**Calculation of the secondary fluorescence for a layered specimen**

Let consider a characteristic X-ray of energy E2 of element A located in the layer a between mass depth and , being fluoresced by the characteristic X-rays of energy E1 emitted by element B located in layer b between mass depth and (see Figure 1).



Figure 1: Characteristic secondary fluorescence in a multilayer sample.

The secondary fluorescence of characteristic X-rays from element A produced by characteristic X-rays coming from element B is given by:

where is the ionization depth distribution of electron shell m in element B present in the material at mass depth . The probability of producing an electron vacancy in the shell m by ionization is given by where is the ionization cross section of shell m of element B by electron impact of energy E0 and represents the enhancement factor which takes into account the fact that vacancies in the considered m shell of element B can be created not only by direct electron impact but also by migration of vacancies between subshells of the same shell through non-radiative transitions (Coster-Kronig and super-Coster-Kronig transitions) as well as by radiative and non-radiative transitions to most inner shells The product of the ionization cross sections, relaxation parameters, and enhancement factor is known as the X-ray production cross section. A given characteristic X-ray of energy is emitted during the relaxation of the ionized atom B by transition of an electron from the shell n to the shell m. These are represented by the fluorescence yield the radiative transition probability for an electron to transition from shell n to shell m, , and the total radiative width for all possible transitions to the shell, . The characteristic X-rays of energy travel from layer b to layer a and undergo absorption along their path. The absorption of the photons, emitted with a direction , along their path in material is represented by the MAC and taken into account by the exponential term . The product of the ionization depth distribution by the absorption exponential is integrated over the mass thickness of the layer b, from to . The photons exiting the layer b pass through the k layers between the layers a and b, of thicknesses and material , and undergo absorption taken into account by the MAC and the thicknesses of each of the k layers tk. When the photons reach the layer a, the X-rays are then absorbed by the material , taken into account through the MAC . At mass depth , in the infinitely small distance , the probability for the X-rays to interact with the atoms of element A through photoelectric interaction and to ionize the electron shell i of element A, is given by , where is the photoelectric cross section and represent the radiative, non-radiative, Coster-Kronig and super-Coster-Kronig contributions to the production of an ionization in shell i. The production of the studied characteristic X-ray of energy produced by element A during the relaxation process from which an electron from the shell j falls into the shell i is taken into account by the fluorescence yield and radiative transition probabilities and . The attenuation of these X-rays, emitted towards the detector with an angle , in the layer a itself and in the layers q (of thickness ) above the layer a, is taken into account by the exponentials with MACs and , respectively. ε and Ω are the intrinsic detection efficiency and the solid angle of collection of the detector, respectively.

Starting from the previous equation

Constant quantities can be regrouped:

with

Terms depending on z can be grouped together:

The first integration is performed on :

This leads to the following equation:

Then, by separating the two terms of the subtraction:

Then, by regrouping the exponential arguments depending of 1/cosθ:

The equation can be written

where:

Equations of the form can be solved using the following method:

By using the following change of variable:

Then, for the first integral, by doing the change of variable , we obtain:

The with the following change of variable :

And finally, with the simple change of variable the equation takes the form:

We can recognize in the integrals the exponential form defining the exponential integral:

Hence, the equation becomes:

By replacing it in , we obtain:

By noting that :

and to exhibit the z0 dependency by imposing we have:

By repeating the same procedure for , we have:

Also:

and by imposing , we have:

Finally, gives:

The secondary fluorescence equation can then be written:

Using the PAP model, can be written into a polynomial form .

We can show, by integrating by parts, that

These 6 integral forms can be used to entirely solve . Similar solutions can be found in the case were element B is in a layer above the layer a containing the fluoresced element A and in the case where elements A and B are in the same layer.