ \alpha_Q^2$. Moreover $X \notin f_1(Y)$. So we get from Definition 15 that $SE(P \star_1 Q) = \{(Y, Y)\}$. On the other hand, since $X \in f_2(Y)$ we get that $SE(P \star_2 Q) = \{(X, Y), (Y, Y)\}$. Therefore, $Y \in AS(P \star_1 Q)$ and $Y \notin AS(P \star_2 Q)$. This contradicts $AS(P \star_1 Q) \subseteq AS(P \star_2 Q)$. □

Proposition 8
The skeptical GLP revision operators are the only GLP revision operators $\star$ such that for all GLPs P, Q, whenever $P + Q$ is inconsistent, we have $AS(P \star Q) \subseteq AS(Q)$.

Proof
Let $\circ$ be a KM revision operator and $\star_S^\circ$ be the corresponding skeptical GLP revision operator. We first show that for all GLPs P, Q such that $P + Q$ is inconsistent, we have $AS(P \star_S^\circ Q) \subseteq AS(Q)$. $\star_S^\circ$ corresponds to the propositional-based revision GLP operator $\star^{\circ,f}$ such that for every interpretation Y, $f(Y) = 2^Y$. Let P, Q be two GLPs such that $P + Q$ is inconsistent. Let $Y \in AS(P \star_S^\circ Q)$. We have $(Y, Y) \in SE(P \star_S^\circ Q)$, so by Definition 15 we get that $(Y, Y) \in SE(Q)$. Now, assume toward a contradiction that $Y \notin AS(Q)$. This means that there exists $X \subsetneq Y$ such that $(X, Y) \in SE(Q)$. Yet $f(Y) = 2^Y$, so $X \in f(Y)$, thus by Definition 15 this implies that $(X, Y) \in SE(P \star_S^\circ Q)$, this contradicts $Y \in AS(P \star_S^\circ Q)$. Therefore, $Y \in AS(Q)$. Hence, $AS(P \star_S^\circ Q) \subseteq AS(Q)$.
We now show that for some any revision operator $\star^{\circ,f}$, if we have $AS(P \star Q) \subseteq AS(Q)$ for all GLPs P, Q such that $P + Q$ is inconsistent, then $\star^{\circ,f}$ corresponds to the skeptical GLP revision operator $\star_S^\circ$. Let us show the contraposite, that is, assume that $\star^{\circ,f}$ is not a skeptical GLP revision operator. This means that there exists an interpretation Y such that $f(Y) \neq 2^Y$, i.e., there exists $X \subsetneq Y$ such that $X \notin f(Y)$. Then, consider a GLP Q such that $SE(Q) = \{(X, Y), (Y, Y)\}$ and any

GLP P such that $Y \not\models P$. Since Y is the only interpretation satisfying $Y \models Q$, from postulates (R1) and (R3) of a KM revision operator we have $Y \models \alpha^2_P \circ \alpha^2_Q$. On the one hand, since $SE(Q) = \{(X, Y), (Y, Y)\}$ we have $Y \notin AS(Q)$. On the other hand, since $X \notin f(Y)$ we get from Definition 15 that $SE(P \star^{\circ,f} Q) = \{(Y, Y)\}$, that is, $Y \in AS(P \star^{\circ,f} Q)$. Therefore, $AS(P \star^{\circ,f} Q) \not\subseteq AS(Q)$. □

Proposition 9
The brave GLP revision operators are the only GLP revision operators $\star^{\circ,f}$ such that for all GLPs P, Q, whenever $P + Q$ is inconsistent, we have $AS(P \star^{\circ,f} Q) = mod(\alpha^2_P \circ \alpha^2_Q)$.

