# Online Appendix

## A Details of the Baseline Model in Section 2

In this section, I add the government spending ingredient to Beaudry et al. (2022)'s aggregate supply-side economics with the real cost channel. To be more specific, each monopolist will use only the basic input $Y_t^B$ for production and follow the one-to-one technology. Therefore, the price of this basic input is the marginal cost. The basic input is produced by representative firms with the following Leontief production function:

$$Y_t^B = \min(aN_t, bM_t),$$

where $M_t$ is the final goods, and $N_t$ is the labor.
The unit price of the final goods attached to the production is $P_t$. As in Beaudry et al. (2022), we assume that the basic input representative should borrow $D_{t+1}$ to pay for the input $M_t$ at the risk-free nominal rate $i_t$ for the production, *i.e.* borrowing costs.[39] In this case, firms should produce, sell the product, pay wages $W_tP_t$, pay back the debt in the previous period, and distribute the dividends $\Pi_t$. One can show the budget constraint of firms at time $t$ by simply assuming zero profits in equilibrium below:

$$D_{t+1} + P_t^B Y_t^B = W_t P_t N_t + (1 + i_{t-1})D_t + P_t M_t,$$

where $P_t^B$ is the basic input price, and $D_{t+1} = P_t M_t$.
In that way, the profit $\Pi_t$ can be shown as:

$$\Pi_t = P_t^B Y_t^B - W_t P_t N_t - (1 + i_{t-1})P_{t-1}M_{t-1}.$$

We further assume that firms maximize the expected discounted sum of real profit $\frac{\Pi_t}{P_t}$ with a discount parameter $\beta$. In this case, the first-order condition can be shown:

$$P_t^B = \left(\frac{1}{a}W_t + \frac{\beta}{b}\mathbb{E}_t\frac{1+i_t}{1+\pi_{t+1}}\right)P_t,$$

Where $\pi_{t+1}$ is the next period's inflation rate. Thus, one can obtain the (real) marginal cost of the basic input:

$$MC_t = \frac{W_t}{a} + \frac{\beta}{b}\mathbb{E}\left[\frac{1+i_t}{1+\pi_{t+1}}\right].$$

[39] The borrowing cost is crucial in modeling since it introduces the real cost channel in the Phillips Curve. The advantage of this introduced real cost channel method as in Beaudry et al. (2022) is that it allows setting arbitrarily the elasticity of marginal cost rate with regard to wage and interest rate. Please see Beaudry et al. (2022) for a comprehensive comparison between the model with the nominal and the real cost channel.

In logs, one can show the linearized equilibrium

$$mc_t = \hat{\gamma}_y(w_t) + \gamma_r(R_t + \log(\beta) - \mathbb{E}\pi_{t+1}),$$

where $\hat{\gamma}_y = \frac{\frac{1}{a}W}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$, $\gamma_r = \frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}W + \frac{\beta}{b}\frac{1+i}{1+\pi}}$, and $R_t$ is the nominal interest rate in level.
On the other hand, the optimal labor supply reads:

$$\frac{v'(N_t)}{u'(C_t)} = W_t.$$

The linearized resource constraint in this economy is:

$$y_t = (1 - s_g)c_t + g_t,$$

where $y_t$ is the output gap, $s_g$ is the fraction of government spending in total production, and $g_t$ is government spending.[40]
By using the linearized production function $y_t = n_t$, the marginal cost can be rewritten as

$$\begin{aligned} mc_t &= \hat{\gamma}_y \frac{Nv''(N)}{v'(N)} y_t - \hat{\gamma}_y \frac{Cu''(C)}{u'(C)} \left(\frac{y_t - g_t}{1 - s_g}\right) + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}) \\ &= \gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1}), \end{aligned}$$

where $\gamma_y = \hat{\gamma}_y \left(\frac{Nv''(N)}{v'(N)} - \frac{Cu''(C)}{u'(C)(1-s_g)}\right)$, and $\gamma_g = \hat{\gamma}_y \frac{Cu''(C)}{u'(C)(1-s_g)}$.
The rest is standard and we have the Phillips Curve:

$$\pi_t = \kappa mc_t + \beta\mathbb{E}_t\pi_{t+1}.$$

Therefore, the Phillips Curve with the real cost channel and government spending is

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left[\gamma_y y_t + \gamma_g g_t + \gamma_r(R_t + \log(\beta) - \mathbb{E}_t\pi_{t+1})\right].$$

# B Proof for Proposition 1

If I consider the demand shock and it is assumed that the demand shock $r^n_S$ can put the economy into liquidity traps with one enough (negatively) big shock ($r^n_S < \underline{r^n_S}$), one

[40] Following Christiano et al. (2011), I define $g_t = (G_t - G)/Y$.

can rewrite the Phillips Curve as

$$y_S = \begin{cases} \frac{1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi}{\kappa\gamma_y}\pi_S & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{1-\beta p+\kappa\gamma_r p}{\kappa\gamma_y}\pi_S - \frac{\gamma_r}{\gamma_y}\log(\beta) & \text{if } r_S^n < \underline{r_S^n}. \end{cases}$$

