Appendix to 'Asset Pricing with Costly and Delayed Firm Entry'

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1 Appendix A—Data for the empirical exercise

To motivate our model we run a recursively identified VAR(4) with the following ordering of the variables: TFP, output per capita, a measure of the number of active firms, and the excess return on stocks. We use the following

set of variables from St. Louis FRED database:

- HOANBS Nonfarm Business Sector: Hours of All Persons (index 2012)

- GDPC1 Real Gross Domestic Product. Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate.

- GDPDEF GDP deflator base = 2012

- PCECC96 Real Personal Consumption Expenditures Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate

Detrending variables. All data is in real terms and divided by US population. Per capita real variables are logged. In line with the cited papers we use a second-order poliynomial to detrend each variables. Stock returns deliver the same results even without detrending. We check that detrended data do not feature a unit root with a ADF test.

To construct the TFP residual from the production function we need quarterly data on capital and hours. Hours worked is non-farm business hours and is available in quarterly terms. We also need quarterly time series for capital. Capital is based on the Private Non-Farm capital services index 2012 downloaded from historical BLS accounts https://www.bls.gov/mfp/mprdload.htm#Capital%20Tables. We obtain quarterly capital from annual data by the Denton-Cholette method (linear extrapolation). After removing the variety effect the TFP is constructed in the usual way i.e. the residual from the linear production function: $a_t = y_t - \alpha k_t - (1 - \alpha)l_t$ where we assume that the labour share is

$1 - \alpha = 0.64$.

To have quarterly stock returns we take a geometric average of monthly S&P 500 yields from Shiller's database. Excess return is the quarterly stock return minus the 3-month T-bill rate.

To calculate the active number of firms we follow the strategy of Savagar (2021). We take the number of establishments at 01-01-1993 using the census' business dynamics statistics which is available at annual frequency. Then we take BLS' Business Employment Dynamics data available at quarterly frequency and adding up the number of active firms (firm birth minus firm death) cumulatively. Hence, our time window is restricted by the availability of net firm entry data: 1992Q3-2018Q4.

Here we provide robustness checks of the empirical exercise such as different orderings of the variables and including consumption instead of output in the VAR, see Figures (1)-(3).

Figure 1: Different ordering of the variables 1.

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Response to Cholesky One S.D. Innovations 2 S.E.

