Transition Systems for Model Generators — A Unifying Approach

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Appendix: Proofs

Proof of Proposition 2

We start with some additional notation and several lemmas.

Let N be a set of literals. By $|N|$ we denote a set of atoms occurring in N. For instance $|\{a,\neg b,c\}| = \{a,b,c\}$. Further, by $ch(N)$ we denote a set of rules of the form $a \leftarrow$ *not not* a, where $a \in |N|$.

By a *program literal* we mean expressions a, *not* a and *not not* a, where a is an atom. For a program literal l, we set $s(l) = a$, if $l = a$ or $l =$ *not not* a, and $s(l) = \neg a$, if $l =$ *not* a. For a set B of body literals, we define $s(B) = \{s(l) \mid l \in B\}$. If Π is a program and N is a set of literals, by $\Pi(N)$ we denote the program obtained from Π by removing each rule whose body contains a program literal l such that $\overline{s(l)} \in N$, and deleting from the bodies of all rules in Π every program literal l such that $s(l) \in N$.

Lemma 1
Let Π be a logic program and N a consistent set of literals such that $|N| \cap Head(\Pi) = \emptyset$. For every consistent set M of literals such that $|N| \cap |M| = \emptyset$,

$$\{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } s(B) \subseteq M \cup N\} \setminus N = \{a \mid a \leftarrow B \in \Pi(N) \text{ and } s(B) \subseteq N\}\cdot$$

Proof
Let $c \in \{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } s(B) \subseteq M \cup N\} \setminus N$. Let $c \in |N|$. The only rule in $\Pi \cup ch(N)$ with c as the head is $c \leftarrow$ *not not* c. It follows that $c \in M \cup N$. Since $|N| \cap |M| = \emptyset$, $c \in N$, a contradiction. Thus, $c \notin |N|$ and there is a rule $c \leftarrow B \in \Pi$ such that $s(B) \subseteq M \cup N$. Let B' be what remains when we remove from B all expressions l such that $s(l) \in N$. The rule $c \leftarrow B' \in \Pi(N)$ and $s(B') \subseteq M$. It follows that $c \in \{a \mid a \leftarrow B \in \Pi(N) \text{ and } s(B') \subseteq M\}$.

Conversely, let $c \in \{a \mid a \leftarrow B \in \Pi(N) \text{ and } s(B) \subseteq M\}$. It follows that $c \notin |N|$ and so, $c \notin N$. Moreover, there is a rule $c \leftarrow B' \in \Pi(N)$ such that $s(B') \subseteq M$. By the definition of $\Pi(N)$, there is a rule $c \leftarrow B \in \Pi$ such that $s(B) \subseteq M \cup N$. Thus, $c \in \{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } s(B) \subseteq M \cup N\} \setminus N$. □

Let N be a set of literals. We define $N^- = \{a \mid \neg a \in N\}$.

Lemma 2

For a logic program Π, a consistent set N of literals such that $|N| \cap Head(\Pi) = \emptyset$, and a consistent set M of literals such that $|M| \cap |N| = \emptyset$, $GUS(M \cup N, \Pi \cup ch(N)) \setminus N^- = GUS(M, \Pi(N))$.

Proof

We note that since the sets M and N are consistent and $|M| \cap |N| = \emptyset$, $M \cup N$ is consistent. Moreover, we note that to prove the claim it suffices to show that U is an unfounded set on $M \cup N$ w.r.t. $\Pi \cup ch(N)$ if and only if $U \setminus N^-$ is an unfounded set on M w.r.t. $\Pi(N)$.

($\Rightarrow$) Let $a \in U \setminus N^-$ and let $D \in Bodies(\Pi(N), a)$. It follows that $a \notin |N|$. It also follows that there is a rule $a \leftarrow B \in \Pi$ such that for every program literal $l \in B$, $\overline{s(l)} \notin N$, and D is obtained by removing from B every program literal l such that $s(l) \in N$.

Since U is an unfounded set on $M \cup N$ w.r.t. $\Pi \cup ch(N)$, it follows that $\overline{s(B)} \cap (M \cup N) \neq \emptyset$ or $U \cap B^+ \neq \emptyset$. In the first case, since for every program literal $l \in B$, $\overline{s(l)} \notin N$, $\overline{s(B)} \cap M \neq \emptyset$ follows. Moreover, D differs from B only in program literals l such that $s(l) \in N$. Since $|M| \cap |N| = \emptyset$, we have $\overline{s(D)} \cap M \neq \emptyset$. Thus, let us consider the second case. Let $a \in U \cap B^+$. Since $a \notin |N|$, $a \notin N^-$. For the same reason, $a \notin N$. Thus, $a \in U \setminus N^-$ and $a \in D^+$. That is, $(U \setminus N^-) \cap D^+ \neq \emptyset$. This proves that $U \setminus N^-$ is an unfounded set on M w.r.t. $\Pi(N)$.

($\Leftarrow$) Let U' be any unfounded set on M w.r.t. $\Pi(N)$. By the definition of an unfounded set, U' contains no atoms from $|N|$ since they do not appear in $\Pi(N)$. We show that $U' \cup N^-$ is an unfounded set on $M \cup N$ w.r.t. $\Pi \cup ch(N)$. Let a be any atom in $U' \cup N^-$.

Case 1. $a \in N^-$. It follows that a occurs in the head of only one rule in $\Pi \cup ch(N)$ namely, $a \leftarrow$ *not not* a. Since $\neg a \in N$, $\overline{s(\textit{not not } a)} \in N$ and, consequently, $\overline{s(\textit{not not } a)} \in M \cup N$.

Case 2. $a \in U'$. It follows that $a \notin N$ and so, $Bodies(\Pi \cup ch(N), a) = Bodies(\Pi, a)$. To complete the argument it suffices to show that for every body $B \in Bodies(\Pi, a)$, $\overline{s(B)} \cap (M \cup N) \neq \emptyset$ or $(U' \cup N^-) \cap B^+ \neq \emptyset$ holds.