Proof
Let $\circ$ be a KM revision operator and $\star^{\circ}_B$ be the corresponding brave GLP revision operator. We first show that for all GLPs P, Q such that $P + Q$ is inconsistent, we have $AS(P \star^{\circ}_B Q) = mod(\alpha^2_P \circ \alpha^2_Q)$. $\star^{\circ}_B$ corresponds to the propositional-based revision GLP operator $\star^{\circ,f}$ such that for every interpretation Y, $f(Y) = \{Y\}$. Let P, Q be two GLPs such that $P + Q$ is inconsistent. For every interpretation Y and every $X \subsetneq Y$, $X \notin f(Y)$, thus from Definition 15 for every interpretation Y, we have $Y \in AS(P \star^{\circ}_B Q)$ if and only if $(Y, Y) \in SE(P \star^{\circ}_B Q)$ if and only if $Y \models \alpha^2_P \circ \alpha^2_Q$. Therefore, $AS(P \star^{\circ}_B Q) = mod(\alpha^2_P \circ \alpha^2_Q)$.
We now show that for some any revision operator $\star^{\circ,f}$, if we have $AS(P \star^{\circ}_B Q) = mod(\alpha^2_P \circ \alpha^2_Q)$ for all GLPs P, Q such that $P + Q$ is inconsistent, then $\star^{\circ,f}$ corresponds to the brave GLP revision operator $\star^{\circ}_B$. Let us show the contraposite, that is, assume that $\star^{\circ,f}$ is not a brave GLP revision operator. This means that there exists an interpretation Y such that $f(Y) \neq \{Y\}$, i.e., there exists $X \subsetneq Y$ such that $X \in f(Y)$. Then, consider a GLP Q such that $SE(Q) = \{(X, Y), (Y, Y)\}$ and any GLP P such that $Y \not\models P$. On the one hand, since Y is the only interpretation satisfying $Y \models Q$, from postulates (R1) and (R3) of a KM revision operator we have $Y \models \alpha^2_P \circ \alpha^2_Q$. On the other hand, since $SE(Q) = \{(X, Y), (Y, Y)\}$ and $X \in f(Y)$, we get from Definition 15 that $SE(P \star^{\circ,f} Q) = \{(X, Y), (Y, Y)\}$, that is, $Y \notin AS(P \star^{\circ,f} Q)$. Therefore, $AS(P \star^{\circ}_B Q) = mod(\alpha^2_P \circ \alpha^2_Q)$. □

Proposition 10
$\mathsf{MC}(\circ_D)$ is coNP-complete.

Proof
Let ϕ, ψ be two formulae and I be an interpretation. In the case where $I \models \phi \land \psi$ or $I \not\models \psi$, to determine whether $I \models \phi \circ_D \psi$ can be checked in polynomial time (the answer is "yes" in the former case, "no" in the latter one). So let us assume that $I \models \neg\phi \land \psi$. Then to determine whether $I \models \phi \circ_D \psi$ comes down to determine whether $\phi \land \psi$ is an inconsistent formula, that can be down using one call to a coNP oracle. Hence, $\mathsf{MC}(\circ_D) \in \mathsf{coNP}$. We prove coNP-hardness by exhibiting a polynomial reduction from the unsatisfiability problem. Consider a propositional formula α over a set of propositional variables $\mathcal{A}$, and let us associate with it in polynomial time:

- the formulae ϕ, ψ defined on $\mathcal{A} \cup \{new, new'\}$ (with $\mathcal{A} \cap \{new, new'\} = \emptyset$) as

$\phi = \alpha \wedge new$ and $\psi = new'$;
• the interpretation I over $\mathcal{A} \cup \{new, new'\}$ defined as $I(p) = 0$ if $p = new$, otherwise $I(p) = 1$.
If α is inconsistent then ϕ is inconsistent, so $\phi \circ_D \psi = \psi = new'$; since $I(new') = 1$, we get that $I \models \psi$, so $I \models \phi \circ_D \psi$. Now, if α is consistent then ϕ is consistent, so $\phi \circ_D \psi = \phi \wedge \psi = \alpha \wedge new \wedge new'$; since $I(new) = 0$, we get that $I \not\models \phi \circ_D \psi$. We just showed that α is inconsistent if and only if $I \models \phi \circ_D \psi$, thus $\mathsf{MC}(\circ_D)$ is coNP-hard. □

Proposition 11
The skeptical GLP revision operators are both DLP revision operators and NLP revision operators.

