

**THE ALPERIN WEIGHT CONJECTURE AND DADE'S
CONJECTURE FOR THE SIMPLE GROUP Fi'_{24}**

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APPENDIX B

DEGREES OF IRREDUCIBLE CHARACTERS OF
CHAIN NORMALIZERS OF $3.\text{Fi}'_{24}$

Let ζ be a faithful irreducible character of the cyclic group $3 = Z(3.\text{Fi}'_{24})$. For a radical p -chain $C \in \mathcal{R}(\text{Fi}'_{24})$ denote by $\text{Irr}(N_{3.\text{Fi}'_{24}}(C) \mid \zeta)$ the irreducible characters of the stabilizer $N_{3.\text{Fi}'_{24}}(C)$ covering the character ζ .

Degree	15	21	72	90	105	210	252	315	336
Number	2	2	1	2	6	2	1	2	2
Degree	378	384	420	630	720	729	756	945	1260
Number	3	4	2	5	1	2	2	2	1
Degree	1440	1458	1512	1890	2268	3780	5103	5376	5670
Number	1	1	1	1	2	1	4	2	4
Degree	7560	7680	8064	9072	10206	11340	11520	12096	13440
Number	3	2	2	2	2	2	1	4	2
Degree	15309	16128	21504	22680	24192	24576	25515	26880	30240
Number	2	4	2	3	2	2	6	2	5
Degree	30618	34020	34560	40320	48384	51030	53760	60480	64512
Number	2	1	3	2	1	2	4	1	1
Degree	69120	76545	80640	92160	120960	161280			
Number	1	4	2	1	2	1			

TABLE B-1. The degrees of characters in $\text{Irr}(3.2^{1+12}.3U_4(3):2 \mid \zeta)$

Degree	3	6	9	12	18	24	36	48	54	72	81	96	108
Number	12	8	4	9	4	6	13	13	8	8	12	8	8
Degree	144	162	192	216	243	288	324	384	486	576	648	768	
Number	6	18	9	4	4	4	14	6	2	5	2	1	

TABLE B-2. The degrees of characters in $\text{Irr}(3.2^3.2^8.3.(S_3 \times S_3 \times S_3):S_3 \mid \zeta)$

This appendix is published online as an add-on to *LMS J. Comput. Math.* 11 (2008) 100–145.

Degree	3	6	84	105	150	168	210	252	300	315
Number	4	1	4	2	4	1	1	2	1	2
Degree	504	525	630	756	900	1050	1260	1575	1701	1800
Number	1	4	2	2	4	5	1	4	2	1
Degree	1890	2100	2520	2916	3150	3402	4032	4200	4725	5040
Number	2	7	2	4	3	1	2	6	2	1
Degree	5103	5832	6300	6720	6804	7560	8064	8400	8505	9450
Number	2	1	5	2	2	2	1	1	2	3
Degree	9600	12096	12288	12600	13440	13608	14175	17010	18225	18900
Number	4	2	4	5	1	1	2	1	4	2
Degree	19200	20160	20412	24576	25200	25515	36450			
Number	1	2	2	1	1	2	1			

TABLE B-3. The degrees of characters in $\text{Irr}(3.(A_4 \times O_8^+(2):3):2)$

Degree	3	6	9	12	18	24	27	36	48	54	72	108	144	216
Number	8	6	16	17	14	14	8	27	3	4	34	8	8	8

TABLE B-4. The degrees of characters in $\text{Irr}(3.2^4.2^3.2^6.(S_3 \times S_3).3)$

Degree	3	6	9	12	18	24	36	48	72	96	144
Number	24	26	8	43	14	44	17	23	12	4	4

TABLE B-5. The degrees of characters in $\text{Irr}(3.2^3.2^8.2^3.3^2.2)$

Degree	3	6	9	15	18	21	24	27	36	42	45
Number	2	1	5	1	4	2	2	2	1	1	2
Degree	48	54	63	72	90	105	120	210	315	420	630
Number	1	1	1	1	1	5	1	4	8	1	6
Degree	672	945	1260	1344	1890	2016	2520	2688	3360	3780	4032
Number	2	4	1	1	6	6	6	2	2	1	2
Degree	5040	5376	6048	6720	7560	8064	10080	20160			
Number	2	1	4	7	2	2	2	2			