Let B be any body in $Bodies(\Pi, a)$. It follows that Π contains the rule $a \leftarrow B$. If there is a program literal l in B such that $\overline{s(l)} \in N$, then the first condition above holds. Thus, let us assume that for every program literal $l \in B$, $\overline{s(l)} \notin N$. Let D be obtained from B by removing from it every program literal l such that $s(l) \in N$. It follows that $a \leftarrow D \in \Pi(N)$. Since U' is unfounded on M w.r.t. $\Pi(N)$, there is l in D such that $\overline{s(l)} \in M$ or $U' \cap D^+ \neq \emptyset$. In the first case, we have $\overline{s(B)} \cap (M \cup N) \neq \emptyset$. In the second case, we have $(U' \cup N^-) \cap B^+ \neq \emptyset$. $\square$

By $W^i_\Pi(M)$ we will denote the i-fold application of the W_Π operator on the set M of literals. By convention, we assume that $W^0_\Pi(M) = M$.

Lemma 3

For a normal logic program Π and a consistent set N of literals such that $|N| \cap Head(\Pi) = \emptyset$,

$$W^i_{\Pi \cup ch(N)}(N) = W^i_{\Pi(N)}(\emptyset) \cup N.$$

Proof
We proceed by induction on i. For $i = 0$, since N is consistent, we have

$$W^0_{\Pi\cup ch(N)}(N) = N = \emptyset \cup N = W^0_{\Pi(N)}(\emptyset) \cup N.$$

Let us assume that the identity holds for some $i \geq 0$. We show that it holds for $i+1$.

Let M denote $W^i_{\Pi(N)}(\emptyset)$. We recall that $W^{fix}_{\Pi(N)}(\emptyset)$ is the well-founded model of the normal program $\Pi(N)$. Consequently, the sets $W^{fix}_{\Pi(N)}(\emptyset)$ and $W^j_{\Pi(N)}(\emptyset)$, $j \geq 0$, are consistent (Van Gelder et al. 1991). In particular, M is consistent. Moreover, since $|N| \cap |W^{fix}_{\Pi(N)}(\emptyset)| = \emptyset$, the sets $W^j_{\Pi(N)}(\emptyset) \cup N$, $j \geq 0$, are consistent, too. Thus, we have

$$\begin{aligned} W^{i+1}_{\Pi(N)}(\emptyset) \cup N &= N \cup W_{\Pi(N)}(W^i_{\Pi(N)}(\emptyset)) = N \cup W_{\Pi(N)}(M) \\ &= N \cup M \cup \{a \mid a \leftarrow B \in \Pi(N) \text{ and } B \subseteq M\} \cup \overline{GUS\big(M, \Pi(N)\big)}. \end{aligned}$$

Since $|N| \cap |W^{fix}_{\Pi(N)}(\emptyset)| = \emptyset$, $|M| \cap |N| = \emptyset$. We also observed that M is consistent. By Lemmas 1 and 2 and the fact that $\{\neg a \mid a \in N^-\} \subseteq N$, we have

$$\begin{aligned} W^{i+1}_{\Pi(N)}(\emptyset) \cup N &= N \cup (M \cup \{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } B \subseteq M \cup N\} \setminus N) \\ &\quad \cup \overline{GUS\big(M \cup N, \Pi \cup ch(N)\big)} \setminus N^- \\ &= N \cup (M \cup \{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } B \subseteq M \cup N\} \setminus N) \\ &\quad \cup (\overline{GUS\big(M \cup N, \Pi \cup ch(N)\big)} \setminus \{\neg a \mid a \in N^-\}) \\ &= N \cup M \cup \{a \mid a \leftarrow B \in \Pi \cup ch(N) \text{ and } B \subseteq M \cup N\} \\ &\quad \cup \overline{GUS\big(M \cup N, \Pi \cup ch(N)\big)}. \end{aligned}$$

Since this last set is consistent, it is equal to $W_{\Pi\cup ch(N)}(M \cup N) = W_{\Pi\cup ch(N)}(W^i_{\Pi(N)}(\emptyset) \cup N)$. Applying the induction hypothesis, the inductive step follows. □

Proposition 2
For a PC(ID) theory (F, Π) such that Π is a normal program, M is a model of (F, Π) if and only if M is a model of (F, Π) according to the definition by Denecker (2000).[1]

Proof
Let (F, Π) be a PC(ID) theory. Denecker (2000) defines that a consistent and complete (over $At(F \cup \Pi)$) set M of literals is a model of (F, Π) if

(i) M is a model of F, and
(ii) $M = W^{fix}_{\Pi(M^{O^\Pi})}(\emptyset) \cup M^{O^\Pi}$.

To prove the assertion it is sufficient to show that for any model M of F such that $|M| = At(\Pi \cup F)$, $M = W^{fix}_{\Pi^o}(M^{O^\Pi})$ if and only if $M = W^{fix}_{\Pi(M^{O^\Pi})}(\emptyset) \cup M^{O^\Pi}$. Let $N = M^{O^\Pi}$. The definitions of O^Π and Π^o directly imply that $|N| \cap Head(\Pi) = \emptyset$ and that $\Pi^o = \Pi \cup ch(N)$. Thus, the property follows from Lemma 3. □

[1] For the bibliography we refer to the main paper.

Proofs of Results from Section 3

Proposition 3
For a logic program Π and a set X of atoms,

(a) $X \subseteq \mathit{Head}(\Pi)$ and X is an input answer set of Π if and only if X is an answer set of Π.
(b) if $(X \setminus \mathit{Head}(\Pi)) \cap \mathit{At}(\Pi) = \emptyset$, then X is an input answer set of Π if and only if $X \cap \mathit{Head}(\Pi)$ is an answer set of Π.

Proof
The proof of part (a) is straightforward and follows directly from the definition of an input answer set. To prove (b), let us assume first that X is an input answer set of Π. By the definition, X is an answer set of $\Pi \cup (X \setminus \mathit{Head}(\Pi))$. Thus, X is the least model of the reduct $[\Pi \cup (X \setminus \mathit{Head}(\Pi))]^X$. Clearly, we have $[\Pi \cup (X \setminus \mathit{Head}(\Pi))]^X = \Pi^X \cup (X \setminus \mathit{Head}(\Pi))$. Since $(X \setminus \mathit{Head}(\Pi)) \cap \mathit{At}(\Pi) = \emptyset$, $\Pi^X = \Pi^{X \cap \mathit{Head}(\Pi)}$. It follows that X is the least model of $\Pi^{X \cap \mathit{Head}(\Pi)} \cup (X \setminus \mathit{Head}(\Pi))$. Using again the assumption $(X \setminus \mathit{Head}(\Pi)) \cap \mathit{At}(\Pi) = \emptyset$, one can show that $X \cap \mathit{Head}(\Pi)$ is the least model of $\Pi^{X \cap \mathit{Head}(\Pi)}$. Thus, $X \cap \mathit{Head}(\Pi)$ is an answer set of Π