Similarly, one can rewrite the Euler equations as follows:

$$y_S = \begin{cases} -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n}{1-p} & \text{if } r_S^n \geq \underline{r_S^n} \\ \frac{\frac{1}{\sigma}p}{1-p}\pi_S + \frac{1}{\sigma}\frac{r_S^n - \log(\beta)}{1-p} & \text{if } r_S^n < \underline{r_S^n}. \end{cases}$$

I combine the first questions of Euler equation and Phillips Curve to obtain the exact expression for $\underline{r_S^n}$ which can be written as:

$$\underline{r_S^n} = \left[\frac{(1-\beta p+\kappa\gamma_r p-\kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

One can show the exact boundary condition for $\underline{r_S^n}$ in the standard NK model without the real cost channel:

$$\underline{r_S^{n,B}} = \left[\frac{(1-\beta p)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

Likewise, the exact boundary condition for $\underline{r_S^n}$ in the model with the nominal cost channel:

$$\underline{r_S^{n,N}} = \left[\frac{(1-\beta p-\kappa\gamma_r\phi_\pi)(1-p)}{\kappa\gamma_y\frac{1}{\sigma}} + (\phi_\pi - p)\right]\frac{\log(\beta)}{\phi_\pi} < 0.$$

In this case, I have

$$\underline{r_S^n} - \underline{r_S^{n,B}} = \frac{\kappa\gamma_r(p-\phi_\pi)}{\kappa\gamma_y\frac{1}{\sigma}}\frac{\log(\beta)}{\phi_\pi} > 0.$$

And further one can show

$$\underline{r_S^n} - \underline{r_S^{n,N}} < 0.$$

One can use this to obtain the result in the main text.

# C Proof for Proposition 2

I show the solutions for the output gap and inflation multipliers below:

$$\mathcal{M}^{O}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(\phi_\pi-p)} + \frac{\sigma(1-p)}{\phi_\pi-p}}$$
$$= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)] - \kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}$$

$$\mathcal{M}^{I}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(\phi_\pi-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(\phi_\pi-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

For the normal cost channel case, the item $[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]$ will switch to $[1-\beta p-\kappa\gamma_r\phi_\pi]$. In this case, one can easily prove that the inflation multiplier $\mathcal{M}^{I,N}_{S,N}$ can be larger with the nominal cost channel due to a smaller denominator. However, the output gap multiplier with the nominal cost channel $\mathcal{M}^{O,N}_{S,N}$ can be less. For the output gap multiplier, the numerator is less than the denominator. That is,

$$\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)] - \kappa\gamma_g(\phi_\pi-p) < \kappa\gamma_y(\phi_\pi-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)].$$

Thus, the output gap multiplier is less than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}^{O,B}_{S,N} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p) + \sigma(1-p)[1-\beta p]}.$$

One can compare this expression to the previous one with the real cost channel and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.

For the inflation multiplier without the real cost channel:

$$\mathcal{M}^{I,B}_{S,N} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p) + \sigma(1-p)[1-\beta p]}.$$

In this case, the denominator of the inflation multiplier with the real cost channel is lower due to a negative item and thus the inflation multiplier is higher. For the output gap multiplier:

$$\frac{\partial \mathcal{M}^O_{S,N}}{\partial \gamma_r} = -\sigma(1-p)(\phi_\pi - p)\kappa \frac{\mathcal{D}-\mathcal{N}}{\mathcal{D}^2} < 0,$$

where $\mathcal{D}_N = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)]$ and $\mathcal{N}_N = \sigma(1-p)[1-\beta p - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)$. Thus, the output gap multiplier in normal times decreases in the increased strength of the real cost channel.

For the inflation multiplier, it is easily observed that the higher the strength of the real cost channel $\gamma_r$, the lower the denominator of this multiplier. In other words, the inflation multiplier increases with the increased strength of the real cost channel.

# D Proof for Proposition 3

The output gap and inflation multipliers at the ZLB are reproduced here:

$$\mathcal{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\frac{\sigma(1-p)}{-p} - \frac{\kappa\gamma_g}{1-\beta p-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p-\kappa\gamma_r(-p)} + \frac{\sigma(1-p)}{-p}}$$
$$= \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}$$

$$\mathcal{M}^I_{S,Z} = \frac{\partial \pi_S}{\partial g_S} = \frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p-\kappa\gamma_r(-p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p)}(-p)}$$
$$= \frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

The numerator and denominator of the output and inflation multipliers (we assume $\gamma_r$ is far greater than $\gamma_y$ and the denominator is positive) are both positive here and thus we have positive spending multipliers. The output gap multiplier can be rewritten as:

$$\mathcal{M}^O_{S,Z} = \frac{\partial y_S}{\partial g_S} = 1 + \frac{\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)] + \kappa\gamma_g p + \kappa\gamma_y p}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

This output gap multiplier is larger than one. For the output gap multiplier without the real cost channel:

$$\mathcal{M}^{O,B}_{S,Z} = \frac{\partial y_S}{\partial g_S} = \frac{\sigma(1-p)[1-\beta p] - \kappa\gamma_g(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