Proof
We show that every skeptical GLP revision operator $\star^{\circ,f} = \star^{\circ}_{S}$ is a DLP revision operator. We have to prove that for all DLP P, Q, $P \star^{\circ,f} Q$ is a DLP, i.e., that $SE(P \star^{\circ}_{S} Q)$ is a complete set of SE interpretations. This is trivial when $P + Q$ is consistent since in this case, $P \star^{\circ}_{S} Q = P + Q$ and expansion preserves completeness of SE models, so assume that $P + Q$ is inconsistent. Let X, Y, Z s.t. $Y \subseteq Z$, $(X, Y), (Z, Z) \in SE(P \star^{\circ,f} Q)$, and let us show that $(X, Z) \in SE(P \star^{\circ,f} Q)$. By definition of a propositional-based LP revision operator, we know that $(X, Y), (Z, Z) \in SE(Q)$. Yet Q is a DLP, thus $(X, Z) \in SE(Q)$. Since $(Z, Z) \in SE(P \star^{\circ,f} Q)$, we get that $Z \models \alpha^2_P \circ \alpha^2_Q$. Moreover, $X \in f(Z)$ since $\star^{\circ}_{S}$ is a skeptical GLP revision operator. Hence, by definition of a propositional-based LP revision operator we get that $(X, Z) \in SE(P \star^{\circ,f} Q)$.

One can prove that every skeptical GLP revision operator $\star^{\circ,f} = \star^{\circ}_{S}$ is a NLP revision operator is a similar way, by augmenting the above conditions of completeness on SE interpretations with the condition of closeness under here-intersection. □

Proposition 12
An LP operator $\star$ is a DLP (resp. NLP) revision operator if and only if there exists a DLP (resp. NLP) parted assignment (Φ, Ψ_Φ), where Φ associates with every DLP (resp. NLP) P a total preorder $\leq_P$, Ψ_Φ is a Φ-based complete (resp. normal) assignment which associates with every DLP (resp. NLP) P and every interpretation Y a set of interpretations $P_\Phi(Y)$, and such that for all DLPs (resp. NLPs) P, Q,

$$SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P_\Phi(Y)\}\cdot$$

Proof
Let us first prove the representation of DLP revision operators.
(Only if part) The proof is identical to the one of Proposition 3 (i.e., our representation theorem for GLP revision operators), except that we now consider that for every well-defined set of SE interpretations S, $\mathsf{lp}(S)$ denotes any DLP R whose set of SE models is the smallest (w.r.t. the set inclusion) superset of S, i.e., $S \subseteq SE(R)$

and there is no DLP R' such that $S \subseteq SE(R')$ and $SE(R') \subsetneq SE(R)$. Remark here that given some set S, the DLP $\mathsf{lp}(S)$ is uniquely defined (modulo strong equivalence): to determine $SE(\mathsf{lp}(S))$, it is enough to add to S all SE interpretations (X, Z) which are missing from S to ensure its completeness, i.e., those SE interpretations (X, Z) such that $(X, Y), (Z, Z) \in S$ for some interpretation $Y \subseteq Z$. Also when S is of the form $\{(Y, Y) \mid Y \in E\}$ for some set of interpretations E, we write $\mathsf{lp}(E)$ instead of $\mathsf{lp}(S)$.

Obviously enough, Remark 2 and 4 from the proof of Proposition 3 still hold. We show now that Remark 3 from the proof of Proposition 3 also holds, i.e., that for all sets of interpretations E, F, $\mathsf{lp}(E) + \mathsf{lp}(F) \equiv_s \mathsf{lp}(E \cap F)$. First, let us show the following intermediate result, that is, for every set E of interpretations and every SE interpretation (X, Z),

$$(X, Z) \in SE(\mathsf{lp}(E)) \text{ if and only if } (X, X), (Z, Z) \in SE(\mathsf{lp}(E))\cdot \tag{1}$$