The proof in the other direction is similar. Let us assume that $X \cap \mathit{Head}(\Pi)$ is an answer set of Π. It follows that $X \cap \mathit{Head}(\Pi)$ is the least model of $\Pi^{X \cap \mathit{Head}(\Pi)}$. Since $(X \setminus \mathit{Head}(\Pi)) \cap \mathit{At}(\Pi) = \emptyset$, X is the least model of $\Pi^{X \cap \mathit{Head}(\Pi)} \cup (X \setminus \mathit{Head}(\Pi))$. Moreover, since $\Pi^{X \cap \mathit{Head}(\Pi)} = \Pi^X$, X is the least model of $\Pi^X \cup (X \setminus \mathit{Head}(\Pi)) = [\Pi \cup (X \setminus \mathit{Head}(\Pi))]^X$. Thus, X is an input answer set of Π. $\square$

Proposition 4
A set of literals M is a model of an SM(ASP) theory $[F, \Pi]$ if and only if M is a model of an SM(ASP) theory $[F, \Pi^o]$.

Proof
Proceeding in each direction, we can assume that M is a complete (over $\mathit{At}(F \cup \Pi)$) and consistent set of literals such that $|M| = |\mathit{At}(F \cup \Pi)|$. It follows that to prove the assertion it suffices to show that for every such set M, M^+ is an input answer set of Π if and only if M^+ is an input answer set of Π^o.

We note that $\Pi^o = \Pi \cup \{a \leftarrow \mathit{not}\ \mathit{not}\ a \mid a \in \mathit{At}(F \cup \Pi) \setminus \mathit{Head}(\Pi)\}$. Thus, $M^+ \subseteq \mathit{Head}(\Pi)$ and so, by Proposition 3, M^+ is an input answer set of Π^o if and only if M^+ is an answer set of Π^o. It follows that to complete the argument, it suffices to show that under our assumptions about M, M^+ is an answer set of $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$ if and only if M^+ is an answer set of Π^o. This statement is evident once we observe that the reducts of $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$ and Π^o with respect to M^+ are equal (they are both equal to $\Pi^{M^+} \cup (M^+ \setminus \mathit{Head}(\Pi))$). $\square$

Proposition 5
For any SM(ASP) theory $[F, \Pi]$ that is Π-safe, a set X of atoms is an answer set of Π if and only if $X = M^+ \cap \mathit{At}(\Pi)$, for some model M of $[F, \Pi]$.

Proof
(⇒) Let X be an answer set of Π. Since $[F,\Pi]$ is Π-safe, there is a model M of F such that $X = M^+ \cap Head(\Pi)$. Moreover, again by the Π-safety of $[F,\Pi]$, $\{\neg a \mid a \in O_\Pi\} \subseteq M$. It follows that $X = M^+ \cap At(\Pi)$ and $(M^+ \setminus Head(\Pi)) \cap At(\Pi) = \emptyset$. By Proposition 3(b), M^+ is an input answer set of Π.

(⇐) Let $X = M^+ \cap At(\Pi)$, where M is a model of $[F,\Pi]$. It follows that M is a model of F. By the Π-safety of $[F,\Pi]$, we have $\{\neg a \mid a \in O_\Pi\} \subseteq M$. As above, it follows that $(M^+ \setminus Head(\Pi)) \cap At(\Pi) = \emptyset$. Since M^+ is an input answer set of Π, Proposition 3(b) implies that $M^+ \cap Head(\Pi)$ is an answer set of Π. From the identity $(M^+ \setminus Head(\Pi)) \cap At(\Pi) = \emptyset$, it follows that $M^+ \cap Head(\Pi) = M^+ \cap At(\Pi)$. Thus, X is an answer set of Π. □

Corollary 1 follows immediately from Proposition 5. We omit its proof and move on to Proposition 6. We start by proving two simple auxiliary results.

Lemma 4
For a logic program Π, and a consistent and complete set M of literals over $At(\Pi)$, if $M = W_\Pi(M)$, then M is a model of Π.

Proof
It is sufficient to show that for every rule $a \leftarrow B \in \Pi$ if $s(B) \subseteq M$ then $a \in M$. This follows from the definition of the operator W_Π and the fact that $M = W_\Pi(M)$. □

Lemma 5
For a logic program Π and a consistent and complete set M of literals over $At(\Pi)$, if $M = W_\Pi(M)$ then M^+ does not have any non-empty subset that is unfounded on M with respect to Π.

Proof
Let us assume that U is a non-empty subset of M^+ that is unfounded on M with respect to Π. It follows that $\overline{U} \subseteq M$. Since $U \neq \emptyset$, M is inconsistent, a contradiction. □

Next, we recall the following generalization of a well-known characterization of answer sets in terms of unfounded sets due to Leone et al. (1997). The generalization extended the characterization to the case of programs with double negation.

Theorem on Unfounded Sets (Lee 2005)
For a set M of literals, M^+ is an answer set of a program Π if and only if M is a model of Π and M^+ does not have any non-empty subset that is unfounded on M with respect to Π.

Proposition 6
For a total PC(ID) theory (F,Π) and a set M of literals over the set $At(F \cup \Pi)$ of atoms, the following conditions are equivalent:

(a) M is a model of (F,Π)
(b) M is a model of an SM(ASP) theory $[F,\Pi]$
(c) M is a model of an SM(ASP) theory $[Comp(\Pi_{At(\Pi)}) \cup F,\Pi]$
(d) for some model M' of an SM(ASP) theory $[ED\text{-}Comp(\Pi_{At(\Pi)}) \cup F,\Pi]$, $M = M' \cap At(F \cap \Pi)$.