Similar with the case in normal times, one can compare this expression to the previous one with the real cost channel, and it is easy, after some arrangements, to obtain the output gap multiplier with the real cost channel that is lower.
For the inflation multiplier without the real cost channel:

$$\mathcal{M}_{S,Z}^{I,B} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right] \kappa\gamma_y\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p]}.$$

One can observe that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In this case, it can be lower with the real cost channel.
For the output gap multiplier:

$$\frac{\partial \mathcal{M}_{S,Z}^{O}}{\partial \gamma_r} = -\sigma(1-p)(-p)\kappa\frac{\mathcal{D}-\mathcal{N}}{\mathcal{D}^2} < 0,$$

where $\mathcal{D}_Z = \kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]$ and $\mathcal{N}_Z = \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \kappa\gamma_g(-p)$. Thus, the output gap multiplier at the ZLB reduces with the increased strength of the real cost channel.
For the inflation multiplier, it is easily observed that the higher the strength of the real cost channel $\gamma_r$, the higher the denominator of this multiplier. In other words, the inflation multiplier decreases with the increased strength of the real cost channel.
The nominal cost channel multipliers $\mathcal{M}_{S,Z}^{O,N}$ and $\mathcal{M}_{S,Z}^{I,N}$ can be invariant with the standard NK model since the nominal channel in liquidity traps cannot be included in the partial derivative of government spending to the output gap/inflation in the calculation of fiscal multipliers.

# E The Derivation of the Condition for $\gamma_y$

As in Nie (2021), for brevity, one can yield a condition for $\gamma_y$ to ensure $\mathcal{D}_Z > 0$:

$$\begin{aligned}
\mathcal{D}_Z &= (1-p)(1-\beta p + \kappa\gamma_r p) - \sigma_r p\kappa\gamma_y \\
&> \left(1 - \frac{1-\kappa\gamma_r\phi_\pi}{\beta - \kappa\gamma_r}\right)\left(1 - \beta\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r} + \kappa\gamma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\right) - \sigma_r\frac{1-\kappa\gamma_r\phi_\pi}{\beta-\kappa\gamma_r}\kappa\gamma_y \\
&= (\beta - \kappa\gamma_r - 1 + \kappa\gamma_r\phi_\pi)[\beta\kappa\gamma_r\phi_\pi - \kappa\gamma_r + \kappa\gamma_r(1-\kappa\gamma_r\phi_\pi)] - \sigma_r(1-\kappa\gamma_r\phi_\pi)\kappa\gamma_y > 0 \\
\gamma_y &< \frac{(\beta - \kappa\gamma_r - 1 + \kappa\gamma_r\phi_\pi)(\beta\gamma_r\phi_\pi - \kappa\gamma_r^2\phi_\pi)}{\sigma_r(1-\kappa\gamma_r\phi_\pi)} = \Gamma(\gamma_r),
\end{aligned}$$

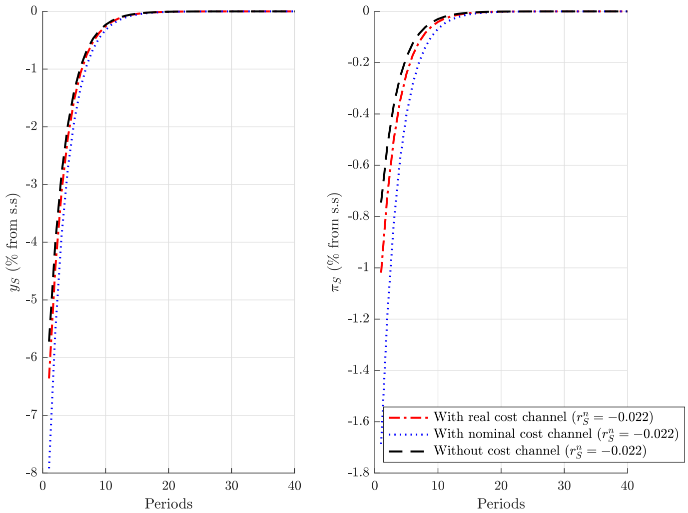
where the second line we assume $p = \bar{p}^c$ due to monotonicity.
In addition, one can check the monotonicity of $\Gamma(\gamma_r)$ w.r.t. $\gamma_r$:

$$\frac{\partial \Gamma(\gamma_r)}{\partial \gamma_r} \propto \frac{\partial \frac{\beta - \kappa\gamma_r}{1 - \kappa\gamma_r\phi_\pi}}{\partial \gamma_r} > 0.$$

Therefore $\Gamma(\gamma_r)$ increases in $\gamma_r$.