TABLE B-6. The degrees of characters in $\text{Irr}(3.2^{3+12}.(A_6 \times L_3(2)) | \zeta)$

Degree	3	6	9	12	15	18	27	30	45	60	90
Number	4	4	6	1	6	3	2	5	10	1	12
Degree	135	180	270	288	360	384	540	576	720	768	864
Number	4	4	16	4	6	2	6	1	6	1	8
Degree	1080	1152	1440	1536	1728	1920	2160	2304	2880	3072	3840
Number	6	8	8	2	2	2	1	3	10	1	7

TABLE B-7. The degrees of characters in $\text{Irr}(3.2^{1+12}.2^4.(S_3 \times A_6) | \zeta)$

Degree	3	780	900	1560	2457	4914	6825	8424
Number	2	4	2	2	2	1	4	2
Degree	13650	16380	28350	32760	52650	54600	70200	72576
Number	1	4	2	2	2	6	4	2
Degree	81900	87360	109200	122850	139776	140400	147420	163800
Number	2	4	1	4	2	2	4	6
Degree	174720	199017	245700	279552	294840	327600	398034	491400
Number	2	2	2	1	2	2	1	2
Degree	552825	568620	656100	698880	737100	786240	873600	982800
Number	4	4	2	4	2	4	6	1
Degree	998400	1105650	1137240	1257984	1397760	1474200	1572480	1594323
Number	4	1	2	2	4	2	2	2
Degree	1747200	1996800	2150400	2515968	2795520			
Number	1	1	4	1	1			

TABLE B-8. The degrees of characters in $\text{Irr}(3.(3 \times O_8^+(3):3):2 \mid \zeta)$

Degree	3	42	192	234	273	312	504	546
Number	2	2	4	2	6	2	2	4
Degree	819	1344	1638	2184	2187	2457	2496	
Number	4	4	4	4	2	2	2	

TABLE B-9. The degrees of characters in $\text{Irr}(3.(3^2:2 \times G_2(3)) \mid \zeta)$

Degree	3	6	9	12	18	24	36	48	54	72	144
Number	4	6	4	2	6	8	6	10	2	8	6

TABLE B-10. The degrees of characters in $\text{Irr}(3.(3^2:2 \times (3^2 \times 3^{1+2}):2S_4) \mid \zeta)$

Degree	3	6	9	12	18	24	27	36	48	54
Number	4	10	4	4	6	4	4	5	10	4
Degree	81	96	108	288	384	486	576	768	864	972
Number	4	4	1	8	4	18	16	10	8	9
Degree	1152	1458	1536	1728	2304	2592	2916	3456	3888	4374
Number	17	6	4	8	10	8	12	2	18	6
Degree	4608	5184	7776	8748	10368	13122				
Number	6	2	9	3	4	2				

TABLE B-11. The degrees of characters in $\text{Irr}(3.(3 \times 3^{1+8}).2(A_4 \times A_4 \times A_4).2:3:2 \mid \zeta)$

Degree	3	6	12	18	24	36	54	72	108	162	216	324	486	648	972
Number	8	24	18	20	4	32	28	20	34	60	10	48	36	12	18

TABLE B-12. The degrees of characters in $\text{Irr}(3.(3 \times 3^{1+8}).3^3.3.2^3 \mid \zeta)$

Degree	3	6	9	12	24	48	54	96	108	144	162
Number	4	6	4	2	20	18	14	4	24	10	18
Degree	216	288	324	432	486	648	864	972	1296	2592	
Number	10	4	18	14	4	39	10	1	36	9	

TABLE B-13. The degrees of characters in $\text{Irr}(3.3^5.3^4.3^4.2S_4.2 \mid \zeta)$