Proof

(a)$\Rightarrow$(b) It is sufficient to show that M^+ is an input answer set of Π, that is, an answer set of $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. Since M is a model of the PC(ID) theory (F, Π), M is a complete and consistent set of literals over $At(F \cup \Pi)$ and $M = W^{\mathit{fix}}_{\Pi^o}(M^{O^\Pi})$. It follows that $M = W_{\Pi^o}(M)$. Since $At(\Pi^o) = At(F \cup \Pi)$, by Lemma 4 it follows that M is a model of Π^o. Consequently, M is a model of $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. By Theorem on Unfounded Sets, it is sufficient to show that M^+ does not have any non-empty subset that is unfounded on M with respect to $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. For a contradiction, let us assume that there is a nonempty set $U \subseteq M^+$ that is unfounded on M with respect to $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. Let $a \in U$. It follows that $a \in M^+$. If $a \notin \mathit{Head}(\Pi)$, then a is a fact in $\Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. This is a contradiction with the unfoundedness of U. Thus, $a \in \mathit{Head}(\Pi)$. By the definition of Π^o, $\mathit{Bodies}(\Pi^o, a) = \mathit{Bodies}(\Pi, a)$. It follows that for every $B \in \mathit{Bodies}(\Pi^o, a)$, $\overline{s(B)} \cap M \neq \emptyset$ or $U \cap B^+ \neq \emptyset$. This shows that U is unfounded on M with respect to Π^o. This contradicts Lemma 5.

(a)$\Leftarrow$(b) Since M is a model of $[F, \Pi]$, M is a complete and consistent set of literals over $At(F \cup \Pi)$. By the assumption, M^+ is an answer set of $\Pi' = \Pi \cup (M^+ \setminus \mathit{Head}(\Pi))$. Since Π' and Π have the same reducts with respect to M^+, M^+ is an answer set of Π^o.

Since $M^{O^\Pi} \subseteq M$, $W_{\Pi^o}(M^{O^\Pi}) \subseteq W^o_\Pi(M)$. Let $l \in W^o_\Pi(M)$. If $l = a$, where a is an atom in Π^o, then there is a rule $a \leftarrow B$ in Π^o such that $s(B) \subseteq M$. Since M is a model of Π^o (it is so since M^+ is an answer set of Π^o), $a \in M$. If $l = \neg a$, then $a \in \mathit{GUS}(M, \Pi^o)$.

Let us assume that $a \in M^+$ and let us define $U = M^+ \cap \mathit{GUS}(M, \Pi^o)$. Clearly, $U \neq \emptyset$ and $U \subseteq \mathit{GUS}(M, \Pi^o)$. Let $b \in U$ and let $B \in \mathit{Bodies}(\Pi^o, b)$. Let us assume that $\overline{s(B)} \mathscr{M} = \emptyset$. By the completeness of M, $s(B) \subseteq M$. Since $b \in \mathit{GUS}(M, \Pi^o)$, there is an element $\mathit{GUS}(M, \Pi^o) \cap B^+ \neq \emptyset$. Let us assume that $c \in \mathit{GUS}(M, \Pi^o) \cap B^+$. It follows that $c \in M^+$ and so, $c \in U$. Thus, U is a nonempty set contained in M^+ and unfounded on M with respect to Π^o. By Theorem on Unfounded Sets, this contradicts the fact that M^+ is an answer set of Π^o. it follows that $a \notin M^+$. By the completeness of M, $\neg a \in M$. Thus, $W^o_\Pi(M) \subseteq M$ and, consequently, $W_{\Pi^o}(M^{O^\Pi}) \subseteq M$. By iterating, we obtain that $W^{\mathit{fix}}_{\Pi^o}(M^{O^\Pi}) \subseteq M$. Since (F, Π) is total, $W^{\mathit{fix}}_{\Pi^o}(M^{O^\Pi}) = M$. Thus, (a) follows.

(b)$\Leftrightarrow$(c) It is sufficient to show that M is a model of F if and only if M is a model of $\mathit{Comp}(\Pi^o) \cup F$ given that M^+ is an input answer set of Π or, equivalently, that M^+ is an answer set of $\Pi \cup M^+ \setminus \mathit{Head}(\Pi)$. The “if” part is obvious. For the “only if” part, we proceed as follows. First, reasoning as above we observe that M^+ is an answer set of Π^o. Thus, M is the model of the completion $\mathit{Comp}(\Pi^o)$ and so, M is a model of $\mathit{Comp}(\Pi^o) \cup F$, which we needed to show.

(b)$\Leftrightarrow$(d) The equivalence follows from the fact that $\mathit{ED\text{-}Comp}(\Pi_{At(\Pi)})$ is a conservative extension of $\mathit{Comp}(\Pi_{At(\Pi)})$. $\square$

We now proceed to the proof of Proposition 7. We first recall a result proved by Lierler (2011) (using a slightly modified notation).

Lemma 6 (*Lemma 4 (Lierler 2011)*)

For any unfounded set U on a consistent set M of literals with respect to a program Π, and any assignment N, if $N \models M$ and $N \cap U \neq \emptyset$, then N^+ is not an answer set for Π.

It is well known that for any consistent and complete set M of literals over $At(\Pi)$ (*assignment* on $At(\Pi)$), if M^+ is an answer set for a program Π, then M is a model of Π^{cl}. The property has a counterpart for SM(ASP) theories. The proof is straightforward and we omit it.

Lemma 7
For every SM(ASP) theory $[F,\Pi]$, if M is a model of $[F,\Pi]$, then M is a model of $F \cup \Pi^{cl}$.

Next, we prove the following auxiliary result.

Lemma 8
For every SM(ASP) theory $[F,\Pi]$, every state M other than *FailState* reachable from $\emptyset$ in $\text{SM(ASP)}_{F,\Pi}$, and every model N of $[F,\Pi]$, if N satisfies all decision literals in M, then N satisfies M.

Proof
We proceed by induction on $n = |M|$. The property trivially holds for $n = 0$. Let us assume that the property holds for all states with $k' \leq k$ elements that are reachable from $\emptyset$. For the inductive step, let us consider a state $M = l_1 \ldots l_k$ such that every model N of $[F,\Pi]$ that satisfies all decision literals l_j with $j \leq j$ satisfies M. We need to prove that applying any transition rule of $\text{SM(ASP)}_{F,\Pi}$ in the state $l_1 \ldots l_k$, leads to a state $M' = l_1 \ldots l_k, l_{k+1}$ such that if N is a model of $[F,\Pi]$ and N satisfies every decision literal l_j with $j \leq k+1$, then N satisfies M'.