# F Impulse Response Function

Figure 9: Impulse response to a contractionary natural rate shock



# G Baseline Multiplier Figures w.r.t. $p$

Figure 10: Spending multipliers in normal times

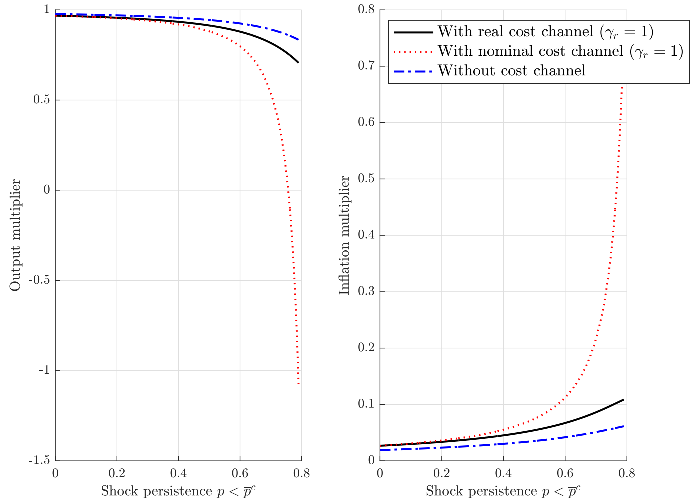
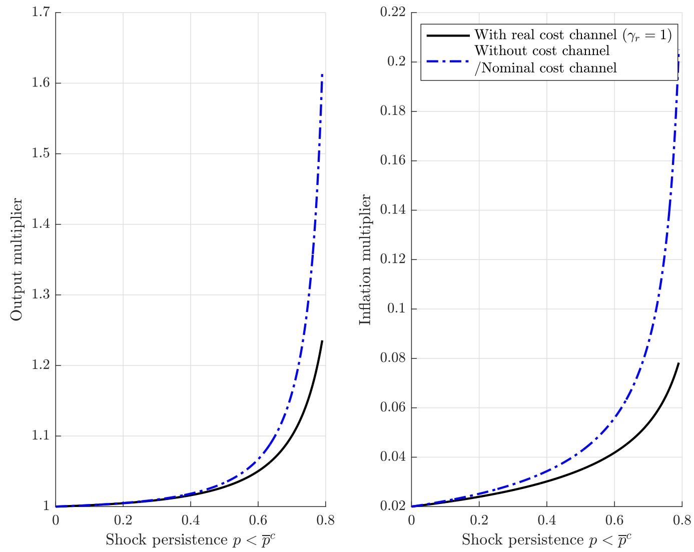
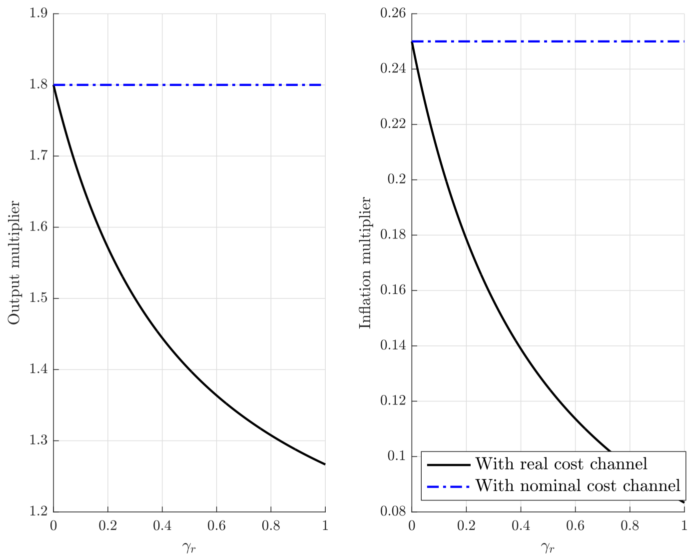



Figure 11: Spending multipliers at ZLB



# H Baseline Spending Multiplier Figure at ZLB ($p = 0.8$)

Figure 12: Spending multipliers at ZLB ($p = 0.8$)




# I Multipliers in the Medium Run

In the medium run, the economy experiences no shocks to the natural rate, and one can show medium-run government spending with the persistence $q$ as follows

$$\begin{aligned} y_M &= \mathcal{M}_M^O \times g_M \\ &= \frac{\sigma(1-q)[1-\beta p - \kappa\gamma_r(\phi_\pi^q - q)] - \kappa\gamma_g(\phi_\pi^q - q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} g_M. \end{aligned}$$

$$\begin{aligned} \pi_M &= \mathcal{M}_M^I \times g_M \\ &= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]} g_M. \end{aligned}$$

# J Euler Equation and Phillips Curve with Long-run Government Spending Policy

If the short-run economy is in normal times, the Euler equation can be re-derived with consideration of long-run government spending using a three-state Markov chain:

$$y_S = -\frac{1}{\sigma}\frac{\phi_\pi - p}{1-p}\pi_S + \Theta_{AD} g_s$$
$$\Theta_{AD} = q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1) + 1,$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation, $\zeta$ is the policy discount parameter, $\mathcal{M}_M^O$ and $\mathcal{M}_M^I$ are the medium-run policy multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to 1, which can nest the case in our baseline model in Section 2. The new items in this shift are from rational expectations of the output gap, inflation, and medium-run spending shock. Note that the first new term is from future wealth effects (higher expected output gap in the future) as in Bouakez et al. (2017) and households have consumption incentives due to consumption smoothing. The second term comes from the fact that government spending, in the long run, can increase firms' marginal costs and inflation. The third term is due to the direct demand effect from future government spending. See appendix J.1; it turns out that $q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1)$ is negative which means the overall expected effects from longer spending can crowd out the present output. In addition, the effects of long-run government spending can be controlled by the product of policy parameters $q\zeta$.