Equation 1 trivially holds when $X = Z$, so assume $X \subsetneq Z$. The if part comes from the fact that $SE(\mathsf{lp}(E))$ is complete. Let us prove the only if part. On the one hand, $(Z, Z) \in SE(\mathsf{lp}(E))$ since $SE(\mathsf{lp}(E))$ is well-defined. On the other hand, $SE(\mathsf{lp}(E))$ is complete and minimal w.r.t. the set inclusion, which means that there necessarily exists $Y \subsetneq Z$, $X \subseteq Y$ such that $(X, Y) \in SE(\mathsf{lp}(E))$. If now $X \subsetneq Y$, then the reasoning can be repeated recursively (by setting $Z = Y$ each time). Then after a finite number of steps we get that $X = Y$ since we deal with a finite set of atoms, that is, $(X, X) \in SE(\mathsf{lp}(E))$ which proves that Equation 1 holds. Now, for every SE interpretation (X, Z), we have that

$(X, Z) \in SE(\mathsf{lp}(E) + \mathsf{lp}(F))$
if and only if $(X, Z) \in SE(\mathsf{lp}(E)) \cap SE(\mathsf{lp}(F))$
if and only if $(X, X), (Z, Z) \in SE(\mathsf{lp}(E)) \cap SE(\mathsf{lp}(F))$ (by Equation 1)
if and only if $X, Z \in E \cap F$
if and only if $(X, X), (Z, Z) \in SE(\mathsf{lp}(E \cap F))$
if and only if $(X, Z) \in SE(\mathsf{lp}(E \cap F))$ (by Equation 1).

This shows that Remark 3 from the proof of Proposition 3 also holds here, i.e., that for all sets of interpretations E, F, $\mathsf{lp}(E) + \mathsf{lp}(F) \equiv_s \mathsf{lp}(E \cap F)$.

Consider now a DLP revision operator $\star$. We associate with $\star$ a DLP parted assignment (Φ, Ψ_Φ) which uses the same construction as for a GLP parted assignment in the proof of Proposition 3: define for every DLP P the relation $\leq_P$ over interpretations such that $\forall Y, Y' \in \Omega$, $Y \leq_P Y'$ iff $Y \models P \star \mathsf{lp}(\{Y, Y'\})$, and by defining for every DLP P and every $Y \in \Omega$ the set $P_\Phi(Y)$ as $P_\Phi(Y) = \{X \subseteq Y \mid (X, Y) \in SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))\}$. Then the same proof as for Proposition 3 can be used to show that:

(i) for every DLP P, $\leq_P$ is a total preorder;
(ii) for all DLPs P, Q, $SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P_\Phi(Y)\}$;
(iii) conditions (1 - 3) of the faithful assignment Φ and conditions (a - e) of the Φ-based complete assignment Ψ_Φ are satisfied.

It remains to show that the condition (f) of Ψ_Φ is satisfied. Let P be a DLP, X, Y, Z be interpretations such that $Y \subseteq Z$, $Y \simeq_P Z$ and $X \in P_\Phi(Y)$. Assume toward a contradiction that $X \notin P_\Phi(Z)$. By (ii) we get that $SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y), (Z, Z), (X, Z)\})) = \{(X, Y), (Y, Y), (Z, Z)\}$, which is not a complete set of SE interpretations since (X, Z) does not belong to it. This contradicts the fact that $P \star \mathsf{lp}(\{(X, Y), (Y, Y), (Z, Z), (X, Z)\})$ is a DLP, i.e., that $\star$ is a DLP revision operator.

(If part) We consider a faithful assignment Φ that associates with every DLP P a total preorder $\leq_P$ and a Φ-based complete assignment Ψ_Φ that associates with every DLP P and every interpretation Y a set $P_\Phi(Y) \subseteq \Omega$. For all DLPs P, Q, let $\mathsf{S}(P, Q)$ be the set of SE interpretations defined as $\mathsf{S}(P, Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P_\Phi(Y)\}$. Let P, Q be two GLPs. The proof that $\mathsf{S}(P, Q)$ is well-defined is given in the proof of Proposition 3, by using condition (a) of the Φ-based complete assignment Ψ_Φ. We show that $\mathsf{S}(P, Q)$ is complete by using condition (f). Let $(X, Y), (Z, Z)$ be two SE interpretations such that $Y \subseteq Z$ and $(X, Y), (Z, Z) \in \mathsf{S}(P, Q)$. By definition of $\mathsf{S}(P, Q)$ we get that $Y, Z \in \min(mod(Q), \leq_P)$, which means that $Y \simeq_P Z$, and we also get that $X \in P_\Phi(Y)$. Thus condition (f) implies that also $X \in P_\Phi(Z)$. Therefore, $(X, Z) \in \mathsf{S}(P, Q)$ which means that $\mathsf{S}(P, Q)$ is complete. Then we define an operator $\star$ associating two DLPs P, Q with a new DLP $P \star Q$ such that for all DLPs P, Q, $SE(P \star Q) = \mathsf{S}(P, Q)$. The proof that $\star$ satisfies postulates (RA1 - RA6) is identical to the one of Proposition 3.