Unit Propagate: By the definition of *Unit Propagate*, there is a clause $C \vee l \in F \cup \Pi^{cl}$ such that $\overline{C} \subseteq M$ and $M' = M\,l$. Let N be any model of $[F,\Pi]$ that satisfies all decision literals $l_j \in M\,l$. It follows that N satisfies all decision literals in M. By the induction hypothesis, $N \models M$. Since $N \models C \vee l$ and $\overline{C} \subseteq M$, Lemma 7 implies that $N \models l$.

Decide: In this case, $M' = M\,l^d$ (l is a decision literal). If N is a model of the theory $[F,\Pi]$ and it satisfies all decision literals in M', then N satisfies l (by the assumption) and N satisfies every decision literal in M. By the induction hypothesis, the latter implies that $N \models M$. Thus, $N \models M'$.

Fail: If this rule is applicable, M has no decision literals and is inconsistent. If $[F,\Pi]$ has a model N, then by the induction hypothesis, $N \models M$, a contradiction. It follows that $[F,\Pi]$ has no models and the assertion is trivially true.

Backtrack: If this rule is applied, it follows that M has the form $P l_i^d Q$, where Q contains no decision literals, and $M' = P\overline{l_i}$. Let N be a model of $[F,\Pi]$ such that N satisfies all decision literals in $P\overline{l_i}$. It follows that N satisfies all decision literals in P and so, by the induction hypothesis, $N \models P$. Let us assume that $N \models l_i$. Then, N satisfies all decision literals in M and, consequently, $N \models M$, a contradiction as M is inconsistent. Thus, $N \models \overline{l_i}$ and so, $N \models M'$.

Unfounded: If M' is obtained from M by an application of the *Unfounded* rule, then M is consistent and $M' = M\neg a$, for some $a \in U$, where U is an unfounded set on M with respect to Π^o. Let N be any model N of $[F,\Pi]$ such that N satisfies all decision literals in M'. It follows that N satisfies all decision literals in M and so, by the inductive hypothesis, $N \models M$. By the definition of a model of $[F,\Pi]$, N^+ is an input answer set of Π. Consequently,

N^+ is an answer set of $\Pi \cup (N^+ \setminus Head(\Pi))$. Arguing as as before, we obtain that N^+ is an answer set of Π^o. By Lemma 6, $a \notin N^+$, that is, $N \models \neg a$. □

Proposition 7

For any SM(ASP) theory $[F, \Pi]$,

(a) graph $\text{SM(ASP)}_{F,\Pi}$ is finite and acyclic,
(b) for any terminal state M of $\text{SM(ASP)}_{F,\Pi}$ other than *FailState*, M is a model of $[F, \Pi]$
(c) *FailState* is reachable from $\emptyset$ in $\text{SM(ASP)}_{F,\Pi}$ if and only if $[F, \Pi]$ has no models.

Proof

Parts (a) and (c) are proved as in the proof of Proposition 1 (Lierler 2011, Proposition 1) using Lemma 8.

(b) Let M be a terminal state. It follows that none of the rules are applicable. From the fact that *Decide* is not applicable, we derive that M assigns all literals. Since neither *Backtrack* nor *Fail* are applicable, M is consistent. Since *Unit Propagate* is not applicable, it follows that for every clause $C \vee a \in F \cup \Pi^{cl}$ if $\overline{C} \subseteq M$ then $a \in M$. Consequently, if $M \models \overline{C}$ then $M \models a$. Thus, M is a model of $F \cup \Pi^{cl}$. Consequently, M is a model of F.

Next, we show that M^+ is an input answer set of Π, that is, that M^+ is an answer set of $\Pi \cup (M^+ \setminus Head(\Pi))$. To this end, it is sufficient to show that M^+ is an answer set of Π^o (we again exploit here the fact that M^+ is an answer set of $\Pi \cup (M^+ \setminus Head(\Pi))$ if and only if M^+ is an answer set of Π^o). Since M is a model of $F \cup \Pi^{cl}$, M is a model of Π^o.

Let us assume that M^+ is not an answer set of Π^o. By Theorem on Unfounded Sets, it follows that there is a non-empty unfounded set U on M with respect to Π^o such that $U \subseteq M^+$. Then *Unfounded* can be applied for some $a \in U$. If $\neg a \notin M$, M is not terminal, a contradiction. Thus, $\neg a \in M$. Since M is consistent, $a \notin M^+$, a contradiction (as $U \subseteq M^+$). It follows that M^+ is an answer set of Π^o, as required. □

Finally, we sketch a proof for Proposition 8.

Proposition 8

For every program Π, the graphs SM^-_Π and $\text{SM(ASP)}^-{}_{Comp(\Pi),\Pi}$ are equal.

Proof

Sketch: First we show that the states of the graphs SM^-_Π and $\text{SM(ASP)}^-{}_{Comp(\Pi),\Pi}$ coincide. In view of Proposition 3 stated and proved by Lierler (2011) it is sufficient to show that there is a non-singular edge $M \Longrightarrow M'$ in SM_Π justified by the transition *Unfounded* (defined for SM) if and only if there is a non-singular edge $M \Longrightarrow M'$ in $\text{SM(ASP)}_{Comp(\Pi),\Pi}$ justified by *Unfounded* (defined for SM(ASP)). We conclude by proving the last statement. □

Proof of Proposition 9

We first extend Lemma 8 to the "learning" version of the graph $\text{SM(ASP)}_{F,\Pi}$.

Lemma 9
For every SM(ASP) theory $[F,\Pi]$, every state $M||\Gamma$ reachable from $\emptyset||\emptyset$ in $\text{SM(ASP)}_{F,\Pi}$, and every model N of $[F,\Pi]$, if N satisfies all decision literals in M, then N satisfies M.

Proof
The proof is by induction on $n = |M|$ and proceeds similarly as that of Lemma 8. In particular, the property trivially holds for $n = 0$. Let us assume that the property holds for all states $M||\Gamma$, where $|M| \leq k$, that are reachable from $\emptyset||\emptyset$. For the inductive step, let us consider a state $M||\Gamma$, with $M = l_1 \dots l_k$, such that every model N of $[F,\Pi]$ that satisfies all decision literals l_j with $j \leq k$ satisfies M. We need to prove that applying any transition rule of $\text{SM(ASP)}_{F,\Pi}$ in the state $M||\Gamma$, leads to a state $M'||\Gamma'$, where $M' = M\,l_{k+1}$, such that if N is a model of $[F,\Pi]$ and N satisfies every decision literal l_j with $j \leq k+1$, then N satisfies M'.