On the other hand, the Phillips Curve with long-run government spending can be shown below:

$$\pi_S = \kappa\frac{\gamma_y}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}y_S + \frac{1}{1-\beta p - \kappa\gamma_r(\phi_\pi - p)}\Theta_{AS} g_s$$
$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I + \kappa\gamma_g,$$

where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve, $\zeta$ is the policy discount parameter, $\mathcal{M}_M^I$ is the medium run inflation multiplier as in equation (24). For reference, this shift without long-run government spending will collapse to $\kappa\gamma_g$, which is the same as in our baseline model in Section 2. The new items in this shift are from rational expectations of inflation and long-run government spending. As seen in appendix J.1, it turns out that long-run spending can increase firms' marginal costs and inflation. Similar to the case in the Euler equation, the effects of long-run policy can be controlled by the product of policy parameters $q\zeta$.[41]

[41] In this paper, I assume that $\kappa$ is very small in our theoretical analysis which is in line with Eggertsson (2011), Gabaix (2020) and Budianto et al. (2020). In this sense, one can assume $\beta - \kappa\gamma_r > 0$ such that

If the short-run economy is at the ZLB, the Euler equation with long-run spending is given by:

$$y_S = -\frac{1}{\sigma(1-p)}[\log(\beta) - p\pi_S] + \Theta_{AD} g_s$$
$$\Theta_{AD} = q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1) + 1.$$

where $\Theta_{AD}$ is the government spending shock shift in the Euler equation. The long-run government spending terms $q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1)$ are negative and can be controlled by the product of policy parameters $q\zeta$.
In addition, I now move to describe the Phillips Curve with long-run government spending as:

$$\pi_S = \kappa \frac{\gamma_y + \gamma_r \log(\beta)}{1 - \beta p + \kappa\gamma_r p} y_S + \frac{1}{1 - \beta p + \kappa\gamma_r p}\Theta_{AS} g_s$$
$$\Theta_{AS} = (\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I + \kappa\gamma_g.$$

where $\Theta_{AS}$ is the government spending shock shift in the Phillips Curve and can be controlled by the product of policy parameters $q\zeta$.

### J.1 Euler and Phillips Shift

The long-run government spending in the Euler equation shift:

$$q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1) = q\zeta \frac{\kappa(\gamma_y + \gamma_g)(1 - \phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1 - \beta q - \kappa\gamma_r(\phi_\pi^q - q)]} < 0.$$

The long run government spending in the Phillips Curve shift

$$(\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I > 0,$$

where we assume that $\beta - \kappa\gamma_r > 0$ since $\kappa$ is very small in our theoretical analysis.

long-run government spending can increase inflation here. However, if $\beta - \kappa\gamma_r < 0$, this might resolve the fiscal price puzzle (FPP) as in Han et al. (2020) that a long-run fiscal stimulus can lower inflation .

# K Long-run Government Spending Effects in Normal Times

I can use the new Euler equation and the Phillips Curve to reproduce the spending multipliers:

$$\mathcal{M}_{S,N}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]-\Theta_{AS}(\phi_\pi-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}$$

$$\mathcal{M}_{S,N}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD}+\Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

For the output gap multipliers, as in appendix J.1, one can see that the long-run government spending shock can lead to a lower $\Theta_{AD}$ but a higher $\Theta_{AS}$. In this case, the multiplier should be lower.
For inflation multiplier,

$$\kappa\gamma_y q\zeta(\mathcal{M}_M^O+\frac{1}{\sigma}\mathcal{M}_M^I-1)+(\beta-\kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I$$

$$=q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y+\gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}+q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}>0,$$

where we assume $\kappa$ is minor in our theoretical analysis as in theoretical analysis and the first item has an addition multiplier $\kappa$. One can use this to prove the result in the main text. Since there is no $\gamma_r$ in $\Theta_{AD}$, we only focus on the $\Theta_{AS}$'s effects. Since the term with $\Theta_{AD}$ reduces in $\gamma_r$, we only need to show the other terms with $\Theta_{AS}$ also decrease in $\gamma_r$. Thus one can prove the output gap multiplier decreases in $\gamma_r$. The output gap multiplier can be reduced below:

$$-\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

Since the term with $\Theta_{AD}$ increases in $\gamma_r$, we only need to show the other terms with $\Theta_{AS}$ also increase in $\gamma_r$. Thus one can prove the inflation multiplier increases in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(\phi_\pi-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{\sigma(1-p)\kappa(\beta\phi_\pi - 1)\mathcal{D}_{1,N} + \sigma(1-q)\kappa(\beta\phi_\pi^q - 1)\mathcal{D}_{2,N} - \mathcal{O}(\kappa^2)}{\mathcal{D}_{3,N}^2}.$$