Response of AHAT to AHAT

Response of YHAT to AHAT

Response of RETHAT to AHAT

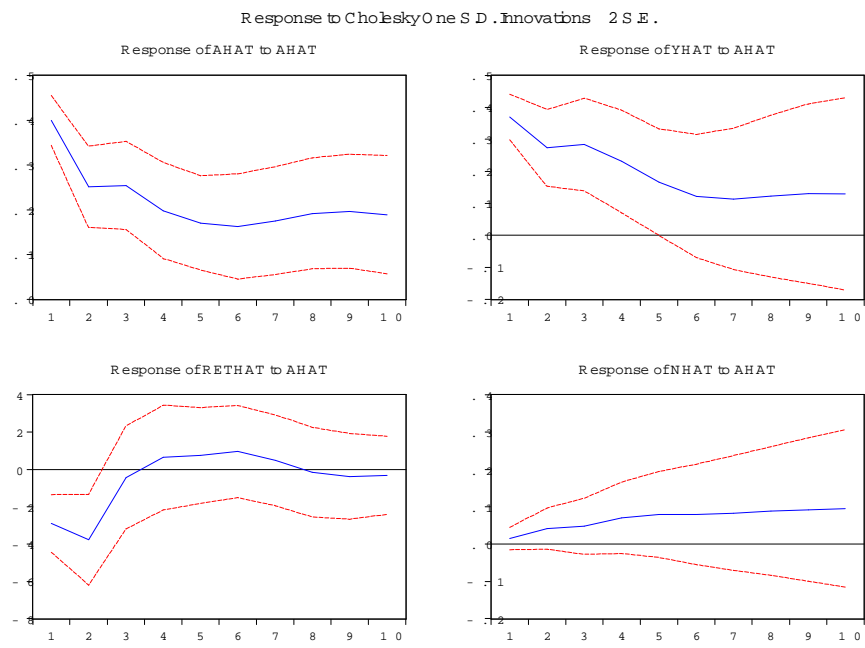Response of NHAT to AHAT

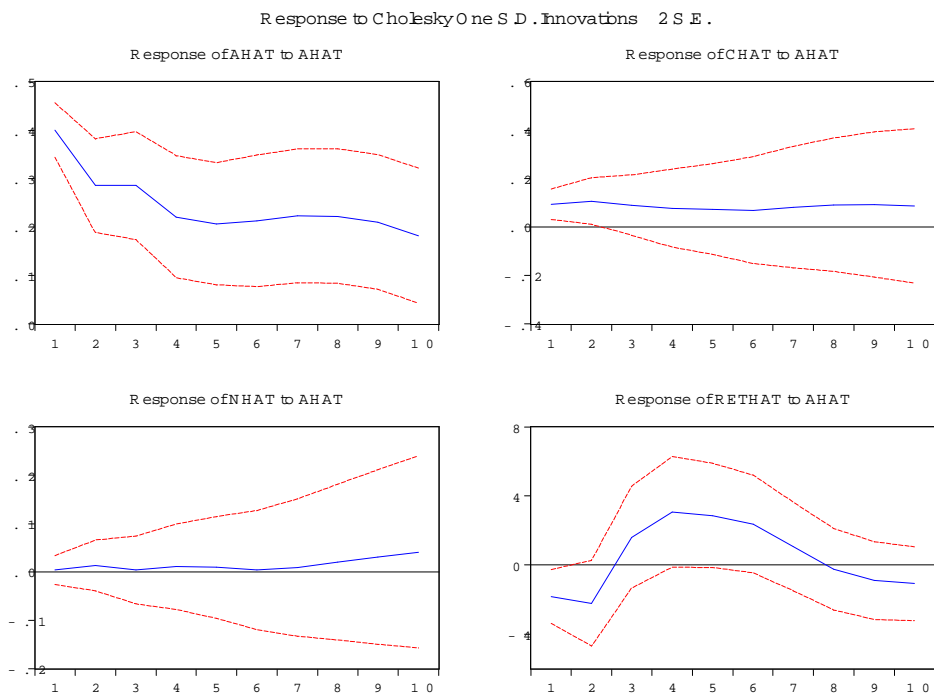Figure 2: Different ordering of the variables 2.

Figure 3: Including consumption in the VAR instead of output.

2 Appendix B—List of equilibrium conditions, steady-state and further moments

2.1 The costly firm entry model

The entry model contains 24 variables:

$$U, V_t, \tilde{V}_t, MUC_t, M_{t,t+1}, Z_t, Y_t, C_t, L_t, N_t, N_{E,t}, d_t, AD_t, \rho_t, \mu_t,$$

$$MC_t, \Pi_t, \Pi_t^C, W_t/P_t, S_t^{AD}, R_t, R_{AD,t}, EQPR_t, R_{f,t}$$

and 24 equations listed below.

Recursive formulation for utility:

$$V_t = U_t^{1-\sigma} + \beta \left[\tilde{V}_t \right]^{\frac{1-\sigma}{1-\gamma}} . \tag{1}$$

The utility function:

$$U(C_t, L_t) = \left[\frac{\left(C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{1-\sigma}}{1-\sigma} \right]^{\frac{1}{1-\sigma}} . \tag{2}$$

Continuation value:

$$\tilde{V}_t \equiv E_t V_{t+1}^{\frac{1-\gamma}{1-\sigma}} . \tag{3}$$

The marginal utility of consumption:

$$MUC_t \equiv \left(C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)^{-\sigma}. \tag{4}$$

The stochastic discount factor:

$$M_{t,t+1} = \beta \frac{MUC_{t+1}}{MUC_t} \left(\frac{V_{t+1}^{1/(1-\sigma)}}{\tilde{V}_t^{1/(1-\gamma)}} \right)^{\sigma-\gamma}. \tag{5}$$