The rules *Decide*, *Fail* and *Unfounded* can be dealt with as before (with only minor notational adjustments to account for extended states). Thus, we move on to the rules *Unit Propagate Learn*, *Backjump*, and *Learn*.

Unit Propagate Learn: We recall that Γ is a set of clauses entailed by F and Π. In other words, any model of $[F,\Pi]$ is also a model of Γ. We now proceed as in the case of the rule *Unit Propagate* in the proof of Proposition 8 with $F \cup \Pi^{cl}$ replaced by $F \cup \Pi^{cl} \cup \Gamma$.

Backjump: The argument is similar to that used in the case of the transition rule *Backtrack* in the proof of Lemma 8.

Learn: This case is trivially true. $\square$

We now recall several concepts we will need in the proofs. Given a set A of atoms, we define $Bodies(\Pi,A) = \bigcup_{a\in A} Bodies(\Pi,a)$. Let Π be a program and Y a set of atoms. We call the formula

$$\bigvee_{a\in Y} a \rightarrow \bigvee\{B \mid B \in Bodies(\Pi,Y) \text{ and } B^{pos} \cap Y = \emptyset\} \quad (1)$$

the *loop formula* for Y (Lin and Zhao 2004). We can rewrite the loop formula (1) as the disjunction

$$(\bigwedge_{a\in Y} \neg a) \vee \bigvee\{B \mid B \in Bodies(\Pi,Y) \text{ and } B^{pos} \cap Y = \emptyset\}. \quad (2)$$

The *Main Theorem* (Lee 2005) implies the following property loop formulas. In its statement we refer to the concept of a program entailing a formula. The notion is defined as follows. A program Π entails a formula F (over the set of atoms in Π) if for every interpretation M (over the set of atoms in Π) such that M^+ is an answer set of Π, M is a model of F.

Lemma 10 (*Lemma on Loop Formulas*)
For every program Π and every set Y of atoms, $Y \subseteq At(\Pi)$, Π entails the loop formula (2) for Y.

For an SM(ASP) theory $[F,\Pi]$ and a list PlQ of literals, we say that a clause $C \vee l$ is a *reason* for l to be in PlQ with respect to $[F,\Pi]$ if

1. $P \models \neg C$, and
2. $F, \Pi^o \models C \vee l$.

Lemma 11
Let $[F,\Pi]$ be an SM(ASP) theory. For every state $M||\Gamma$ reachable from $\emptyset||\emptyset$ in the graph $\text{SML(ASP)}_{F,\Pi}$, every literal l in M is either a decision literal or has a reason to be in M with respect to $[F,\Pi]$.

Proof
We proceed by induction on the length of a path from $\emptyset||\emptyset$ to $M||\Gamma$ in the graph $\text{SML(ASP)}_{F,\Pi}$. Since the property trivially holds in the initial state $\emptyset||\emptyset$, we only need to prove that every transition rule of $\text{SML(ASP)}_{F,\Pi}$ preserves it.

Let us consider an edge $M||\Gamma \Longrightarrow M'||\Gamma'$, where M is a sequence $l_1 \dots l_k$ such that every l_i, $1 \le i \le k$, is either a decision literal or has a reason to be in M with respect to $[F,\Pi]$. It is evident that transition rules *Backjump*, *Decide*, *Learn*, and *Fail* preserve the property (the last one trivially, as *FailState* contains no literals).

Unit Propagate Learn: The edge $M||\Gamma \Longrightarrow M'||\Gamma'$ is justified by the rule *Unit Propagate Learn*. That is, there is a clause $C \vee l \in F \cup \Pi^{cl} \cup \Gamma$ such that $\overline{C} \subseteq M$ and $M' = Ml$. By the inductive hypothesis, the property holds for every literal in M. We now show that a clause $C \vee l$ is a reason for l to be in Ml. By the applicability conditions of *Unit Propagate Learn*, $\overline{C} \subseteq M$. Consequently, $M \models \overline{C}$. It remains to show that $F, \Pi^o \models C \vee l$.

Case 1. $C \vee l \in F$. Then, clearly, $F \models C \vee l$ and, consequently, $F, \Pi^o \models C \vee l$.

Case 2. $C \vee l \in \Pi^{cl}$. Since $\Pi^{cl} \subseteq (\Pi^o)^c l$, $C \vee l \in (\Pi^o)^{cl}$. Let M be a model of $[F,\Pi^o]$. It follows that M^+ is an answer set of Π^o. Thus, $M \models (\Pi^o)^{cl}$ and so, $M \models C \vee l$. Thus, $F, \Pi^o \models C \vee l$.

Case 3. $C \vee l \in \Gamma$. We recall that $F, \Pi^o \models \Gamma$ by the definition of an augmented state. Consequently, $F, \Pi^o \models C \vee l$.

Unfounded: We have that M is consistent, and that there is an unfounded set U on M with respect to Π^o and $a \in U$ such that $M' = M\neg a$. By the inductive hypothesis, the property holds for every literal in M. We need to show that $\neg a$ has a reason to be in $M\neg a$ with respect to $[F,\Pi]$.

Let $B \in \mathit{Bodies}(\Pi^o, U)$ be such that $U \cap B^{pos} = \emptyset$. By the definition of an unfounded set, it follows that $\overline{s(B)} \cap M \neq \emptyset$. Consequently, $s(B)$ contains a literal from $\overline{M}$. We pick an arbitrary one and call it $f(B)$. The clause

$$C = \neg a \vee \bigvee \{f(B) \mid B \in \mathit{Bodies}(\Pi^o, U) \text{ and } B^{pos} \cap U = \emptyset\}, \tag{3}$$

is a reason for $\neg a$ to be in $M\neg a$ with respect to $[F,\Pi]$.