where $\mathcal{D}_{1,N} = \kappa\gamma_y(\phi_\pi - q) + \sigma(1-q)[1 - \beta q - \kappa\gamma_r(\phi_\pi^q - q)]$, $\mathcal{D}_{2,N} = \kappa\gamma_y(\phi_\pi - p) + \sigma(1-p)[1 - \beta p - \kappa\gamma_r(\phi_\pi - p)]$, $\mathcal{D}_{3,N} = \mathcal{D}_{1,N} \cdot \mathcal{D}_{2,N}$ and $\mathcal{O}(\kappa^2)$ is the residual of order two since we assume that $\kappa$ is trivial in our theoretical analysis, it is easy to check that the derivative with regard to $\gamma_r$ is positive. In this case, one can use this to prove the result in the main text.

# L Long-run Government Spending Effects at ZLB

One can produce the output gap and inflation multipliers below

$$\mathcal{M}_{S,Z}^{O,long} = \frac{\partial y_S}{\partial g_S} = \frac{\Theta_{AD}\sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)] - \Theta_{AS}(-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}$$

$$\mathcal{M}_{S,Z}^{I,long} = \frac{\partial \pi_S}{\partial g_S} = \frac{\left[\kappa\gamma_y\Theta_{AD} + \Theta_{AS}\right]\sigma(1-p)}{\kappa\gamma_y(-p) + \sigma(1-p)[1-\beta p - \kappa\gamma_r(-p)]}.$$

For the output gap multipliers, the numerator with medium run spending policy can be decomposed into the following two parts. The first part:

$$\begin{aligned}
&\kappa\gamma_y q\zeta(\mathcal{M}_M^O + \frac{1}{\sigma}\mathcal{M}_M^I - 1)\sigma(1-p)[1-\beta p + \kappa\gamma_r p] \\
&= q\zeta\frac{\kappa\gamma_y\kappa(\gamma_y + \gamma_g)(1-\phi_\pi^q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}\sigma(1-p)[1-\beta p + \kappa\gamma_r p].
\end{aligned}$$

The second part:

$$\begin{aligned}
&(\beta - \kappa\gamma_r)(1-p)q\zeta\mathcal{M}_M^I p \\
&= q\zeta\frac{(\beta-\kappa\gamma_r)(1-p)q\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-q)}{\kappa\gamma_y(\phi_\pi^q - q) + \sigma(1-q)[1-\beta q - \kappa\gamma_r(\phi_\pi^q - q)]}p.
\end{aligned}$$

To simplify the proof, one can add the two items and show the sum is positive if we assume that $\kappa$ is very small in our theoretical analysis. Similar to the inflation multiplier in normal times, we can have a higher long-run inflation multiplier at the ZLB. Since there is no $\gamma_r$ in $\Theta_{AD}$, we focus only on the $\Theta_{AS}$'s effects. For the main result, since the term with $\Theta_{AD}$ reduces in $\gamma_r$, we only need to show the other terms with $\Theta_{AS}$ also decrease in $\gamma_r$. Thus one can see the output gap multiplier decreases in

$\gamma_r$. In this case, the output gap multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

For the main result, since the term with $\Theta_{AD}$ decreases in $\gamma_r$, we only need to show the other terms with $\Theta_{AS}$ also decrease in $\gamma_r$. Thus the inflation multiplier will decrease in $\gamma_r$. The inflation multiplier can be reduced below:

$$\frac{\Theta_{AS}}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}.$$

One can differentiate this common term with regard to $\gamma_r$ and this common term can be reduced further as:

$$\frac{\beta-\kappa\gamma_r}{\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]}\frac{1}{\kappa\gamma_y(\phi_\pi^q-q)+\sigma(1-q)[1-\beta q-\kappa\gamma_r(\phi_\pi^q-q)]}.$$

I differentiate the above term with regard to $\gamma_r$ to obtain:

$$\frac{-\sigma(1-p)\kappa\mathcal{D}_{1,N}+\sigma(1-q)\kappa(\beta\phi_\pi^q-1)\mathcal{D}_{2,Z}-\mathcal{O}(\kappa^2)}{\mathcal{D}_{3,Z}^2}.$$

where $\mathcal{D}_{2,Z}=\kappa\gamma_y(-p)+\sigma(1-p)[1-\beta p-\kappa\gamma_r(-p)]$, $\mathcal{D}_{3,Z}=\mathcal{D}_{1,N}\cdot\mathcal{D}_{2,Z}$ and $\mathcal{O}(\kappa^2)$ is the residual of order two. One can reduce this expression as:

$$-(1-p)\sigma(1-q)(1-\beta q)+(1-q)(\beta\phi_\pi^q-1)\sigma(1-p)(1-\beta p)-\mathcal{O}(\kappa^2)<0,$$

where I use the general condition $\phi_\pi^q\beta-1<1$ and the short run period should be longer or almost equal to the long run period, in reality, such that $p\geq q$. In this case, one can use this to prove the result in the main text.