Degree	27	351	378	756	1728	2457	2808	4914
Number	1	2	1	1	2	3	2	2
Degree	5265	7020	7371	7560	9477	9828	12285	14742
Number	1	1	2	2	1	1	5	1
Degree	16848	19656	19683	22464	24192	25272	29484	31590
Number	1	5	1	2	1	1	1	2
Degree	39312	49140	58968	63180	68040	78624	88452	98280
Number	1	2	2	1	1	1	5	4
Degree	132678	157248	176904	189540	196560	221130	265356	294840
Number	1	2	2	2	5	3	1	2
Degree	314496	353808	379080	393120	442260	483840	505440	511758
Number	4	2	1	1	4	4	2	1
Degree	530712	551124	589680	628992	663390	786240	884520	943488
Number	1	1	1	2	1	3	1	1

TABLE B-14. The degrees of characters in $\text{Irr}(3.3^7.O_7(3) \mid \zeta)$

Degree	27	54	108	135	162	216	270	405	432	486
Number	2	1	2	4	3	5	11	2	2	3
Degree	540	810	864	1080	1296	1620	1728	2160	2430	2592
Number	8	5	4	7	2	2	2	6	8	1
Degree	2916	4860	6480	7290	7776	8640	9720	11664	12960	14580
Number	3	6	8	6	4	2	11	3	11	9
Degree	15552	17280	19440	21870	25920	29160	31104	39366		
Number	4	4	12	6	2	3	3	3		

TABLE B-15. The degrees of characters in $\text{Irr}(3.3^{1+10}.3:U_4(2):2 \mid \zeta)$

Degree	27	54	81	108	162	216	432	486
Number	6	18	6	12	3	12	24	9
Degree	648	972	1296	1458	1944	2916	3888	4374
Number	18	9	21	21	48	18	9	6

TABLE B-16. The degrees of characters in $\text{Irr}(3.3^7.3^5.3^{1+2}.2S_4 \mid \zeta)$

Degree	27	324	351	432	702	729	1053
Number	3	3	3	12	21	3	3
Degree	1404	2106	5616	6318	12636	16848	18954
Number	12	4	21	24	24	2	8

TABLE B-17. The degrees of characters in $\text{Irr}(3.3^7.3^{3+3}.L_3(3) \mid \zeta)$

Degree	27	54	81	108	162	216	432	486	648	972	1296	1458	1944	3888
Number	6	12	6	6	1	24	24	33	6	33	17	11	48	27

TABLE B-18. The degrees of characters in $\text{Irr}(3.3^7.3^{1+6}.3.2S_4 \mid \zeta)$

Degree	27	54	81	108	162	216	324	432	486	648	864	972
Number	4	9	4	7	1	11	2	15	4	2	10	12
Degree	1296	1458	1728	1944	2592	2916	3888	5184	5832	7776	15552	
Number	8	4	4	24	4	2	21	9	2	20	3	

TABLE B-19. The degrees of characters in $\text{Irr}(3.3^7.3^{1+6}.(2A_4 \times A_4).2 \mid \zeta)$

Degree	27	54	108	162	216	324	432	486
Number	8	18	18	10	12	18	4	24
Degree	648	972	1296	1458	1944	2916	3888	5832
Number	14	34	12	6	44	12	9	6

TABLE B-20. The degrees of characters in $\text{Irr}(3.3^7.3^5.3^3.(S_4 \times 2) \mid \zeta)$

Degree	27	54	108	162	324	486	972	1458	2916
Number	12	36	24	30	36	132	36	36	3

TABLE B-21. The degrees of characters in $\text{Irr}(3.3^2.3^4.3^8.3^2.2 \mid \zeta)$