The proof in the NLP case is very similar to the DLP one and uses the same construction, by adapting the structures accordingly and considering the additional condition (g) involved in a NLP parted assignment.
(Only if part) For every well-defined set of SE interpretations S, $\mathsf{lp}(S)$ denotes any NLP R (which is uniquely defined modulo equivalence) whose set of SE models is the smallest (w.r.t. the set inclusion) superset of S. And when S is of the form $\{(Y, Y) \mid Y \in E\}$ for some set of interpretations E, we write $\mathsf{lp}(E)$ instead of $\mathsf{lp}(S)$.

Remark 2 and 4 from the proof of Proposition 3 still hold, but we need to show that Remark 3 from the proof of Proposition 3 also holds, i.e., that for all sets of interpretations E, F, $\mathsf{lp}(E) + \mathsf{lp}(F) \equiv_s \mathsf{lp}(E \cap F)$. For this purpose, we prove an adaptation of Equation 1 previously given in this proof for DLPs, to the case of NLPs; that is, for every set E of interpretations and every SE interpretation (X, Z),

$$\begin{array}{l} (X, Z) \in SE(\mathsf{lp}(E)) \text{ if and only if one of the two following conditions holds:} \\ \quad (i) \quad (X, X), (Z, Z) \in SE(\mathsf{lp}(E)) \\ \quad (ii) \quad \text{there is a set of interpretations } \mathcal{Y} \text{ such that } \bigcap_{Y \in \mathcal{Y}} Y = X, |\mathcal{Y}| \geq 2, \\ \qquad\quad \text{and } \forall Y \in \mathcal{Y}, (Y, Y) \in SE(\mathsf{lp}(E)). \end{array} \tag{2}$$

Equation 2 trivially holds when $X = Z$, so assume $X \subsetneq Z$. The if part comes from the fact that $SE(\mathsf{lp}(E))$ is complete and closed under here-intersection. Let us prove the only if part. Assume that it does not hold that $(X, X), (Z, Z) \in$

$SE(\mathsf{lp}(E))$. Then by Equation 1, (X, Z) belongs to $SE(\mathsf{lp}(E))$ because its condition specific for closure under here-intersection, i.e., $\exists Y, Y' \subseteq Z,\ Y \cap Y' = X,\ Y \neq Y',\ (Y, Z), (Y', Z) \in SE(\mathsf{lp}(E))$. By applying this reasoning recursively, since we are dealing with a finite set of atoms there must exist a finite set $\mathcal{Y}$ of at least two interpretations such that $\bigcap_{Y \in \mathcal{Y}} Y = X$, and such that all (Y, Z) such that $Y \in \mathcal{Y}$ belong $SE(\mathsf{lp}(E))$ because the condition of completeness, which means by Equation 1 that for every $Y \in \mathcal{Y}$, $(Y, Y) \in SE(\mathsf{lp}(E))$.

Now, for every SE interpretation (X, Z), we have that

$$
\begin{aligned}
&(X, Z) \in SE(\mathsf{lp}(E) + \mathsf{lp}(F)) \\
&\quad \text{if and only if } (X, Z) \in SE(\mathsf{lp}(E)) \cap SE(\mathsf{lp}(F)) \\
&\quad \text{if and only either (i) or (ii) from Equation 2 holds for both } E \text{ and } F\cdot
\end{aligned}
$$