# M Multiplier Figures with Long-run Spending Policy w.r.t. $p$

Figure 13: Spending multipliers with long-run spending policy in normal times

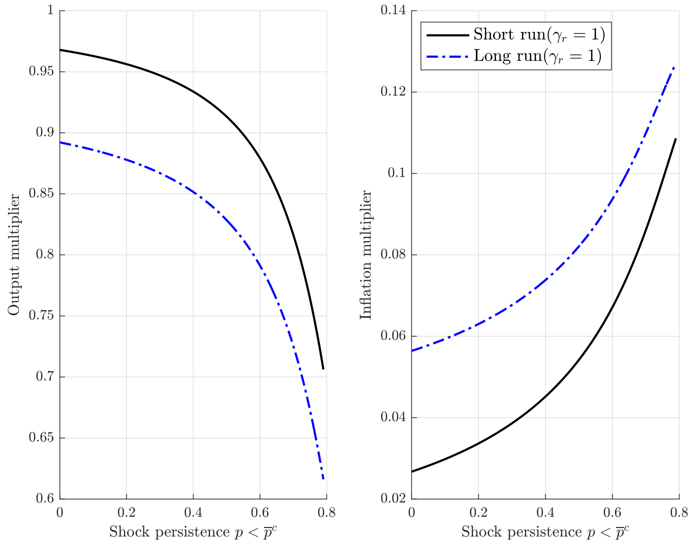
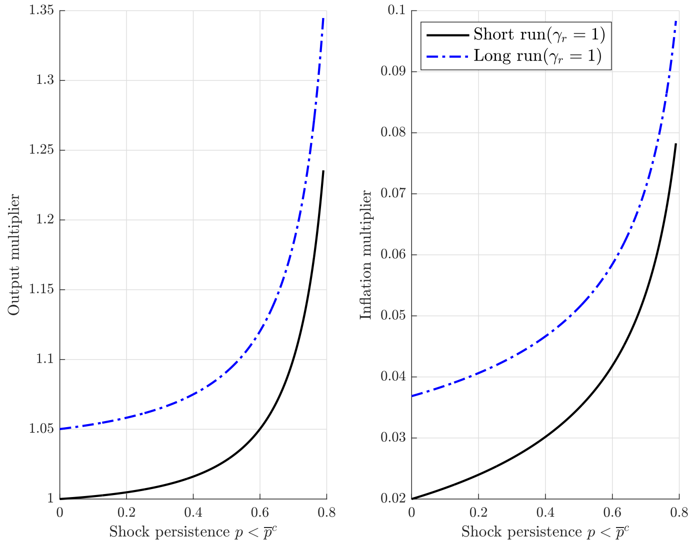



Figure 14: Spending multipliers with long-run spending policy at ZLB



# N Euler Equation and Phillips Curve with Bounded Rationality

The Euler equation and Phillips Curve with bounded rationality in normal times:

$$
\begin{aligned}
y_S &= -\frac{1}{\sigma(1-\alpha_{EE}p)}(\phi_\pi - p)\pi_S + g_S \\
\pi_S &= \kappa \frac{\gamma_y}{1-\beta p \alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} y_S + \kappa \frac{\gamma_g}{1-\beta p \alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} g_S.
\end{aligned}
$$

The Euler equation and Phillips Curve with bounded rationality can be elaborated at the ZLB:

$$
\begin{aligned}
y_S &= -\frac{1}{\sigma(1-p\alpha_{EE})}[\log(\beta) - p\pi_S] + g_S \\
\pi_S &= \frac{\kappa\gamma_y y_S + \kappa\gamma_r \log(\beta)}{1-\beta p\alpha_{PC} + \kappa\gamma_r p} + \kappa\frac{\gamma_g}{1-\beta p\alpha_{PC} + \kappa\gamma_r p} g_S.
\end{aligned}
$$

# O Government Spending Effects with Bounded Rationality in Normal Times

I reproduce the multipliers here. For simplicity, we define $\alpha_{EE} = \bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m}) = \bar{m}[\varphi + \frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$
\begin{aligned}
\mathcal{M}_{S,N}^{O,BR} = \frac{\partial y_S}{\partial g_S} &= \frac{\frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p} - \frac{\kappa\gamma_g}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)} + \frac{\sigma(1-p\alpha_{EE})}{\phi_\pi - p}} \\
&= \frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)] - \kappa\gamma_g(\phi_\pi - p)}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{S,N}^{I,BR} = \frac{\partial \pi_S}{\partial g_S} &= \frac{1 + \frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)}{\kappa\gamma_y} + \frac{1}{\sigma(1-p\alpha_{EE})}(\phi_\pi - p)} \\
&= \frac{\left[1 + \frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(\phi_\pi - p) + \sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC} - \kappa\gamma_r(\phi_\pi - p)]}.
\end{aligned}
$$

where $\alpha_{EE}$ and $\alpha_{PC}$ increase in the cognitive discounting parameter $\bar{m}$. One can differentiate the output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y+\gamma_g)(\phi_\pi-p)f_N'(\bar{m})}{\mathcal{D}_{BN}^2}>0,$$

where $\mathcal{D}_{BN}=\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ and $f_N'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ with regard to $\bar{m}$ which is positive.