Degree	297	486	594	1188	1485	1782	2376	2970	4455
Number	2	1	1	2	2	3	1	3	2
Degree	4860	5346	5940	8910	11880	13365	17820	20736	21384
Number	1	2	4	3	4	2	3	2	1
Degree	23760	26730	32076	35640	38016	47520	53460	58320	76032
Number	4	3	2	1	4	1	5	1	2
Degree	80190	85536	106920	128304	152064	155520	160380	190080	207360
Number	1	1	4	2	4	1	2	4	5
Degree	213840	228096	240570	248832	304128	320760	342144	380160	427680
Number	3	4	4	4	2	1	2	2	2
Degree	433026	456192	481140	497664	590490				
Number	3	1	2	1	2				

TABLE B-22. The degrees of characters in $\text{Irr}(3.3^{1+10}.U_5(2):2 \mid \zeta)$

Degree	135	162	270	324	486	540	648	810	972	1080	1296
Number	8	2	18	1	10	14	2	10	7	2	1
Degree	1620	1944	2430	2916	3240	4860	6480	7290	9720	14580	19440
Number	23	8	14	1	10	28	2	4	10	4	1

TABLE B-23. The degrees of characters in $\text{Irr}(3.3^{1+10}.3^4.(2 \times A_5):2 \mid \zeta)$

Degree	135	270	405	486	540	648	810	972	1080	1296
Number	4	9	4	4	5	2	1	6	8	1
Degree	1458	1944	2160	2592	3240	3888	4320	4860	5184	5832
Number	4	5	14	2	2	6	2	6	1	1
Degree	6480	9720	12960	14580	19440	25920	38880			
Number	13	14	8	2	6	2	4			

TABLE B–24. The degrees of characters in $\text{Irr}(3.3^2.3^4.3^8.(2A_4 \times A_5):2 \mid \zeta)$

Degree	3	6	12	18	24	36	72	108
Number	8	10	16	8	2	16	6	4

TABLE B–25. The degrees of characters in $\text{Irr}(3.(3^2:2 \times (3^2 \times 3^{1+2}).3.2^2).2 \mid \zeta)$

Degree	3	42	192	234	273	312	504	546
Number	4	4	8	4	4	4	4	2
Degree	1092	1638	2187	2457	2496	2688	3276	4368
Number	2	2	4	4	4	2	2	2

TABLE B–26. The degrees of characters in $\text{Irr}(3.(3^2:2 \times G_2(3)).2 \mid \zeta)$

Degree	27	54	81	108	162	216	432	486	648
Number	4	14	4	11	4	17	28	18	12
Degree	864	972	1296	1458	1944	2592	2916	3888	7776
Number	7	30	14	6	36	5	4	32	10

TABLE B–27. The degrees of characters in $\text{Irr}(3.3^2.3^4.3^8.3.(2 \times 2S_4) \mid \zeta)$

Degree	27	54	324	351	432	648	702	729
Number	2	1	2	2	8	1	19	2
Degree	864	1053	1404	1458	2106	2808	4212	5616
Number	4	2	18	1	5	3	1	18
Degree	6318	11232	12636	18954	25272	33696	37908	
Number	12	6	21	4	9	1	3	

TABLE B–28. The degrees of characters in $\text{Irr}(3.3^3.3^4.3^3.3^3.(L_3(3) \times 2) \mid \zeta)$

Degree	27	54	81	108	162	216	324	432	486	648	864
Number	4	14	4	14	4	20	1	22	6	12	7
Degree	972	1296	1458	1944	2592	2916	3888	4374	5832	7776	8748
Number	9	16	14	27	8	19	24	4	6	3	2

TABLE B-29. The degrees of characters in $\text{Irr}(3.3^{1+10}.(3 \times 3^{1+2}).(2 \times 2S_4) | \zeta)$

Degree	27	54	108	162	216	324	486	648	972	1458	1944	2916	5832
Number	8	36	30	28	7	32	72	12	68	24	13	14	1

TABLE B-30. The degrees of characters in $\text{Irr}(3.3^2.3^4.3^8.3^2.2^3 | \zeta)$

Degree	3	6	9	12	24	48	72
Number	12	12	4	2	4	2	4

TABLE B-31. The degrees of characters in $\text{Irr}(3.(5^2:4A_4 \times A_4).2 | \zeta)$

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