First, by the choice of $f(B)$, for every $B \in \mathit{Bodies}(\Pi^o, U)$ and $B^{pos} \cap U = \emptyset$, $\overline{f(B)} \in M$. Consequently,

$$M \models \neg \bigvee \{f(B) \mid B \in \mathit{Bodies}(\Pi^o, U) \text{ and } B^{pos} \cap U = \emptyset\}\cdot \tag{4}$$

Second, since $f(B) \in B$, the loop formula

$$(\bigwedge_{u \in U} \neg u) \vee \bigvee \{B \mid B \in \mathit{Bodies}(\Pi, U) \text{ and } B^{pos} \cap U = \emptyset\} \tag{5}$$

entails C. By Lemma on Loop Formulas, it follows that Π^o entails C. Consequently, $F, \Pi^o \models C$. $\square$

For a list M of literals, by *consistent*(M) we denote the longest consistent prefix of M. For example, *consistent*$(abc\neg bd) = abc$. A clause C is *conflicting* on a list M of literals with respect to an SM(ASP) theory $[F,\Pi]$ if *consistent*$(M) \models \neg C$ and $F,\Pi^o \models C$.

For a state $M||\Gamma$ reachable from $\emptyset||\emptyset$ in SML(ASP)$_{F,\Pi}$, by r_M we denote a function that maps every non-decision literal in M to its reason to be in M (with respect to $[F,\Pi]$). By $\mathbf{R}_M$ we denote the set consisting of the clauses $r_M(l)$, for each non-decision literal $l \in$ *consistent*(M).

A *resolution derivation of a clause C from a sequence of clauses* $C_1,\ldots,C_m$ is a sequence $C_1,\ldots,C_m,\ldots,C_n$, where $C \equiv C_l$ for some $l \leq n$, and each clause C_i in the sequence is either a clause from $C_1,\ldots,C_m$ or is derived by applying the resolution rule to clauses C_j and C_k, where $j,k < i$ (we call such clauses *derived*). We say that a clause C is *derived by a resolution derivation* from a sequence of clauses $C_1,\ldots,C_m$ if there is a resolution derivation of a clause C from $C_1,\ldots,C_m$.

Lemma 12
Let $[F,\Pi]$ be an SM(ASP) theory, $M||\Gamma$ a state in the graph SM(ASP)$_{F,\Pi}$ such that M is inconsistent, and C_1 a clause in $\mathbf{R}_M$. If clause C_2 is conflicting on M with respect to $[F,\Pi]$, then every clause C derived from C_1 and C_2 is also a conflicting clause on M with respect to $[F,\Pi]$.

Proof
Let us assume that C is derived from C_1 and C_2 by resolving on some literal $l \in C_1$. Then, C_2 is of the form $\bar{l} \vee C_2'$.

From the fact that $C_1 \in \mathbf{R}_M$, it follows that $F,\Pi^o \models C_1$ and that C_1 has the form $c_1 \vee C_1'$, where *consistent*$(M) \models \neg C_1'$. Since C_2 is conflicting, *consistent*$(M) \models \neg C_2$ and $F,\Pi^o \models C_2$. By the consistency of *consistent*(M), there is no literal in C_1' such that its complement occurs in C_2. Therefore $l = c_1$ and, consequently, $C = C_1' \vee C_2'$. It follows that *consistent*$(M) \models \neg C$. Moreover, since $F,\Pi^o \models C_1$ and $F,\Pi^o \models C_2$ and C results from C_1 and C_2 by resolution, $F,\Pi^o \models C$. □

For an SM(ASP) theory $[F,\Pi]$ and a node $M||\Gamma$ in SM(ASP)$_{F,\Pi}$, a resolution derivation $C_1,\ldots,C_n$ is *trivial* on M with respect to $[F,\Pi]$[2] if

(1) $\{C_1,\ldots,C_i\} = \mathbf{R}_M$
(2) C_{i+1} is a conflicting clause on M with respect to $[F,\Pi]$
(3) C_j, $j > i+1$, is derived from C_{j-1} and a clause C_k, where $k \leq i$ (that is, $C_k \in \mathbf{R}_M$), by resolving on some non-decision literal of *consistent*(M).

For a record $M_0 l_1 M_1 \ldots l_k M_k$, where l_i are all the decision literals of the record, we say that the literals of $l_i M_i$ *belong to a decision level i*. For a state $M l M' l' M''$, we say that l is *older* than l'. We say that a state is a *backjump* state if it is inconsistent, contains a decision literal, and is reachable from $\emptyset||\emptyset$ in SML(ASP)$_{F,\Pi}$.

Lemma 13
For every SM(ASP) theory $[F,\Pi]$, the transition rule *Backjump* is applicable in every backjump state in SM(ASP)$_{F,\Pi}$.

[2] This definition is related to the definition of a *trivial* resolution derivation (Beame et al. 2004).

Proof
Let $M||\Gamma$ be a backjump state in $\mathrm{SM(ASP)}_{F,\Pi}$. We will show that M has the form $Pl^{\Delta}Q$ and that there is a literal l' that has a reason to be in Pl' with respect to $[F,\Pi]$.

Since $M||\Gamma$ is a backjump state, it follows that M has the form $consistent(M)\,l\,N$. It is clear that l is not a decision literal (otherwise $consistent(M)\,l$ would be consistent). By Lemma 11, there is a reason, say R for l to be in M. We denote this reason by R. Since $consistent(M)\,l$ is inconsistent, $\bar{l} \in consistent(M)$. This observation and the definition of a reason imply that $consistent(M) \models \neg R$. Moreover, since $F,\Pi^o \models R$ (as R is a reason), R is a conflicting clause.

Let dec be the largest of the decision levels of the complements of the literals in R (each of them occurs in $consistent(M)$). Let D be the set of all non-decision literals in $consistent(M)$. By D^{dec} we denote a subset of D that contains all the literals that belong to decision level dec.

It is clear that $C_1,\dots,C_i,C_{i+1}$, where $\{C_1,\dots C_i\} = \mathbf{R}_M$ and $C_{i+1} = R$, is a trivial resolution derivation with respect to M and $consistent(M) \models \neg C_{i+1}$. Let us consider a trivial resolution derivation with respect to M of the form $C_1,\dots,C_i,C_{i+1},\dots,C_n$, where $n \geq i+1$ and $consistent(M) \models \neg C_n$. Let us assume that there is a literal $l \in D$ such that $\bar{l}$ in C_n. It follows that $C_n = \bar{l} \vee C'_n$, for some clause C'_n.