One can differentiate inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha_{EE}'\mathcal{D}_{BN}-f_N'(\bar{m})(1-p\alpha_{EE})}{\mathcal{D}_{BN}^2}<0,$$

where $\mathcal{D}_{BN}=\kappa\gamma_y(\phi_\pi-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ and $f_N'(\bar{m})$ is the derivative of $\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(\phi_\pi-p)]$ with regard to $\bar{m}$ which is positive. The strength of the real cost channel $\gamma_r$ is independent of the new ingredient (bounded rationality). See appendix C: The output gap multiplier decreases in the increased strength of the real cost channel $\gamma_r$, and the inflation multiplier increases in the increased strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# P Government Spending Effects with Bounded Rationality at ZLB

The spending multipliers at the ZLB are shown below. For simplicity, we define $\alpha_{EE}=\bar{m}(1-s_g)$ and $\alpha_{PC}(\bar{m})=\bar{m}[\varphi+\frac{1-\beta\varphi}{1-\beta\varphi\bar{m}}(1-\varphi)]$.

$$\begin{aligned}\mathcal{M}_{S,Z}^{O,BR}=\frac{\partial y_S}{\partial g_S}&=\frac{\frac{\sigma(1-p\alpha_{EE})}{-p}-\frac{\kappa\gamma_g}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}}{\frac{\kappa\gamma_y}{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}+\frac{\sigma(1-p\alpha_{EE})}{-p}}\\&=\frac{\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]-\kappa\gamma_g(-p)}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{S,Z}^{I,BR}=\frac{\partial\pi_S}{\partial g_S}&=\frac{1+\frac{\gamma_g}{\gamma_y}}{\frac{1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)}{\kappa\gamma_y}+\frac{1}{\sigma(1-p\alpha_{EE})}(-p)}\\&=\frac{\left[1+\frac{\gamma_g}{\gamma_y}\right]\kappa\gamma_y\sigma(1-p\alpha_{EE})}{\kappa\gamma_y(-p)+\sigma(1-p\alpha_{EE})[1-\beta p\alpha_{PC}-\kappa\gamma_r(-p)]}.\end{aligned}$$

One can differentiate the output gap multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{\kappa(\gamma_y+\gamma_g)(-p)f_Z'(\bar{m})}{\mathcal{D}_{BZ}^2}<0,$$

where $\mathcal{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.
One can differentiate the inflation multiplier with regard to $\bar{m}$ and after some arrangements we have:

$$\frac{-p\alpha'_{EE}\mathcal{D}_{BZ} - f'_Z(\bar{m})(1 - p\alpha_{EE})}{\mathcal{D}^2_{BN}} < 0,$$

where $\mathcal{D}_{BZ} = \kappa\gamma_y(-p) + \sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ and $f'_Z(\bar{m})$ is the derivative of $\sigma(1 - p\alpha_{EE})[1 - \beta p\alpha_{PC} - \kappa\gamma_r(-p)]$ with regard to $\bar{m}$ which is positive.
The strength of the real cost channel $\gamma_r$ is independent of the new ingredient that is bounded rationality. See appendix D: the output gap and inflation multipliers decrease in the increased strength of the real cost channel $\gamma_r$. One can use this to prove the main text.

# Q Multiplier Figures with Bounded Rationality w.r.t. $p$

Figure 15: Spending multipliers with bounded rationality in normal times

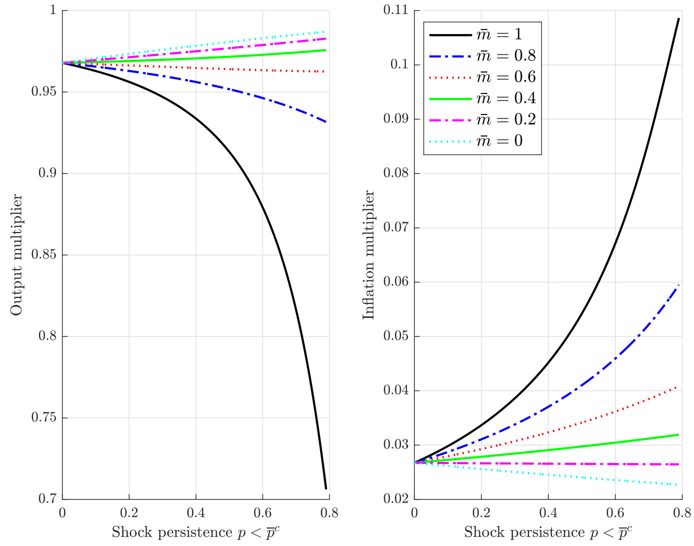



Figure 16: Spending multipliers with bounded rationality at ZLB