Bond Euler equation:

$$1 = E_t \left[M_{t,t+1} \frac{R_t}{\Pi_{t+1}^C} \right]. \tag{6}$$

Intratemporal condition:

$$\frac{W_t}{P_t} = \chi L_t^{1/\varphi}, \tag{7}$$

Firm level dividends:

$$d_t = \left(1 - \frac{1}{\mu_t} - \frac{\phi}{2}\pi_t^2 Y_t \right) \frac{Y_t}{N_t}. \tag{8}$$

Aggregate dividends:

$$AD_t = N_t d_t. \tag{9}$$

Share Euler (including free entry condition $S_t^{ex} = f_E$):

$$1 = (1-\delta)E_t \left(M_{t,t+1} \frac{f_E + d_{t+1}}{f_E} \right). \tag{10}$$

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Price of the aggregate real dividend claim:

$$S_t^{AD} = AD_t + E_t \left[M_{t,t+1} S_{t+1}^{AD} \right] \tag{11}$$

Return on the aggregate real dividend claim:

$$R_{AD,t} = \frac{S_t^{AD}}{S_{t-1}^{AD} - AD_{t-1}} \tag{12}$$

The risk free rate:

$$R_{f,t} = \frac{1}{E_t M_{t,t+1}} \tag{13}$$

Levered excess return as in Croce (2014):

$$EQPR_t = (1 + \phi_{lev})(\ln(R_{AD,t}) - \ln(R_{f,t-1})) \tag{14}$$

Pricing condition:

$$\rho_t = \mu_t MC_t, \tag{15}$$

where

$$\rho_t \equiv N_t^{\frac{1}{\theta - 1}} \tag{16}$$

and

$$MC_t = \frac{W_t/P_t}{Z_t} \tag{17}$$

is the real marginal cost. μ_t denotes the gross markup *function*:

$$\mu_t \equiv \frac{\theta}{(\theta - 1)\left(1 - \frac{\phi}{2}\pi_t^2\right) + \phi\pi_t(1 + \pi_t) - \beta E_t\{\Upsilon_{t+1}\}} \quad (18)$$

$$\text{with } \Upsilon_{t+1} \equiv (1 - \delta)M_{t,t+1}\phi\pi_{t+1}\frac{Y_{t+1}}{Y_t}\frac{N_t}{N_{t+1}}(1 + \pi_{t+1})$$

The connection between producer price (Π_t) and consumer price (Π_t^C) inflation comes from the variety effect:

$$\frac{\Pi_t}{\Pi_t^C} = \left(\frac{N_t}{N_{t-1}}\right)^{\frac{1}{\theta - 1}}. \quad (19)$$

Temporary technology shock:

$$\log Z_t = \rho_z \log Z_{t-1} + \sigma_Z \varepsilon_t. \quad (20)$$

The aggregate market clearing reads as:

$$Y_t = \frac{W_t}{P_t}L_t + N_t d_t \quad (21)$$

Goods market clearing:

$$Y_t = C_t + \frac{\phi}{2}\pi_t^2 Y_t. \quad (22)$$

Monetary policy is described by an interest rate rule:

$$R_t = \frac{1}{\beta}R_{t-1}^\rho\left[(\Pi_t)^{\phi_\pi}\left(Y_t^R/Y_{t-1}^R\right)^{\phi_Y}\right]^{1-\rho}. \quad (23)$$

The number of firms producing at period t, N_t, is described by:

$$N_t = (1 - \delta)(N_{t-1} + N_{E,t-1}). \tag{24}$$

The aggregate production function is already used to calculate firm level dividends but is only mentioned here for completeness:

$$Y_t = \rho_t Z_t L_t.$$

2.2   Steady-state of the entry model

The combination of firm level dividends and the Euler equation (including the free entry condition) yields:

$$f_E = \beta(1 - \delta) \left[f_E + \left(1 - \frac{1}{\mu} \right) \frac{Y}{N} \right].$$

The combination of the variety effects and the pricing condition yields:

$$N^{\frac{1}{\theta-1}} = \mu \frac{w}{Z}.$$

The production function is given by:

$$Y = Z \rho L.$$

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We choose χ such that it pins downs steady-state hours worked through the intratemporal condition

$$w = \chi L.$$

For given μ, f_E, L and Z we are using the non-linear solver of matlab to solve for: N, χ, Y and w (4 unknows) using the 4 equations listed above.