Yet on the one hand, condition (i) from Equation 2 holds for both E and F if and only if $X, Z \in E \cap F$ if and only if $(X, X), (Z, Z) \in SE(\mathsf{lp}(E \cap F))$. On the other hand, condition (ii) from Equation 2 holds for both E and F if and only if there is a set of interpretations $\mathcal{Y}$ such that $\bigcap_{Y \in \mathcal{Y}} Y = X$, $|\mathcal{Y}| \geq 2$ and $\forall Y \in \mathcal{Y}, (Y, Y) \in SE(\mathsf{lp}(E)) \cap SE(\mathsf{lp}(F))$, if and only there is a set of interpretations $\mathcal{Y}$ such that $\bigcap_{Y \in \mathcal{Y}} Y = X$, $|\mathcal{Y}| \geq 2$ and $\forall Y \in \mathcal{Y}$, $Y \in E \cap F$, if and only if there is a set of interpretations $\mathcal{Y}$ such that $\bigcap_{Y \in \mathcal{Y}} Y = X$, $|\mathcal{Y}| \geq 2$ and $\forall Y \in \mathcal{Y}$, $(Y, Y) \in SE(\mathsf{lp}(E \cap F))$. Therefore, by Equation 2 we get that either (i) or (ii) from Equation 2 holds for both E and F if and only if $(X, Z) \in SE(\mathsf{lp}(E \cap F))$. This shows that Remark 3 from the proof of Proposition 3 also holds here, i.e., that for all sets of interpretations E, F, $\mathsf{lp}(E) + \mathsf{lp}(F) \equiv_s \mathsf{lp}(E \cap F)$.

Consider now a NLP revision operator $\star$, and similarly to the case of DLPs, we associate with $\star$ the following NLP parted assignment (Φ, Ψ_Φ): we define for every DLP P the relation $\leq_P$ over interpretations such that $\forall Y, Y' \in \Omega$, $Y \leq_P Y'$ iff $Y \models P \star \mathsf{lp}(\{Y, Y'\})$, and for every GLP P and every $Y \in \Omega$ the set $P_\Phi(Y)$ as $P_\Phi(Y) = \{X \subseteq Y \mid (X, Y) \in SE(P \star \mathsf{lp}(\{(X, Y), (Y, Y)\}))\}$. Then the same proof as for Proposition 3 can be used to show that:

(i) for every NLP P, $\leq_P$ is a total preorder;
(ii) for all NLPs P, Q, $SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P_\Phi(Y)\}$;
(iii) conditions (1 - 3) of the faithful assignment Φ and conditions (a - e) of the Φ-based complete assignment Ψ_Φ are satisfied.

Additionally, we can use the same proof as for DLPs to show that condition (f). It remains to show that the condition (g) of Ψ_Φ is satisfied. Let P be a DLP, X, Y, Z be interpretations such that $X, Y \in P_\Phi(Z)$. Assume toward a contradiction that $X \cap Y \notin P_\Phi(Z)$. By (ii) we get that $SE(P \star \mathsf{lp}(\{(X, Z), (Y, Z), (Z, Z), (X \cap Y, Z)\})) = \{(X, Y), (Y, Y), (Z, Z)\}$, which is not closed under here-intersection since $(X \cap Y, Z)$ does not belong to it. This contradicts the fact that $P \star \mathsf{lp}(\{(X, Z), (Y, Z), (Z, Z), (X \cap Y, Z)\})$ is a NLP, i.e., that $\star$ is a NLP revision operator.

(If part) We consider a NLP parted assignment (Φ, Ψ_Φ) defined as the DLP parted assignment in the if part of the proof for the DLP case. Then defined an operator

$\star$ associating two NLPs P, Q with a new NLP $P \star Q$ such that for all NLPs P, Q, $SE(P \star Q) = \{(X, Y) \mid (X, Y) \in SE(Q), Y \in \min(mod(Q), \leq_P), X \in P_\Phi(Y)\}$. We already showed that $SE(P \star Q)$ is well-defined and complete, and condition (g) of the Φ-based normal assignment Ψ_Φ directly implies that $SE(P \star Q)$ is closed under here-intersection. Therefore, $P \star Q$ is a NLP. The proof that $\star$ satisfies postulates (RA1 - RA6) is identical to the one of Proposition 3. $\Box$