Since $l \in D$ (is a non-decision literal in $consistent(M)$), the set R_M contains the clause $r_M(l)$, which is a reason for l to be in M. The clause $r_M(l)$ is of the form $l \vee l_1 \vee \dots \vee l_m$, where literals $\overline{l_1},\dots,\overline{l_m}$ are older than l and $consistent(M) \models \neg(l_1 \vee \dots \vee l_m)$. Resolving C_n and $r_M(l)$ yields the clause $C_{n+1} = C'_n \vee l_1 \vee \dots \vee l_m$. Clearly, $C_1,\dots,C_{n+1}$ is a trivial resolution derivation with respect to M and $conistent(M) \models \neg C_{n+1}$.

If we apply this construction selecting at each step a non-decision literal $l \in D^{dec}$ such that $\bar{l} \in R$, then at some point we obtain a clause C_n that contains exactly one literal whose complement belongs to decision level dec (the reason is that in each step of the construction, the literal with respect we perform the resolution is replaced by older ones).

By Lemma 12, the clause $C = C_n$ is conflicting on M with respect to $[F,\Pi]$, that is, $consistent(M) \models \neg C$ and $F,\Pi^o \models C$. By the construction, $C = l' \vee C'$, where l' is the only literal whose complement belongs to the decision level dec and the complements of all literals in C' belong to lower decision levels.

Case 1. $dec = 0$. Since for every literal $l \in C'$, the decision level of $\bar{l}$ is strictly lower than dec, $C' = \bot$. Since $M||\Gamma$ is a backjump state, M contains a decision literal. Then M can be written as $Pl^{\Delta}Q$, where P contains no decision literals (in other words P consists of all literals in $consistent(M)$ of decision level $dec = 0$) and $\overline{l'} \in P$. Clearly, $P \models \neg C'$ (as $C' = \bot$). Since $F,\Pi^o \models C (= l' \vee C')$, C is a reason for l' to be in Pl'.

Case 2. $dec \geq 1$. Let l be the decision literal in M that starts the decision level dec. Then, M can be written as $Pl^{\Delta}Q$. By the construction of the clause C, the complement of every literal in C' belongs to a decision level smaller than dec, that is, to P. It follows that $P \models \neg C'$. Thus, as before, we conclude that C is a reason for l' to be in Pl'. □

Proposition 9
For any SM(ASP) theory $[F,\Pi]$,

(a) every path in $\mathrm{SML(ASP)}_{F,\Pi}$ contains only finitely many edges justified by basic transition rules,

(b) for any semi-terminal state $M||\Gamma$ of $\text{SML}(\text{ASP})_{F,\Pi}$ reachable from $\emptyset||\emptyset$, M is a model of $[F,\Pi]$,

(c) *FailState* is reachable from $\emptyset||\emptyset$ in $\text{SML}(\text{ASP})_{F,\Pi}$ if and only if $[F,\Pi]$ has no models.

Proof

Part (a) is proved as in the proof of Proposition 13↑ (Lierler 2010) (we preserve the notation used in that work).

(b) Let $M||G$ be a semi-terminal state reachable from $\emptyset||\emptyset$ (that is, none of the basic rules are applicable.) Since *Decide* is not applicable, M assigns all literals. Next, M is consistent. Indeed, if M were inconsistent then, since *Fail* is not applicable, M would contain a decision literal. Consequently, $M||\Gamma$ would be a backjump state. By Lemma 13, the transition rule *Backjump* would be applicable in $M||\Gamma$, contradicting our assumption that $M||\Gamma$ is semi-terminal. We now proceed as in the proof of Proposition 7 (b) to show M is a model of F and M^+ is an input answer set of Π.

(c) If *FailState* is reachable from $\emptyset||\emptyset$ in $\text{SML}(\text{ASP})_{F,\Pi}$, then there is a state $M||\Gamma$ reachable from $\emptyset||\emptyset$ in $\text{SML}(\text{ASP})_{F,\Pi}$ such that there is an edge between $M||\Gamma$ and *FailState*. By the definition of $\text{SML}(\text{ASP})_{F,\Pi}$, this edge is due to the transition rule *Fail*. Thus, M is inconsistent and contains no decision literals. By Lemma 9, every model N of $[F,\Pi]$ satisfies M. Since M is inconsistent, $[F,\Pi]$ has no models.

Conversely, if $[F,\Pi]$ has no models, let us consider a maximal path in $\text{SML}(\text{ASP})_{F,\Pi}$ starting in $\emptyset||\emptyset$ and consisting of basic transition rules. By (a), it follows that such a path is finite and ends in a semi-terminal state. By (b), this semi-terminal must be *FailState*, because $[F,\Pi]$ has no models. □

Proofs of Results from Section 6

Proposition 10

For a total PC(ID) theory (F,Π) and a consistent and complete (over $At(F\cup\Pi)$) set M of literals, M is a model of (F,Π) if and only if M^+ is an answer set of $\pi(F,\Pi)$.

Proof

By Proposition 6, it is enough to prove that M is a model of the SM(ASP) theory $[F,\Pi]$ if and only if M^+ is an answer set of $\pi(F,\Pi)$. By the definition of $\pi(F,\Pi)$, M^+ is an answer set of $\pi(F,\Pi)$ if and only if M^+ is an answer set of Π^o and a model of F. Since M^+ is a subset of $Head(\Pi^o)$ (since $Head(\Pi^o) = At(F\cup\Pi)$), Proposition 3(a) implies that M^+ is an answer set of Π^o if and only if M^+ is an input answer set of Π^o. It follows that M^+ is an answer set of $\pi(F,\Pi)$ if and only if M is a model of the SM(ASP) theory $[F,\Pi^o]$. The assertion follows now from Proposition 4. □

Proposition 11

For a PC(ID) theory (F,Π), we have

$$\text{SML}(\text{ASP})_{\textit{ED-Comp}(\Pi^o)\cup F,\Pi^o} = \text{SML}(\text{ASP})_{\textit{ED-Comp}(\pi(F,\Pi)),\pi(F,\Pi)}.$$

Proof
We recall that $\pi(F,\Pi) = F^r \cup \Pi^o$. From the construction of *ED-Comp*, it is easy to see that

$$ED\text{-}Comp(\Pi^o) \cup F = ED\text{-}Comp(\pi(F,\Pi))\cdot$$

Furthermore, from the definition of an unfounded set it follows that for any consistent set M of literals and a set U of atoms, U is unfounded on M with respect to Π^o if and only if U is unfounded on M with respect to $\pi(F,\Pi)$. $\square$