2.3 Model without entry

The noentry model contains 19 variables (ρ_t, Π_t^C, $N_{E,t}$, AD_t, and N_t drop):

$$U, V_t, \tilde{V}_t, MUC_t, M_{t,t+1}, Z_t, Y_t, C_t, d_t, L_t, \mu_t, MC_t, \Pi_t, W_t/P_t, S_t^d, R_t, R_{d,t}, EQPR_t, R_{f,t}$$

The aggregate production function is given by:

$$Y_t = Z_t L_t$$

The pricing condition is given by:

$$1 = \mu_t MC_t$$

where definition of the markup is given by:

$$\mu_t \equiv \frac{\theta}{(\theta - 1)\left(1 - \frac{\phi}{2}\pi_t^2\right) + \phi\pi_t(1 + \pi_t) - \beta E_t\left\{\Upsilon_{t+1}\right\}}$$
$$\text{with } \Upsilon_{t+1} \equiv M_{t,t+1}\phi\pi_{t+1}\frac{Y_{t+1}}{Y_t}(1 + \pi_{t+1})$$

There is no distinction between CPI and PPI inflation and the variety effect equation also drops.

The market clearing in equation (21) changes to:

$$Y_t = \left(1 - \frac{\phi}{2}\pi_t^2\right)^{-1} C_t$$

There is no distinction between individual firm and aggregate dividends as the number of firms are fixed:

$$d_t = AD_t = Y_t - \frac{W_t}{P_t}L_t$$

The share Euler equation does not contain $1 - \delta$ in the no entry model.

The rest of the equations are identical to those in the entry-exit model.

3 Further moments

We present moments related to the standard deviation of output growth as well as relative standard deviations and correlations with output growth. We make it subject to future research of how the standard deviations of output and labour growth can be raised without creating excess variation in consumption growth. The firm entry model generates higher second moments in most cases relative to the noentry model.

| | Data | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | | Entry | No entry | Entry | Entry | Entry | Entry | No entry |
		benchm.	benchm.	IES<1	Flex. pr.	Taylor	Sep.	Sep.
$\sigma(\Delta Y)$	2.71	2.87	2.94	4.02	3.87	5.48	1.58	2.56
$\sigma(\Delta C)$	5.12	2.88	2.94	4.01	3.87	5.40	1.58	2.57
$\sigma(\Delta L)$	3.73	2.01	1.75	2.55	2.57	6.67	1.77	1.38
$\sigma(\Delta C)/\sigma(\Delta Y)$	0.60	1.00	1.00	1.00	1.00	0.99	1.00	1.00
$\sigma(\Delta L)/\sigma(\Delta Y)$	0.78	0.70	0.59	0.64	0.66	1.22	1.12	0.54
$corr(\Delta Y, \Delta C)$	0.84	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$corr(\Delta C, \Delta L)$	0.41	0.97	1.00	1.00	1.00	0.96	0.72	1.00
$corr(\Delta Y, \Delta L)$	0.64	0.97	1.00	1.00	1.00	0.96	0.72	1.00
$\sigma(\Pi^C)$	2.52	2.34	0.12	0.50	2.00	2.53	1.42	0.10
$\sigma(\Delta R)$	1.04	1.75	0.19	0.99	1.54	1.77	1.01	0.14
$\sigma(R)$	2.71	0.52	0.04	0.15	0.43	0.44	0.48	0.03

Table 1: Further Macroeconomic Moments. Notes: data moments are based on Favilukis and Lin (2013), US data 1929-2013. The second moments of inflation and the nominal interest rate are taken from Rudebusch and Swanson (2012).